**Chapter 3 – Measuring Yield**

1. **What is the effective annual yield (EAY) if the semiannual periodic interest rate is 4.3%?**

Periodic rate = r = 4.30%; m = 2

EAY = (1 + r)m – 1 = (1.0430)2 – 1 = 8.7849%

1. **What is the yield to maturity of a bond?**

The YTM is the discount rate that equates the cash flows to the price. It is the “promised yield”  
from holding the bond *IF* the bond is held to maturity and the coupons are reinvested at the YTM.

1. **What is the yield to maturity calculated on a bond-equivalent basis?**

Bond equivalent basis or Bond Equivalent Yield (BEY) is the common way to quote YTM. It is double the period rate. Therefore BEY is a semi-annual APR.

1. **Each bond shown in the table below has a par value of $1,000 and pays interest semiannually.**

|  |  |  |  |
| --- | --- | --- | --- |
| **Bond** | **Coupon Rate (%)** | **Number of Years to Maturity** | **Price** |
| **W** | **7** | **5** | **$884.20** |
| **X** | **8** | **7** | **$948.90** |
| **Y** | **9** | **4** | **$967.70** |
| **Z** | **0** | **10** | **$456.39** |

**Show the cash flows and calculate the YTM for each bond.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Period | Cash Flow  for Bond W | Cash Flow  for Bond X | Cash Flow  for Bond Y | Cash Flow  for Bond Z |
| 1 | $35 | $40 | $45 | $0 |
| 2 | $35 | $40 | $45 | $0 |
| 3 | $35 | $40 | $45 | $0 |
| 4 | $35 | $40 | $45 | $0 |
| 5 | $35 | $40 | $45 | $0 |
| 6 | $35 | $40 | $45 | $0 |
| 7 | $35 | $40 | $45 | $0 |
| 8 | $35 | $40 | $1,045 | $0 |
| 9 | $35 | $40 |  | $0 |
| 10 | $1,035 | $40 |  | $0 |
| 11 |  | $40 |  | $0 |
| 12 |  | $40 |  | $0 |
| 13 |  | $40 |  | $0 |
| 14 |  | $1,040 |  | $0 |
| 15 |  |  |  | $0 |
| 16 |  |  |  | $0 |
| 17 |  |  |  | $0 |
| 18 |  |  |  | $0 |
| 19 |  |  |  | $0 |
| 20 |  |  |  | $1,000 |

YTM W = 9.9993% ≈ 10.00%

YTM X = 8.9997% ≈ 9.00%

YTM Y = 9.9995% ≈ 10.00%

YTM Z = 7.9999% ≈ 8.00%

1. **Consider the following $1,000 par value bond with an 11% Coupon rate and 18 years to maturity. The market price is $1,169.00.**

**The bond is callable at par in exactly 13 years and each year after.**

**The bond is putable at par value in exactly five years. This is the only put date.**

1. **Calculate the yield to maturity.**
2. **Calculate the yield to first par call.**
3. **Calculate the yield to put.**
4. **Suppose that in addition to the par call schedule, the bond can also be called at 105.5% of par in eight years. Calculate the yield to worst for this bond. Hint: You must calculate the yield for each call date.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Event | N | PMT | PV | FV | Yield |
| Maturity | 36 | $55 | $1,169 | $1,000 | 9.08% |
| 1st Call | 26 | $55 | $1,169 | $1,000 | 8.79% |
| Call | 27 | $55 | $1,169 | $1,000 | 8.83% |
| Call | 28 | $55 | $1,169 | $1,000 | 8.87% |
| Call | 29 | $55 | $1,169 | $1,000 | 8.90% |
| Call | 30 | $55 | $1,169 | $1,000 | 8.93% |
| Call | 31 | $55 | $1,169 | $1,000 | 8.96% |
| Call | 32 | $55 | $1,169 | $1,000 | 8.99% |
| Call | 33 | $55 | $1,169 | $1,000 | 9.01% |
| Call | 34 | $55 | $1,169 | $1,000 | 9.04% |
| Call | 35 | $55 | $1,169 | $1,000 | 9.06% |
| Premium Call | 16 | $55 | $1,169 | **$1,055** | 8.54% |
| Put | 10 | $55 | $1,169 | $1,000 | 6.94% |

1. YTM = 9.08%
2. YTC = 8.79%
3. YTP = 6.94%
4. YTW = 8.54% not counting the YTP.

Since the put option is held by the holder of the bond, you do not to consider YTP in the YTW analysis. The holder will likely ***not*** exercise the put option (sell the bond to the issuer for $1,000) after five years since the bond holder paid a premium ($1,169) to earn the 11% coupon for 18 years

1. **Answer the below questions.**
2. **What is meant by an amortizing security?**

Amortizing (or self-amortizing) securities are fixed income securities in which the periodic cash flows include interest payments and repayment of the principal (the loan amount). This is a common structure for consumer loans. Note that almost all amortizing loans allow the borrower to prepay the loan at any time. This is equivalent to a call option granted by the lender to the borrower.

For amortizing securities, reinvestment risk is even greater than for non-amortizing securities since the lender must reinvest the periodic principal repayments in addition to the periodic coupon interest payments. Moreover, the cash flows are usually monthly, not semiannually as with most non-amortizing securities. So the lender must reinvest periodic interest payments and principal – and do so more often.

1. **What are the three components of the cash flow for an amortizing security?**

(1) Interest, (2) scheduled repayments and (3) sometimes prepayments

1. **What is meant by a cash flow yield?**

It is the discount rate that equates the present value of the projected cash flows to the market price. It is essentially an IRR. A difficulty in computing a cash flow yield is projecting prepayments in each period.

1. **Suppose that the coupon rate of a floating-rate security resets every six months at a spread of 70 basis points over the reference rate. If the bond is trading at below par value, explain whether the discount margin is greater than or less than 70 basis points (bps).**

If the bond is trading at a discount, the discount margin exceeds the spread.

If the bond is trading at a premium, the discount margin is less than the spread.

Consider the following example. Assume a floater with 10 years to maturity. The coupon is the reference rate plus 70 bps and the reference rate is 5%. Compute the discount margin if the price is 99.00 and 101.00. The discount margin is the YTM less the reference rate.

NPER = 20 PMT = 5.70/2 = 2.85 PV = -99.00 FV = 100 RATE = 2.917 🡺 YTM = 5.83%

Discount Margin = 5.83% - 5.00% = 0.83% = 83 bps > 70 bps

NPER = 20 PMT = 5.70/2 = 2.85 PV = -101.00 FV = 100 RATE = 2.784 🡺 YTM = 5.57%

Discount Margin = 5.57% - 5.00% = 0.57% = 57 bps < 70 bps

1. **An investor is considering the purchase of a 20-year$1,000 par value, 7% coupon bond selling for $816. The yield to maturity for this bond is 9%. Answer the below questions.**
2. **What would be the total future dollars if this investor invested $816 for 20 years earning 9% compounded semiannually?**

9% semi-annual for 20 years means 4.5% for 40 periods:

Pn = P0 (1 + r)n 🡺 P40 = $816(1.045)40 = $4,746.15

NPER = 40 I/Y = 4.5 PV = 816 FV = -4,746.15

1. **What are the total coupon payments over the life of this bond?**

7.00%/2 x 1000 = $35 every six months for 20 years

40 x $35 = $1,400

1. **What would be the total future dollars from the coupon payments and the repayment of principal at the end of 20 years (assuming reinvestment at the YTM)?**

Total future dollars is Coupons + Interest on Interest + Principal

Coupons + Interest on Interest is the future value of an annuity with $35 payments and a 4.5% periodic return:

NPER = 40 RATE = 4.5 PMT = 35 FV = $3,746.06

Principal is $1000

Total futures dollars = $4,746.06

1. **and (e) For the bond to produce the same total future dollars as in part (a), how much must the interest on interest be?**

I on I = $3,746.06 - $1,400 = $2,346.06

1. **What is the total return for a 20-year zero-coupon bond that is offering a yield to maturity of 8% if the bond is held to maturity?**

Since a zero coupon bond has no reinvestment risk, the total return for a zero is equal to the yield to maturity, so the total return is also 8%.

1. **Suppose that an investor with a five-year investment horizon is considering purchasing a seven-year 9% coupon bond selling at par. The investor expects that he can reinvest the coupon payments at an interest rate of 9.4% APR S-A and that at the end of the investment horizon two-year bonds will be selling to offer a yield to maturity of 11.2%. What is the total return for this bond?**

Coupon + I on I = future value of a 10 period, $45, 4.70% annuity

NPER = 10 RATE = 0.047 PMT = 45 FV = 558.14

Sale Price at time 5 is the price of a 2 year, 9% coupon bond with a YTM of 11.2%

NPER = 4 RATE = 0.056 PMT = 45 FV = 1000 PV = 961.53

Total Future Dollars = $558.14 + $961.53 = $1,519.67

Periodic Return over the 10 periods = ($1,519.67/$1,000)1/10 – 1 = 0.04274

Total Return (reported in Bond Annual Yield basis) = 2 x 0.04274 = 8.55%

**Additional Questions:**

1. Define
2. Yield to Maturity (YTM)
3. Bond Equivalent Yield (also called the Bond Annual Yield)
4. EAY
5. Define
6. Current Yield
7. Yield to Maturity
8. Yield to Call
9. Yield to Sinker
10. Yield to Put
11. Yield to Worst
12. Cash Flow Yield
13. Yield Spread
14. Rank by size the Coupon Rate, YTM and Current Yield for a bond priced at a premium, a bond priced at a discount and a bond priced at par.

From the Lecture notes:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| (Variable %) |  | (Fixed %) |  | (Fixed $) |  | (Variable $) |
| YTM | **>** | Coupon Rate |  | Par | **>** | Price |
| YTM | **=** | Coupon Rate |  | Par | **=** | Price |
| YTM | **<** | Coupon Rate |  | Par | **<** | Price |

If the bond is priced at a premium, the holder earns less than the coupon rate. 🡺 YTM < Coup

If the bond is priced at a par, the holder earns the coupon rate 🡺 YTM = Coup

If the bond is priced at a discount, the holder earns more than the coupon rate 🡺 YTM > Coup

Since the YTM accounts for both the coupon yield and the capital gain (or loss) and the current yield is only the coupon yield, the Current Yield is always between the YTM and Coupon Yield.

Premium: Coupon Rate > Current Yield > YTM

Par: Coupon Rate = Current Yield = YTM

Discount: Coupon Rate < Current Yield < YTM