Improving the 0–1 Multi Dimensional Knapsack Lower Bounds with Tabu Search

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1 Introduction

The 0-1 multi dimensional knapsack problem ($01MKP$) is an NP–Hard problem which arises in several practical problems such as capital budgeting, cargo loading, cutting stock problems, and computing processor allocation in large distributed systems. The problem can be stated as follows :

$$01MKP \begin{cases} \text{maximize } c.x \text{ subject to} \\ A.x \leq b \text{ and } x \in \{0,1\}^n \end{cases}$$

where $c \in \mathbb{N}^{*n}$, $A \in \mathbb{N}^{m \times n}$ and $b \in \mathbb{N}^m$. The binary components x_j of x are decision variables: $x_j = 1$ if the item j is selected, 0 otherwise. c_j is the profit associated with selecting item j. A_{ij} is the "cost" (in terms of the i^{th} resource) of selecting item j. b_i is the budget available for resource i.

Due to the problems intrinsic difficulty, which leads to intractable computation time for larger instances, several heuristics have been used to solve it, including simulated annealing, tabu search, genetic algorithms, and many other population based algorithms. Large instances ($n = 500, m = 30$ from OR-Library), for which exact methods fail to prove the optimum, have thus been tackled successfully, *i.e.* lower-bounds with small gap to the fractional optimum value were obtained by these incomplete methods.

2 Resolution Method

In a first study we have implemented an algorithm which combines Linear Programming with Tabu Search [5, 6]. Linear Programming makes it possible to define geometric constraint and cutting planes ($1.x = k$ where k is an integer) and to design the search space and the neighborhood. Instead of the classical $1 - move$ we have used a specific $2 - move$ that keeps the search in a specified hyperplane determined by an equation constraining the sum of variables to a chosen constant. The exploration of the search space avoid the trap of local optimality by using the TS reverse elimination method to provide an exact tabu list management, as proposed by Fred Glover [2] and examined in [3, 4].

Starting from this point, we have intensified the local search around promising zones in order to improve the lower-bounds obtained by this first algorithm. To do so, we have organized the previous algorithm to operate within the context of limited enumeration. This leads to selecting and fixing variables by using "good" points in the search space [7].

The following table shows the improvement obtained by our algorithms on the 30 largest instances of the OR-Library. The whole benchmark is available at `http://people.brunel.ac.uk/~mastjjb/jeb/orlib/mknapinfo.html`. The description of the data, per column is:

- instance Pb.: row number r of the instance. The whole name of the problem is CB$m.n_ r$ where m is the number of constraints and n the number of items;
- upper-bound $\bar{z}$: the optimum value of the integrity relaxed version of the original $01MKP$;
- lower-bound GA_{CB}: obtained by Chu and Beasley's Genetic Algorithm [1];
- lower-bound $LP+TS$: obtained by the first version of our algorithm [5];
- lower-bound $Fix+LP+TS$: obtained by the variables fixing heuristic [7].

Table 1: Improving **CB30.500** lower-bounds

Pb.	$\bar{z}$	GA_{CB}	$LP+TS$	$Fix+LP+TS$
0	116619.0	115868	115991	**116056**
1	115370.1	114667	**114810**	114810
2	117342.5	116661	116683	**116712**
3	115946.4	115237	115301	**115329**
4	117079.3	116353	116435	**116525**
5	116377.6	115604	115694	**115741**
6	114689.7	113952	114003	**114181**
7	114847.8	114199	114213	**114348**
8	115902.6	115247	115288	**115419**
9	117668.8	116947	117055	**117116**
10	218601.5	217995	218068	**218104**
11	215074.7	214534	214562	**214648**
12	216401.1	215854	215903	**215978**
13	218350.5	217836	**217910**	217910
14	216094.5	215566	215596	**215689**
15	216327.4	215762	215842	**215890**
16	216376.3	215772	215838	**215907**
17	217014.1	216336	216419	**216542**
18	217839.2	217290	217305	**217340**
19	215218.5	214624	214671	**214739**
20	302038.8	301627	301643	**301675**
21	300455.0	299985	**300055**	300055
22	305501.2	304995	305028	**305087**
23	302456.2	301935	302004	**302032**
24	304901.4	304404	304411	**304462**
25	297409.4	296894	296961	**297012**
26	303765.9	303233	303328	**303364**
27	307402.5	306944	306999	**307007**
28	303605.9	303057	303080	**303199**
29	301020.6	300460	300532	**300572**

Bold face text highlights the first best values found by one of these three compared methods.

3 Conclusion

The tabu search metaheuristic provides an alternative to exact methods that can perform a similarly rigorous (but more flexible) search space exploration by using an exact tabu list management method, as studied in our approach. We have seen also that such a tabu list management entails a search space design (consisting of neighborhoods, data structures and constraints) that is quite straightforward to implement, yielding a very effective overall method.

Future advances may be produced by identifying improved ways to distribute the second proposed algorithm in order to decrease the computational cost and further improve the lower-bounds. A straightforward way to accomplish this would be to dedicate each sub space defined by a set of fixed variables [7] to a thread on different processing units. An additional promising approach consists in tacking into account the reduced-cost constraint [8] to design a more restricted search space. Particularly useful will be cooperative distributed tabu search algorithms which update the gap value each time one of them improves the lower-bound, thereby providing additional exploitation of the reduced-cost constraints.

References

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