**Chapter 4 – Bond Price Volatility**

1. **The calculated “Price Value of a Basis Point” (PVBP) will be the same regardless if the yield is increased or decreased by 1 basis point. However, the price value of 100 basis points (i.e., the change in price for a 100-basis-point change in interest rates) will not be the same if the yield is increased or decreased by 100 basis points. Why?**

Due convexity or curvature in the price/yield relationship.

Note that the PVBP is *NOT* the exactly the same for 1 bp increase or decrease. It is close – and is only *the same* if we round.

1. **State why you would agree or disagree with the following statement: When interest rates are low, there will be little difference between the Macaulay Duration (Mac D) and Modified Duration (Mod D) measures.**

Mod D = Mac D/(1 + y) 🡺 Mac D = Mod D x (1 + y)

The difference is small for small values of y.

1. **The November 26, 1990, issue of *BondWeek* includes an article, “Van Kampen Merritt Shortens.” The article begins as follows:**

**“Peter Hegel, first v.p. at Van Kampen Merritt Investment Advisory, is shortening his $3 billion portfolio from 110% of his normal duration of 6½ years to 103–105% because he thinks that in the short run the bond rally is near an end.”**

**Explain Hegel’s strategy and the use of the duration measure in this context.**

A “bond rally” is an increase in bond prices (due in this case to a decrease in rates). If Hegal thinks the rally is over, then he thinks rates will not continue to fall. Hegal wants to reduce his portfolio’s interest rate risk (measured by duration) from 110% of 6.5 to 103-105% of 6.5 in case rates rise.

1. **Consider the following two Treasury securities. Which bond will have the greater dollar price volatility for a 25-basis-point change in interest rates?**

|  |  |  |
| --- | --- | --- |
| **Bond** | **Price** | **Mod D** |
| **A** | **$100** | **6** |
| **B** | **$ 80** | **7** |

Mod D = -∂P/∂y (1/P) ≈ ΔP/Δy (1/P)

ΔP ≈ -Mod D x Δy x P

ΔPA ≈ -Mod D x Δy x P = 6 x 0.0025 x $100 = $1.50

ΔPB ≈ -Mod D x Δy x P = 7 x 0.0025 x $80 = $1.40

Note: Mod D ≈ ΔP/P (1/Δy) ≈ %ΔP = percent change in price (for given change in yield)

Bond B has the greater %ΔP but a lower dollar price change than bond A.

1. **What are the limitations of using duration as a measure of a bond’s price sensitivity to interest-rate changes?**
2. Duration is appropriate for small changes in yield since the slope estimate does not capture convexity (change in slope).
3. The Duration formulas assume all cash flows for the bond are discounted at the same discount rate. In essence we are calculating the price change by assuming the yield curve (the term-structures of interest rates or all the interest rates for different maturities) is flat for the bond and shifts to the curve are parallel (all rates change by the same amount). This problem is exacerbated when a bond portfolio is made up of bonds with different maturities and credits since changes in rates may not be identical for all maturities and credits.
4. The Duration equations may not work for bonds that are not option-free. The duration equations assume the cash flows are constant and this may not be the case for bonds with embedded options. For bonds with embedded options, the cash flows may be a function of the interest rates.

Consider a 20 year 10% $1,000 bond callable at par in ten years. If rates decrease the bond might be called in ten years in which case the cash flow at in ten years (at time 20) increases from $50 to $1050 and all subsequent cash flows go to zero. Since the cash flows are not constant, we *cannot* apply the derivative formulas used to calculate duration. Therefore we need to estimate duration using

When change in yields result in changes in the expected cash flow for a bond (which is *not* the case for fixed coupon, non-callable and non-putable bonds) the duration and convexity measures may not be appropriate. (See Question 13.) To account for this, use the approximate duration equation (4.23 on page 84). See Extra Question 3 for an example.

* Price the bond at the current yield
* Price the bond at a higher yield making sure to account for any changes in cash flows associated with embedded options.
* Compute the approximate duration using equation 4.23.
1. **Consider the excerpt from an article titled “Denver Investment to Make $800 Million Treasury Move,” that appeared in the December 9, 1991, issue of *BondWeek*, p. 1:**

**“Denver Investment Advisors will swap $800 million of long zero-coupon Treasuries for intermediate Treasuries. . . . The move would shorten the duration of its $2.5 billion fixed-income portfolio. . . .”**

**Why would the trade described here shorten the duration of the portfolio? Does the manager expect rates to increase or decrease?**

Mod D decreases as the coupon increases and maturity decreases. The manager expects rates to rise.

**Bonus Question: If the manager expects rates to rise, why not just sell all the bonds and “go flat” bonds (or go flat rates)?**

The manager can’t do it. The manager is in the bond portfolio business. Clients expect the manager to hold bonds.

1. **Re-written: Can the duration of a bond exceed its maturity?**

Yes and no. For a *straight bond* (the cash flows are known), Macaulay Duration (D) must be less than or equal to the term to maturity. Consider a zero coupon bond:

Mac D = 

For a zero-coupon bond, only the last term in the numerator is not zero:

Mac D = [nM/(1 + y)n]/P

But for a zero coupon bond we can substitute: M/(1 + y)n = P

Mac D = [nP]/P = n

We also know ∂2P/∂y∂C < 0

So increasing C decreases D.

Since Mod D = Mac D/(1 + *y*), Mod D < Mac D, so Mod D must be less than the term to maturity.

Modified Duration (Mod D) ***is defined as*** the negative of the first derivative of price w.r.t yield, divided by the initial price:

Mod D = -∂P/∂y (1/P)

The above formulations work only for *straight* bonds. For a bond whose cash flows *might change* as yield changes (*levered in yield)*, the price change of that bond for a given yield change *might exceed* the maturity number. Note that for such a bond *we could not use* the duration formulas shown above.

Consider a bond backed by mortgages (a collateralized mortgage obligation or CMO). You might expect that defaults on the underlying mortgages will increase as interest rates increase. Therefore for a given change in rates (∂y or Δy), the expected cash flows from holding the bond decrease *and* the discount rate of the now-lower cash flows increase. The combined effect might cause a change in price (∂P or ΔP) for a given change in yield such that Mod D *exceeds* that maturity of the underlying mortgages and therefore the maturity of the CMO bond.

This also holds for bonds with embedded options – which induce uncertainty into the bond price formula,

**Bonus Question: How can you calculate the duration of a *non-straight* bond?**

Use the Equation 4.23 on page 84. See the discussion starting on page 83, the “Approx duration for Callable Bond” spreadsheet and Question 11 answer (3) above and Extra Question #3 below.

1. **Answer the below questions.**
2. **Suppose that the spread duration for a fixed-rate bond is 2.5. What is the approximate change in the bond’s price if the spread changes by 50 basis points?**

A measure of how a non-Treasury bond’s price will change if the spread sought by the market changes is referred to as spread duration. A spread duration for a fixed-rate security is interpreted as the approximate change in the price of a fixed-rate bond for a 100-basis-point change in the spread. If the change is 2.5% (as given by a duration of 2.5) for 100 basis points then it would be about 1.25% for 50 basis points as shown below in more detail.

Let us begin by noting that

 *=* −Mod D(*dy*).

Substituting spread duration for modified duration to approximate the percentage price change for a given change in the yield we get:

 *=* −**(**Spread Mod D)(*dy*).

Putting in 2.5 for the spread duration and 0.005 for *dy* (since the spread changes by 50 basis points), we get:

 *=* −2.5(0.005)= −0.0125.

Thus, the change in the bond’s price if the spread changes by 50 basis points in percentage terms is −1.25%, which can be interpreted as the approximate percentage change in price for a 50-basis-point change in yield.

1. **What is the spread duration of a Treasury security?**

The spread represents compensation for credit risk. The price of a non-Treasury bond is exposed to a change in the spread that is called credit spread risk. For a Treasury security, there is no credit risk and thus the spread duration for a Treasury security is zero.

1. **Explain why the duration of an inverse floater is a multiple of the duration of the collateral from which the inverse floater is created.**

The formula on page 85 is:

Duration of an inverse floater = (1 + *L*)(duration of collateral) x collateral price/Inverse price

With slightly different notation: Mod DI = (1 + *L*)(Mod DC) × PC/PI

Where *L* = ValueFloater/ValueInverse Floater = VF/VI

In general, a floater and inverse floater are created from a single fixed-rate collateral bond. The floater and inverse floater are structured so that the cash flow from the collateral will be sufficient to satisfy the obligation of the two bonds.

Two assumptions:

First Assumption: Consider a floater and inverse floater as two components of a portfolio that sum to equal the collateral bond. Therefore the duration of the collateral bond (C) equals:

Mod DC = WF × Mod DF + WI × Mod DI

The weights for the floater and inverse floater are the value of the floater divided by the value of the collateral bond and the value of the inverse floater divided by the value of the collateral bond.

Second Assumption: The Duration of a floater is close to zero. In general, if rates change, both the coupon payment and the discount rate change by the same amount.

Therefore ∂P/∂y ≈ 0.

Given these two assumptions, the portfolio duration relationship (Mod DC = WF x Mod DF + WI x Mod DI) yields the Mod DI equation above.

Example: Suppose that the par value of the collateral of $50 million is split as follows: $40 million for the floater and $10 million for the inverse floater. Suppose also that the collateral and inverse are trading at par so that the ratio of the prices is 1 and that the duration for the collateral is 8. For a 100-basis-point change in interest rates, the collateral’s price will decline by 8% or 0.08 ($500 million) = $4 million.

Assuming that the floater’s price does not change when interest rates increase, the $4 million decline must come from the inverse floater. For the inverse floater to realize a decline in value of $4 million when its value is $10 million, the duration must be 40. That is, a duration of 40 will produce a 40% change in value or 0.04($10 million) = $4 million. Thus, the duration is five times the collateral’s duration of 8. Or equivalently, because *L* is 4, it is (1 + 4) times the collateral’s duration.

1. **Consider the following portfolio:**

|  |  |  |
| --- | --- | --- |
| **Bond** | **Market Value** | **Mod D** |
| **W** | **$13 million** | **2** |
| **X** | **$27 million** | **7** |
| **Y** | **$60 million** | **8** |
| **Z** | **$40 million** | **14** |

1. **What is the portfolio’s duration?**

Portfolio Duration is the average weighted duration of the bonds in the portfolio:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Bond | Market Value | Mod D | Weight | Weight x Mod D |
| W | $13 | 2 | 0.09 | 0.19 |
| X | $27 | 7 | 0.19 | 1.35 |
| Y | $60 | 8 | 0.43 | 3.43 |
| Z | $40 | 14 | 0.29 | 4.00 |
| Sum | $140 |  | 1.00 | 8.96 |

The Portfolio’s Mod D = 8.96

1. **If interest rates for all maturities change by 50 basis points, what is the approximate percentage change in the value of the portfolio?**

The same relationship holds for a portfolio: Mod D = -∂P/∂y (1/P)

Percent change in value = ∂P/P = -Mod D x ∂y = 8.96 \* 0.0050 = -4.48%

1. **What is the contribution to portfolio duration for each bond?**

Contribution = Weight x Mod D

See the fifth column of the table in part (a).

**Extra Questions**

1. **Be sure to understand all the formulas, definitions and relationships in Exhibit 4-11 on page 73.**
2. **The price of a bond can be written as either as the sum of series of discounted CFs (Equation 4.1, page 63) or as the sum of the PV of an annuity and the discounted maturity value (Equation 4.9, page 67).**

**Note that the PV of an annuity formula used in Equation 4.9 is derived from the difference between a perpetuity starting at time zero and a perpetuity starting at time n. The difference is an annuity starting at time 0 and ending at time n.**

**Equation 4.3 is the first derivative of price w.r.t. yield (∂P/∂y) using equation 4.1.**

***The numerator* of equation 4.10 is first derivative of the price w.r.t. yield using equation 4.9.**

**Consider either equation 4.3 or the numerator of 4.10. Determine *only the sign* of following second partial derivative and cross-partial derivatives:**

* **∂2P/∂y2 < 0**
* **∂2P/∂y∂C < 0**
* **∂2P/∂y∂N > 0**
1. **Does duration increase or decrease as the initial yield increases?**
2. **Does duration increase or decrease as the coupon increases?**
3. **Does duration increase or decrease as the maturity increases?**
4. **A $1,000 face value bond makes semi-annual coupon payments. It has exactly 10 years to maturity. The yield to maturity is 8.00% and the coupon rate is 8.00%. All rates are given in BEY terms.**
5. **Calculate the price of the bond.**

Coup = YTM 🡺 Price = Par = $1,000

NPER = 10 x 2 = 20; RATE = 0.08/2 = 0.04; PMT = 0.08/2 x 1000 = 40; FV = 1,000; **PV = 1,000**

1. **Compute *BOTH* the new lower price of the bond if the YTM increases by 100 basis points and the new higher price of the bond if the YTM decreases by 100 basis points.**

NPER = 20; RATE = (0.08 + **0.01**)/2 = 0.045; PMT = 40; FV = 1000; PV = 934.96

**P+ = 934.96**

NPER = 20; RATE = (0.08 - **0.01**)/2 = 0.035; PMT = 40; FV = 1000; PV = 1,071.06

**P- = 1,071.06**

1. **Use your calculations from parts (a) and (b) to compute the *approximate* Modified Duration (Mod D) for the bond.**

Mod D ≈ (P- - P+)/(2 x P0 x Δy) = (1,071.06 – 939.46)/(2 x 1000 x 0.01) = **6.81**

1. **Calculate the *actual* dollar change in priceof the bond for a 100 bps increase in yield.**

NPER = 20; RATE = (0.08 + **0.01**)/2 = 0.045; PMT = 40; FV = 1000; PV = 934.96

ΔP = P+ – P0 = 934.46 – 1,000 = **-65.04**

1. **Use the bond’s modified duration calculated in part (c) to *estimate* the dollar change in price of the bond for a 100 bps increase in yield.**

ΔP ≈ -Mod D x P x Δy = -6.81 x 1,000 x 0.01 = **-68.10**

1. **Use your calculations from parts (a) and (b) to compute the approximate Convexity (CX) for the bond.**

CX ≈ (P- + P+ – 2P0)/(P0 x Δy2) = (1,071.06 + 934.96 – 2 x 1,000)/(1,000 x 0.012) = **60.22**

1. **Use the bond’s modified duration calculated in part (c) AND the convexity calculated in part (f) to *estimate* the dollar change in price of the bond for a 100 bps increase in yield.**

ΔP ≈ (-Mod D x P0 x Δy) + (½CX x P0 x Δy2) = (-6.81 x 1,000 x 0.01) + [½(60.22 x 1,000 x (0.01)2]

 = -68.10 + 3.01 = **-65.05**

1. **Compare your estimations from parts (e) and (g) to the actual change computed in part (d).**

**Briefly explain why your estimation from part (g) is better than your estimation from part (e).**

The estimation using both Duration and Convexity from part (g) is better since it corrects for the “curvature” or “change in slope” of the price/yield relationship.

See Exhibit 4-13 on page 75 of the text.