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A comparative study of formulations for a cross-dock door assignment problem

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ABSTRACT

A cross docking facility is a type of warehouse in supply chain management that allows orders to be prepared with or without going through the phase of storing products in the warehouse and subsequently selecting them for delivery. The goods are unloaded from incoming trucks called origins on inbound doors of a cross-docking facility platform and, using a handling device inside the platform such as a forklift, immediately transferred to outbound doors to be loaded into outgoing trucks named destinations or delivery trucks for distribution to customers. Contrary to a traditional warehouse, goods are unloaded and loaded without placing them in temporary storage inside the cross-docking facility. The goal of the cross-docking assignment problem (CDAP) is to assign origins to inbound doors and destinations to outbound doors so that the total cost inside the cross-dock platform is minimized. To the best of our knowledge, there are only three mixed integer programming (MIP) formulations of the CDAP in the literature. We propose eight new MIP models and demonstrate the mathematical equivalence of all 11 models, together with rigorously proving some of their properties. In order to detect which of these 11 models is best, we conduct an extensive comparative analysis on benchmark instances from the literature, which discloses that the best model is one proposed in this paper for the first time.

1. Introduction

A cross docking facility is a type of warehouse in supply chain management that allows orders to be prepared without going through the phase of storing products in the warehouse and subsequently selecting them for delivery. A key difference between a traditional warehouse and a cross-docking warehouse is that, unlike warehouses where products remain (sometimes for long durations) until they are ordered by customers, the products handled by crossdocking are not permitted to remain on the platform beyond 24 hours [1] sometimes are required to be transferred within less than an hour [2]. Several classes of cross-docking problems have been studied in literature, such as [3]: strategic problems which determine a good location for the cross-docking platform and its layout; operational problems which determine the best assignment of truck to door, locations where goods will be temporarily stored, the best synchronization between arriving and departing trucks at the cross-dock doors etc.; and tactical problems which determine the flow of goods through the cross-dock to minimize costs and make supply meet demand. For variants of cross-docking problems and literature reviews, we refer the reader to [3–10].

In this study, we deal with the *Cross-dock Door Assignment Problem* (CDAP) in which a set of incoming trucks (called origins) come from various sources of goods such as suppliers, warehouses, etc., and unload their pallets of goods at a set of inbound doors, at which point unloaded pallets are sorted in a staging area based on their destinations. Finally, the pallets are directly transferred within the cross-docking facility (using material handling devices such as forklifts) to a set of outbound doors where they are consolidated and loaded onto outgoing trucks (called destinations). The goal of the Cross-dock Door Assignment problem is to find the best assignment of origins (origin trucks) to inbound doors and destinations (destination trucks) to outbound doors so that the total cost of transporting pallets from inbound doors to outbound doors within the platform is minimized.

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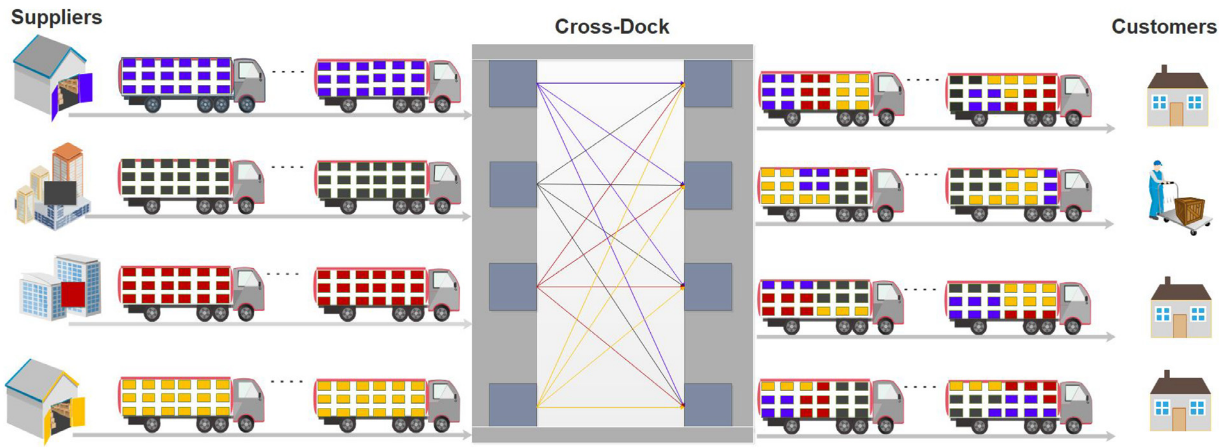

Fig. 1. Cross Docking.

The problem that we consider in this paper belongs to the class of operational problems [3], and specifically to the class of truck to door assignment problems, where the goal is to assign ingoing and outgoing trucks to available doors of the cross-dock in order to minimize costs and improve performance while satisfying a set of constraints. In addition, truck to door assignment problems assume that there are enough doors to accommodate the trucks, so each truck may be assigned to a door and therefore time aspects are not taken into account. Another class of cross-dock problem called the truck scheduling problem considers time aspects when assigning trucks to doors. (See e.g., [4,5,11–15] for literature reviews and relevant works on this problem class.)

Truck to door assignment problems may be classified according to several criteria. The first criterion is based on the allocation strategy. Several types of allocation restrictions are possible: i) each door must serve only one origin/destination and each origin/destination must be assigned to only one inbound/outbound door (see e.g., [16–18]; ii) each inbound door serves only one origin at a time, but the same destination may be assigned to several outbound doors [19]; iii) each door may serve more than one origin/destination [20]. The second criterion considers whether and how capacity constraints are taken into account: In Tsui and Chang [16,17] there are no limitations on the inbound doors' capacities but only on the capacities of outbound doors; In Zhu et al. [20], the authors extended the model of Tsui and Chang [16], and consider capacity constraints on both inbound and outbound doors. The third criterion is based on the layout of a cross-dock as the specification of doors as either inbound or outbound doors. The so-called I-Shape layout is one of the most often considered problems in the literature (see e.g., [16,17,19,21–23]). Fig. 1 describes the I-Shape cross-docking operations in greater detail. An I-shaped cross-dock has a rectangular shape, with receiving doors on one side and outbound doors on the other side. Therefore, rectilinear distances may accurately simulate distances traversed by the forklifts following clearly marked lanes (see Fig. 1). Other layouts considered in the truck to door assignment problems are so-called semi-permanent and dynamic layouts [24–26]. For other shapes of a cross-dock layout considered in the cross-dock literature we refer the reader to [2].

In this paper we consider the CDAP where each door may serve more than one origin/destination, capacity constraints are imposed on each door and I-Shape cross-docking operations are allowed. This variant of the problem was introduced in [20] by Zhu et al., where the authors extended the model of Tsui and Chang [16] to take account of more realistic considerations. Guignard et al. [1] subsequently used the model of Zhu et al. [20] to develop three heuristics, the first two based on local search and the third based on Convex Hull Relaxation. Recently, Nassief et al. [27] proposed a Mixed Integer Programming formulation of the CDAP which is concerned with determining optimal paths for commodities from origins to destinations via inbound and outbound doors. The same paper also proposed some valid inequalities for the problem as well as a Lagrangian Relaxation heuristic to tackle large-scale instances. In 2018, Nasseif et al. [28] presented a study on the standard CDAP (as defined in [20]) with and without load and unload times. They proposed several new formulations and a branch and price solution strategy.

The CDAP includes the Generalized Assignment Problem (GAP) as a subproblem and like the GAP problem is NP-hard [29]. The generalized assignment problem is a well-established field of research in terms of both modeling and solution approaches, and has been extensively studied in papers such as e.g., [30–38]. In addition, several variants of the GAP have been proposed in the literature including the Multi-Resource GAP [39], the multi-level GAP [40], the generalized quadratic assignment problem [41–44], the generalized assignment problem with special ordered sets [45] and the quadratic three-dimensional assignment problem [46]. In [20], the authors establish a relationship between the Generalized Quadratic three-dimensional Assignment Problem (GQ3AP) and the CDAP and show that the CDAP can be solved as a GQ3AP.

Because of its NP-hard character, most of the studies of the CDAP in the literature have been dedicated to developing efficient heuristic solution approaches to cope with large scale instances. On the other hand, to the best of our knowledge, the only integer programming formulations proposed are the standard MIP and the MIP models of Nassief et al. [27,28]. In this paper, we present 11 different MIP formulations of the CDAP, a number of them proposed here for the first time. We further prove the equivalence of these formulations and identify their integrality properties. Finally, we perform an extensive comparative study of their performance on benchmark instances from the literature, reporting the number of instances solved optimally or not, upper bounds they provide, and CPU time consumed by a CPLEX MIP solver applied to each formulation. More precisely, the comparison of performance between the models is not done analytically as in Nassief et al. (2018), but empirically. We have selected CPLEX for these comparisons because it is one of the most effective solvers and because it is a good indicator of model performance, in the respect that if one model performs better than another using CPLEX then the same ranking of the models occurs when applying other leading solvers. Our findings disclose that best MIP formulation among those compared is one of those proposed in this paper for the first time.

The rest of this paper is structured as follows. In Section 2, we describe the standard quadratic model originally proposed in [20] and present the customary approach to linearize this model. In addition, we present some valid inequalities for the problem. In Section 3, we introduce new sets of constraints and build new non standard MIPs for the CDAP. We additionally prove the equivalence of those non standard MIPs as well as their equivalence to the standard linear MIP. In Section 4, we deal with the integrality requirement on the decision variables used to linearize the standard quadratic model and prove that the relaxation of the integrality constraint will not affect the optimal solution. Section 5 provides a comparative analysis of the models on the benchmark data set and identifies the best one. The last section concludes the paper and gives some directions for future work.

2. Standard formulation

In this section we present the standard quadratic formulation of the CDAP due to Zhu et al. [20] together with the standard approach for linearizing this model. In addition, we present some valid equalities and inequalities for the resulting Mixed Integer Programming (MIP) model.

2.1. Standard quadratic formulation

Given a set of incoming trucks (origins) M, a set of outgoing trucks (destinations) N, a set of inbound doors I and a set of outbound doors J, each inbound/outbound door may serve more than one origin/destination respectively subject to the doors' capacity constraints, and each origin/ destination is allocated to one inbound/outbound door respectively. If the origin $m \in M$ is assigned to the inbound door $i \in I$ and the destination $n \in N$ is assigned to the outbound door $j \in J$ a transportation cost is incurred equal to the product of $d_{i,j}$, and $f_{m,n}$, where $d_{i,j}$ is the distance between door i and door j, and $f_{m,n}$ is the number of pallets to be moved from the origin m to the destination n. The total number of pallets delivered to an origin $m \in M$ is $s_m = \sum_{n\in N} f_{m,n}$ and the total number of pallets received at destination $n \in N$ is $r_n = \sum_{m\in M} f_{m,n}$. The capacity of an inbound door $i \in I$ is denoted by S_i and the capacity of an outbound door $j \in J$ is denoted by R_j. In order to formally model the problem we use the binary variable $x_{m,i}$ to indicate whether origin m is assigned to inbound door i or not; and binary variable $y_{n,j}$ to indicate whether destination n is assigned to outbound door j or not.

Using the above notation and decision variables, the CDAP may be formally stated as [20]:

$$(\mathcal{Q}): \begin{cases} \min\ f(x,y) = \sum_{m\in M}\sum_{i\in I} \\ \quad \sum_{n\in N}\sum_{j\in J} d_{i,j} f_{m,n} x_{m,i} y_{n,j} & & (1a) \\ s.t.\ \sum_{i\in I} x_{m,i} = 1, & \forall m \in M & (1b) \\ \quad \sum_{j\in J} y_{n,j} = 1, & \forall n \in N & (1c) \\ \quad \sum_{m\in M} s_m x_{m,i} \le S_i, & \forall i \in I & (1d) \\ \quad \sum_{n\in N} r_n y_{n,j} \le R_j, & \forall j \in J & (1e) \\ \quad x_{m,i}, y_{n,j} \in \{0,1\}, & \forall n \in N, m \in M, \\ & i \in I, j \in J. & (1f) \end{cases}$$

The objective function (1a) minimizes the total transportation cost inside the cross dock. The two sets of constraints (1b) and (1c) ensure that each origin/destination must be allocated to one and only one inbound/outbound door, respectively. The constraints (1d) ((1e)) guarantee that the capacity of each inbound (outbound) door is respected. The last set of constraints (1f) imposes the binary requirement on the decision variables.

2.2. Standard linearization

The quadratic formulation $\mathcal{Q}$ may be linearized using the standard linearization technique that introduces the new binary variable $z_{m,i,n,j}$, such that $z_{m,i,n,j} = x_{m,i} y_{n,j}$, for all $n \in N$, $m \in M$, $i \in I$, $j \in J$. To ensure the variable $z_{m,i,n,j}$ satisfies its required property (i.e., $z_{m,i,n,j} = 1$ iff $x_{m,i} = y_{n,j} = 1$), the following constraints need also to be added to the model:

$$\begin{cases} z_{m,i,n,j} \le x_{m,i}, & \forall n \in N, m \in M, i \in I, j \in J & (2a) \\ z_{m,i,n,j} \le y_{n,j}, & \forall n \in N, m \in M, i \in I, j \in J & (2b) \\ z_{m,i,n,j} \ge y_{n,j} + x_{m,i} - 1, & \forall n \in N, m \in M, i \in I, j \in J. & (2c) \end{cases}$$

So, the resulting Mixed Integer Programming (MIP) model is as follows:

$$(\mathcal{M}^{0,0}): \begin{cases} \min\ g(z) = \sum_{m\in M}\sum_{i\in I} \\ \quad \sum_{n\in N}\sum_{j\in J} d_{i,j} f_{m,n} \\ \quad z_{m,i,n,j} & & (3a) \\ s.t.\ \sum_{i\in I} x_{m,i} = 1, & \forall m \in M & (3b) \\ \quad \sum_{j\in J} y_{n,j} = 1, & \forall n \in N & (3c) \\ \quad z_{m,i,n,j} \le x_{m,i}, & \forall n \in N, m \in M, i \in I, j \in J & (3d) \\ \quad z_{m,i,n,j} \le y_{n,j}, & \forall n \in N, m \in M, i \in I, j \in J & (3e) \\ \quad z_{m,i,n,j} \ge y_{n,j} + x_{m,i} - 1, & \forall n \in N, m \in M, i \in I, j \in J & (3f) \\ \quad \sum_{m\in M} s_m x_{m,i} \le S_i, & \forall i \in I & (3g) \\ \quad \sum_{n\in N} r_n y_{n,j} \le R_j, & \forall j \in J & (3h) \\ \quad x_{m,i}, y_{n,j}, z_{m,i,n,j} \in \{0,1\}, & \forall n \in N, m \in M, i \in I, j \in J. & (3i) \end{cases}$$

The assignment constraints (3b) - (3c) and the capacity constraints (3g) - (3h) are still the same as in Zhu et al. [20] model above. Although, the variables $z_{m,i,n,j}$ are used for linearizing, they may be interpreted in the following way. $z_{m,i,n,j}$ is a binary variable which receives value 1 if and only if the path $< m - i - j - n >$ is established to transfer commodities from origin m to destination n (inbound door i and outbound door j are used as intermediates). With this interpretation the meaning of constraints (3d) - (3f) is as follows. Constraints (3d) and (3e) ensure that if the origin m is not assigned to the receiving door i or the destination n is not assigned to the shipping door j then the path $< m - i - j - n >$ cannot be established. On the other hand, if the origin m is assigned to the receiving door i or the destination n is assigned to the shipping door j then the path $< m - i - j - n >$ is established due to constraints (3f).

The set of constraints that the MIP $\mathcal{M}^{0,0}$ must satisfy can be decomposed into two sets: i) the set of *assignment constraints* (3b) - (3f) and the constraints (3i) on the decision variables which we will refer to as $\mathcal{A}^0$, and ii) the set of *capacity constraints* (3g) - (3h) which we will refer to as $\mathcal{C}^0$.

2.3. Valid inequalities

The next proposition provides some valid equalities for the above model.

Proposition 2.1. *The constraints of the following system*

$$\begin{cases} \sum_{i\in I} z_{m,i,n,j} = y_{n,j}, & \forall m \in M, n \in N, j \in J & (4a) \\ \sum_{j\in J} z_{m,i,n,j} = x_{m,i}, & \forall m \in M, n \in N, i \in I & (4b) \\ \sum_{i\in I}\sum_{j\in J} z_{m,i,n,j} = 1, & \forall m \in M, n \in N & (4c) \end{cases}$$

are valid for the MIP $\mathcal{M}^{0,0}$.

Proof. The valid equalities (4a) and (4b) are directly deduced from constraints (3b) and (3c), multiplying them by $y_{n,j}$ and $x_{m,i}$, respectively. On the other hand, the valid equality (4c) is a direct consequence of the valid equality (4a) ((4b)) taking into account constraints (3b) ((3c)). □

The two sets of valid equalities (4a) and (4b) imply that if the origin m is assigned to the inbound door i, then the commodity from the origin m to the destination n must be routed through inbound door i and some outbound door j; similarly, if the destination n is assigned to an outbound door j, then the commodity from the origin m to the destination n must be routed through outbound door j and some inbound door i. The set of inequalities (4c) imply that the commodity from the origin m to the destination n is routed via unique inbound door i and unique outbound door j.

Starting from capacity constraints (3g) and (3h) gathered into a set $\mathcal{C}^0$:

$$(\mathcal{C}^0):\begin{cases}\sum_{m\in M} s_m x_{m,i} \le S_i, & \forall i\in I\\ \sum_{n\in N} r_n y_{n,j} \le R_j, & \forall j\in J.\end{cases}$$

we may derive the following set of valid inequalities:

$$(\mathcal{C}^1):\begin{cases}\sum_{m\in M} s_m z_{m,i,n,j} \le S_i y_{n,j}, & \forall i\in I, n\in N, j\in J & (5a)\\ \sum_{n\in N} r_n z_{m,i,n,j} \le R_j x_{m,i}, & \forall j\in J, m\in M, i\in I. & (5b)\end{cases}$$

Indeed, these two constraints are obtained multiplying capacity constraints (3g) and (3h) by $y_{n,j}$ and $x_{m,i}$, respectively. The meaning of these newly established constraints is as follows. Constraints (5a) ensure that the total amount of commodities with the destination n routed via the inbound - outbound door pair (i, j) does not exceed the capacity limit of the inbound door i. Similarly, constraints (5b) ensure that the total amount of commodities with the origin m routed via the inbound - outbound door pair (i, j) respects the capacity bound of the outbound door j. In [27] these two constraints are also considered as a valid inequalities.

3. Non standard assignment and capacity constraints

In this section we present three sets of assignment constraints that are deduced from the set $\mathcal{A}^0$ as a result of the valid equalities stated in the preceding section and additionally prove the equivalence of these sets of constraints. We also present a set of capacity constraints deduced directly from the set $\mathcal{C}^0$ and prove the equivalence between the resulting constraint sets.

3.1. Assignment constraints

The first set of assignment constraint that we present here is based on the observation that the nature of the problem implies that the large set of constraints (3f) may be replaced by a smaller one as stated in the next property.

Proposition 3.1. *The constraints (3f) may be replaced by the set of equalities $\sum_{i\in I}\sum_{j\in J} z_{m,i,n,j}=1$ for all $m\in M$, $n\in N$.*

Proof. Constraints (3f) ensure that if $x_{m,i}=y_{n,j}=1$ then $z_{m,i,n,j}=1$ as well, otherwise they are redundant. On the other hand equalities $\sum_{i\in I}\sum_{j\in J} z_{m,i,n,j}=1$ for all $m\in M$, $n\in N$ require that for each m and n there are unique i' and j' so that $z_{m,i',n,j'}=1$. From constraints (3b) and (3c), it follows that for each m and n there are as well unique i'' and j'' so that $x_{m,i''}=y_{n,j''}=1$. Taking into account constraints (3d) and (3e) we have $z_{m,i,n,j}=0$ if $i\neq i''$ or $j\neq j''$ and $z_{m,i,n,j}\le 1$ if $i=i''$ or $j=j''$. This implies that $i'=i''$ and $j'=j''$ and therefore if $x_{m,i''}=y_{n,j''}=1$ then $z_{m,i'',n,j''}=1$ □

As a consequence of the preceding property we obtain the following set of assignment constraints:

Assignment constraints $\mathcal{A}^1$:

$$(\mathcal{A}^1):\begin{cases}\sum_{i\in I} x_{m,i}=1, & \forall m\in M & (6a)\\ \sum_{j\in J} y_{n,j}=1, & \forall n\in N & (6b)\\ z_{m,i,n,j}\le x_{m,i}, & \forall n\in N, m\in M, i\in I, j\in J & (6c)\\ z_{m,i,n,j}\le y_{n,j}, & \forall n\in N, m\in M, i\in I, j\in J & (6d)\\ \sum_{i\in I}\sum_{j\in J} z_{m,i,n,j}=1, & \forall m\in M, n\in N & (6e)\\ x_{m,i}, y_{n,j}, z_{m,i,n,j}\in\{0,1\}, & \forall n\in N, m\in M, i\in I, j\in J. & (6f)\end{cases}$$

The following corollary is a direct consequence of the preceding property.

Corollary 3.1. *Assignment constraints $\mathcal{A}^0$ and $\mathcal{A}^1$ are equivalent.*

Replacing the constraints (6c) and (6d) with equalities $\sum_{j\in J} z_{m,i,n,j}=x_{m,i}$ for all $m\in M$, $n\in N$, $i\in I$ and $\sum_{i\in I} z_{m,i,n,j}=y_{n,j}$ for all $m\in M$, $n\in N$, $j\in J$, respectively we obtain the following set of assignment constraints:

Assignment constraints $\mathcal{A}^2$:

$$(\mathcal{A}^2):\begin{cases}\sum_{i\in I} x_{m,i}=1, & \forall m\in M & (7a)\\ \sum_{j\in J} y_{n,j}=1, & \forall n\in N & (7b)\\ \sum_{i\in I} z_{m,i,n,j}=y_{n,j}, & \forall m\in M, n\in N, j\in J & (7c)\\ \sum_{j\in J} z_{m,i,n,j}=x_{m,i}, & \forall n\in N, m\in M, i\in I & (7d)\\ x_{m,i}, y_{n,j}, z_{m,i,n,j}\in\{0,1\}, & \forall n\in N, m\in M, i\in I, j\in J. & (7e)\end{cases}$$

The inclusion of these equalities make constraints $\sum_{i\in I}\sum_{j\in J} z_{m,i,n,j}=1$ for all $m\in M$, $n\in N$ redundant and therefore we do not include them in the set $\mathcal{A}^2$. The equivalency of the sets of constraints $\mathcal{A}^1$ and $\mathcal{A}^2$ is then formally proved by the next theorem.

Proposition 3.2. *Constraints $\mathcal{A}^1$ and $\mathcal{A}^2$ are equivalent.*

Proof. (⇒) From constraints (6e) we have that for each $m\in M$ and $n\in N$ there are unique $i\in I$ and $j\in J$ so that $z_{m,i,n,j}=1$. This, together with constraints (6a) - (6d), further implies that $x_{m,i}=1$ and $x_{m,i'}=0, i'\in I, i'\neq i$ as well as $y_{n,j}=1$ and $y_{n,j'}=0, j'\in J, j'\neq j$. Therefore we have $\sum_{i'\in I} z_{m,i',n,j'}=y_{n,j'}=0, j'\in J, j'\neq j$ and $\sum_{i'\in I} z_{m,i',n,j}=y_{n,j}=1$. Similarly, we have $\sum_{j'\in J} z_{m,i',n,j'}=x_{m,i'}=0, i'\in I, i'\neq i$ and $\sum_{j'\in J} z_{m,i,n,j'}=x_{m,i}=1$. Consequently, constraints $\mathcal{A}^1$ imply constraints $\mathcal{A}^2$.

(⇐) Constraints (7c) and (7d) imply constraints (6c) and (6d), respectively. On the other hand, constraints (7a) together with constraints (7d) imply constraints (6e) $\forall m\in M$, $n\in N$ and constraint (7b) together with constraint (7c) imply too constraint (6e) $\forall m\in M$, $n\in N$. Consequently, constraints $\mathcal{A}^2$ imply constraints $\mathcal{A}^1$. □

As already pointed out the constraints $\sum_{i\in I}\sum_{j\in J} z_{m,i,n,j}=1$ for all $m\in M$, $n\in N$ are redundant for the set $\mathcal{A}^2$. However, an interesting observation is that replacing constraints (7a) and (7b) by constraints $\sum_{i\in I}\sum_{j\in J} z_{m,i,n,j}=1$ for all $m\in M$, $n\in N$ leads to another valid set of assignment constraints, as follows.

Assignment constraints $\mathcal{A}^3$:

$$(\mathcal{A}^3):\begin{cases}\sum_{i\in I} z_{m,i,n,j}=y_{n,j}, & \forall m\in M, n\in N, j\in J & (8a)\\ \sum_{j\in J} z_{m,i,n,j}=x_{m,i}, & \forall n\in N, m\in M, i\in I & (8b)\\ \sum_{i\in I}\sum_{j\in J} z_{m,i,n,j}=1, & \forall m\in M, n\in N & (8c)\\ x_{m,i}, y_{n,j}, z_{m,i,n,j}\in\{0,1\}, & \forall n\in N, m\in M, i\in I, j\in J. & (8d)\end{cases}$$

Note that this set of constraints is already proposed in the paper of Nassief et al. [27].

Proposition 3.3. *Constraints $\mathcal{A}^2$ and $\mathcal{A}^3$ are equivalent.*

Proof. (⇒) Constraints (7a) and (7d) imply constraints (8c) and therefore constraints $\mathcal{A}^1$ imply constraints $\mathcal{A}^3$.

(⇐) Constraints (8a) and (8c) imply constraints (7a), while constraints (8b) and (8c) imply constraints (7b). Hence, constraints $\mathcal{A}^3$ imply constraints $\mathcal{A}^1$. □

From propositions 3.1, 3.2 and 3.3 we have the following consequence.

Corollary 3.2. *Assignment constraints $\mathcal{A}^0$, $\mathcal{A}^1$, $\mathcal{A}^2$ and $\mathcal{A}^3$ are equivalent.*

3.2. Capacity constraints

As already mentioned in [27] constraints $\mathcal{C}^1$ also provide valid inequalities. In this paper we go further and prove the equivalence between capacity constraints $\mathcal{C}^0$ and $\mathcal{C}^1$ for the CDAP. The proof is based on the fact that $z_{m,i,n,j}=x_{m,i}y_{n,j}$ and the observation that assignment constraints guarantee the existence of $n'\in N$ and $j'\in J$ such that $y_{n',j'}=1$ as well as the existence of $m'\in M$ and $i'\in I$ such that $x_{m',i'}=1$ (due to the problem definition).

Proposition 3.4. *Capacity constraints $\mathcal{C}^0$ and $\mathcal{C}^1$ for the CDAP are equivalent.*

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Proof. (⇒) Multiplying constraints (3g) by $y_{n,j}$ for all $n \in N$, $j \in J$, we obtain $\Sigma_{m \in M} s_m z_{m,i,n,j} \le y_{n,j} S_i$ for all $i \in I$, $n \in N$, $j \in J$ (using the fact that $z_{m,i,n,j} = x_{m,i} y_{n,j}$). Similarly, we show that constraints (3h) imply constraints (5b).

(⇐) If we consider the constraint (5a), we have $\sum_{m\in M} s_m z_{m,i,n,j} = \sum_{m\in M} s_m x_{m,i} y_{n,j} \le S_i y_{n,j}$ for all $i \in I$, $n \in N$, $j \in J$. Keeping in mind that there exist $n' \in N$ and $j' \in J$ such that $y_{n',j'} = 1$ (this follows from assignment constraints) we have $\Sigma_{m \in M} s_m x_{m,i} \le S_i$ for all $i \in I$. Similarly, we can show that constraints (5b) imply constraints (3h). □

4. MIP Models and integrality properties

In this section we present MIP models that may be deduced by combining the assignment and capacity constraints presented in the preceding sections. In addition, we identify the integrality properties of these models.

4.1. Eleven MIP models

Having four equivalent sets of assignment constraints and two equivalent sets of capacity constraints we come up with 8 different MIP formulations. These 8 MIPs may be stated in general form as:

$(\mathcal{M}^{k,h})\ min\{g(z) : \mathcal{A}^k, \mathcal{C}^h\}, \forall k = 0, 1, 2, 3,\ h = 0, 1.$

The following proposition enable us to generate three new MIP models.

Proposition 4.1. *The constraints 3d and 3e are redundant in the MIP models $\mathcal{M}^{0,0}$, $\mathcal{M}^{0,1}$ and $\mathcal{M}^{1,1}$.*

Proof. In models $\mathcal{M}^{0,0}$ and $\mathcal{M}^{0,1}$ constraints (3d) and (3e) are redundant since we seek to minimize the objective function and the objective coefficients in the CDAP are positive. In addition, in both models the equality $z_{m,i,n,j} = x_{m,i} y_{n,j}$ remains true, even if we exclude constraints (3d) and (3e), due to the fact that the $z_{m,i,n,j}$ variables are bounded from below only by constraints (3f). Namely, if $x_{m,i} = y_{n,j} = 1$, then due to constraints (3f) $z_{m,i,n,j}$ will equal 1 as well, while otherwise $z_{m,i,n,j}$ takes the value 0 (again due to the fact that the objective coefficients in the CDAP are positive). The preceding reasoning leads as well as to the conclusion that in models $\mathcal{M}^{0,0}$ and $\mathcal{M}^{0,1}$ with excluded constraints 3d and 3e, the integrality requirement on variables $z_{m,i,n,j}$ may be relaxed.

On the other hand, in the model $\mathcal{M}^{1,1}$ the constraints 3d (resp. 3e) force $z_{m,i,n,j}$ to be zero if $x_{m,i} = 0$ (resp. $y_{n,j} = 0$). Since the parameters $f_{m,n}$ are positive and by consequence the data s_m and r_n are also positive, the capacities constraints 5a and 5b imply $z_{m,i,n,j} = 0$ if $x_{m,i} = 0$ or $y_{n,j} = 0$.

Hence the constraints 3d and 3e are redundant in the MIP models $\mathcal{M}^{0,0}$, $\mathcal{M}^{0,1}$ and $\mathcal{M}^{1,1}$. □

As a consequence of the above proposition, we have three new MIP models $\mathcal{M}'^{0,0}$, $\mathcal{M}'^{0,1}$ and $\mathcal{M}'^{1,1}$ obtained from the corresponding models $\mathcal{M}^{k,h}$ by dropping the constraints 3d and 3e. In the model $\mathcal{M}^{1,0}$ the constraints 3d and 3e cannot be omitted because in this case there will be no connection between variables $z_{m,i,n,j}$ and variables $x_{m,i}$ and $y_{n,j}$.

The 11 MIPs have the same number of binary variables (i.e., $|I||J||M||N| + |I||M| + |J||N|$). Table 1 provides the number of constraints in each of the 11 MIP models $\mathcal{M}^{k,h}$, $\forall k = 0, .., 3, h = 0, 1$ and $\mathcal{M}'^{0,0}$, $\mathcal{M}'^{0,1}$ and $\mathcal{M}'^{1,1}$.

Comparing the number of constraints for each of these 11 MIP models shown in Table 1, it may be inferred that the number of constraints in the model $\mathcal{M}^{2,0}$ is smaller than in any other model.

Table 1
Number of constraints in each MIP model.

MIP	Total number of constraints
$\mathcal{M}^{0,0}$	$3\|I\|\|J\|\|M\|\|N\| + \|I\| + \|J\| + \|M\| + \|N\|$
$\mathcal{M}^{0,1}$	$\|I\|\|J\|(3\|M\|\|N\| + \|M\| + \|N\|) + \|M\| + \|N\|$
$\mathcal{M}^{1,0}$	$(2\|I\|\|J\| + 1)\|M\|\|N\| + \|I\| + \|J\| + \|M\| + \|N\|$
$\mathcal{M}^{1,1}$	$\|I\|\|J\|(2\|M\|\|N\| + \|M\| + \|N\|) + \|M\|\|N\| + \|M\| + \|N\|$
$\mathcal{M}^{2,0}$	$(\|M\|\|N\| + 1)(\|I\| + \|J\|) + \|M\| + \|N\|$
$\mathcal{M}^{2,1}$	$(\|M\| + \|N\|)(1 + \|I\|\|J\|) + \|M\|\|N\|(\|I\| + \|J\|)$
$\mathcal{M}^{3,0}$	$\|M\|\|N\|(\|I\| + \|J\| + 1) + \|I\| + \|J\|$
$\mathcal{M}^{3,1}$	$\|M\|\|N\|(\|I\| + \|J\| + 1) + \|I\|\|J\|(\|M\| + \|N\|)$
$\mathcal{M}'^{0,0}$	$\|M\|\|N\|\|I\|\|J\| + \|I\| + \|J\| + \|M\| + \|N\|$
$\mathcal{M}'^{0,1}$	$\|I\|\|J\|(\|M\|\|N\| + \|M\| + \|N\|) + \|M\| + \|N\|$
$\mathcal{M}'^{1,1}$	$(\|I\|\|J\| + 1)(\|M\|\|N\| + \|M\| + \|N\|)$

4.2. Integrality properties of MIPs

This section provides properties which show that in all our MIP formulations the requirement $z_{m,i,n,j} \in \{0, 1\}$ for all $m \in M$, $n \in N$, $i \in I$, $j \in J$, can be relaxed to require just $z_{m,i,n,j} \in [0, 1]$ for all $m \in M$, $n \in N$, $i \in I$, $j \in J$.

Proposition 4.2. *The integrality requirement on variables $z_{m,i,n,j} \in \{0, 1\}$ for all $m \in M$, $n \in N$, $i \in I$, $j \in J$, in constraints $\mathcal{A}^0$ may be relaxed. Moreover, the binary variables $z_{m,i,n,j} \in \{0, 1\}$ may be replaced by $z_{m,i,n,j} \ge 0$.*

Proof. Suppose $z_{m,i,n,j} = \alpha > 0$ for some $m \in M$, $n \in N$, $i \in I$, $j \in J$. Then, due to constraints (3d) and (3e) we have $x_{m,i} = 1$ and $y_{n,j} = 1$ respectively. This further implies $z_{m,i,n,j} = \alpha \ge 1$ from the constraint (3f) and therefore $\alpha = 1$. The last statement is deduced from constraints (3d) and (3e), and the fact that the variables $x_{m,i}$ and $y_{n,j}$ are binary. □

Proposition 4.3. *The integrality requirement $z_{m,i,n,j} \in \{0, 1\}$ in constraints $\mathcal{A}^1$ may be relaxed.*

Proof. Let suppose that we just impose requirement $z_{m,i,n,j} \ge 0$ and for some $m \in M$ and $n \in N$ and some $i \in I$ and $j \in J$ we have $z_{m,i,n,j} = \alpha > 0$. Because of constraints (6d) we have $\alpha \le 1$. In addition, constraints (6c) and (6d) imply that $y_{n,j} = 1$ and $x_{m,i} = 1$. On the other hand, constraints (6a) and (6b) imply that $y_{n,j'} = 0$ for all $j' \in J$, $j' \ne j$ and $x_{m,i'} = 0$ for all $i' \in I$, $i' \ne i$. This implies in turn that $z_{m,i'',n,j'} = 0$ for all $j' \in J$, $j' \ne j$, $i'' \in I$ (from constraints (6c)) and $z_{m,i',n,j''} = 0$ for all $i' \in I$, $i' \ne i$, $j'' \in J$ (from constraints (6d)). Hence, taking into account constraint (6e) we have $1 = \sum_{i'' \in I} \sum_{j'' \in J} z_{m,i'',n,j''} = z_{m,i,n,j} = \alpha$. Consequently, the integrality requirement $z_{m,i,n,j} \in \{0, 1\}$ in constraints $\mathcal{A}^1$ may be relaxed. □

Proposition 4.4. *The integrality requirement $z_{m,i,n,j} \in \{0, 1\}$ in constraints $\mathcal{A}^2$ may be relaxed.*

Proof. Suppose we impose requirement $z_{m,i,n,j} \ge 0$. Because of constraint (7c) we have $z_{m,i,n,j} \le 1$. Suppose then for some fixed $m \in M$ and $n \in N$ and some $i \in I$ and $j \in J$, we have $z_{m,i,n,j} = \alpha \in \{0, 1\}$. Then, this implies that $y_{n,j} = 1$ and $x_{m,i} = 1$ because of constraints (7c) and (7d). Hence, from constraints (7c) and (7d) follow $\sum_{i' \in I, i' \ne i} z_{m,i',n,j} = 1 - \alpha$ and $\sum_{j' \in J, j' \ne j} z_{m,i,n,j'} = 1 - \alpha$. Taking into account that $\sum_{i\in I} \sum_{j \in J} z_{m,i,n,j} = \sum_{i \in I} x_{m,i} = \sum_{j \in J} y_{n,j} = 1$ (this chain of equalities is deduced by summing the constraints (7c) over set J and the constraints (7d) over set I noting that $\sum_{i \in I} x_{m,i} = 1$ and $\sum_{j \in J} y_{n,j} = 1$) we have

$$1 = \sum_{i\in I} \sum_{j \in J} z_{m,i,n,j} \ge \sum_{i' \in I, i' \ne i} z_{m,i',n,j} + \sum_{j' \in J, j' \ne j} z_{m,i,n,j'} + z_{m,i,n,j} = 2 - \alpha.$$

This implies $\alpha \ge 1$ which is a contradiction. Hence, the integrality requirement may be relaxed. □

Proposition 4.5. *The integrality requirement* $z_{m,i,n,j} \in \{0, 1\}$ *in constraints* $\mathcal{A}^3$ *may be relaxed.*

Proof. Analogous to the proof of Proposition 4.4. □

Note that the preceding property of the $z_{m,i,n,j}$ variables in constraints $\mathcal{A}^3$ has also been detected in [27,28].

Proposition 4.6. *The integrality requirement may be relaxed on the variables* $z_{m,i,n,j} \in \{0, 1\}$ *for all* $m \in M$, $n \in N$, $i \in I$, $j \in J$, *in models* $\mathcal{M}'^{0,0}$, $\mathcal{M}'^{0,1}$ *and* $\mathcal{M}'^{1,1}$.

Proof. The proof is a direct consequence of the preceding propositions and Proposition 4.1, which implies that in each of models $\mathcal{M}'^{0,0}$, $\mathcal{M}'^{0,1}$ and $\mathcal{M}'^{1,1}$ constraints (3d) and (3e) may be deduced from the constraints in a model. □

To the best of our knowledge, the standard MIP formulation $\mathcal{M}^{0,0}$ was already considered in [20], while the models $\mathcal{M}^{3,0}$ and $\mathcal{M}^{3,1}$ were proposed in [27]. On the other hand, the remaining MIPs have not been yet considered for solving the CDAP.

5. Computational results

All tests presented in this section were conducted on a personal computer Intel(R) Core(TM) with i7-6700HQ 2.60GHz CPU and 16GB of RAM, running Windows 10 OS. To solve the MIP formulations we have used the Concert Technology library of CPLEX 12.6.3.0 version in Java IDE. The MIP formulations are compared in terms of the quality of the upper bounds they provide, and the CPU time consumed by a CPLEX to solve an instance. As a maximum CPU time allowed to be consumed by CPLEX we impose a time limit of 2 hours (7200 seconds). For testing purposes 50 benchmark instances[2] proposed by Guignard et al. [1] were used. The authors have generated this data set in the following way. They filled the flow matrix ($f_{m,n}$, $m \in M$, $n \in N$) with randomly generated integer values between 10 and 50 until 25% of the flow matrix was filled. It is assumed that a destination n will receive a flow of at least $f_{m,n}$ from one origin m and an origin m will send at least flow $f_{m,n}$ to one destination n. The process is repeated until all $|M|$ origins and all $|N|$ destinations are accommodated assuming $|M| = |N|$. To generate the distance matrix, the I-Shape cross-docking facility is assumed to have an equal number of inbound and outbound doors, i.e., $|I| = |J|$. According to [2] the cross-docks have width ranging from 60 to 120 feet and doors with a width of 12 feet. Guignard et al. [1] considered the average cross-dock width of 90 feet and doors with a width of 12 feet, which corresponds to an approximate proportion of 8 to 1. Therefore, in all instances distances range from 8 to $8 + |I| - 1$ (see [1]). In addition, the I-shaped cross-dock has a rectangular shape, with receiving doors on one side and outbound doors on the other side. Therefore, rectilinear distances may accurately simulate distances traversed by the forklifts following clearly marked lanes (see [1]). This means that all instances are generated to correspond to a realistic situation. The capacity of each door is set to be equal to the total flow coming from all origins divided by the total number of inbound doors, plus the quotient of p% of the slackness of the total flow, where $p \in \{5, 10, 15, 20, 30\}$. More precisely, the door capacity is calculated using the following formulas:

$$Slack(p\%) = (total\_flow) * p\%$$

$$Door\_capacity = total\_flow / number\_inbound\_doors + Slack(p\%)$$

The number of origins/destinations in the instances ranges from 8 to 20, while the number of inbound/outbound doors is between 4 and 10.

The computational results section consists of two parts. In the first part we test models where integrality requirements on the variables $z_{m,i,n,j}$ are relaxed, while in the second part we keep the integrality requirements. We identify models with relaxed integrality requirements by denoting them as $\mathcal{M}^{k',h}$ where $\mathcal{M}^{k,h}$ is the corresponding model with the integrality requirement intact.

5.1. Comparison of models - integrality requirement on variables $z_{m,i,n,j}$ relaxed

In Tables 2 and 3 we compare the preceding models (with a relaxed integrality requirement on the variables $z_{m,i,n,j}$). By the convention that "solving" an instance means that a feasible solution is found, Table 2 provides summary results in terms of the number of instances solved (row ''# instances''), the number of instances solved to optimality (row ''# optimal''), the average (relative) optimality gap attained by CPLEX (row ''gap''), and the average CPU time consumed by CPLEX to solve an instance (row ''CPU time''). Table 3 provides detailed results for each class of instances for models that succeed in solving all instances. Instances with the same number of origins/destinations and inbound/outbound doors form a class. The number of origins/destinations and inbound/outbound doors in each class is given in the first column of the table in the form $|N| \times |I|$. The remaining columns of the table report for each method the average solution value (column ''value''), the average CPU time (column ''CPU time'') and the average optimality gap (column ''gap'') attained by CPLEX on each class of five instances. The detailed results may be found in Appendix A.

From the reported results we observe that only models $\mathcal{M}^{0',1}$, $\mathcal{M}^{1',0}$, $\mathcal{M}^{2',0}$, $\mathcal{M}^{3',0}$, $\mathcal{M}'^{0',1}$, and $\mathcal{M}'^{1',1}$, enable us to solve all 50 instances using CPLEX. Among them, models $\mathcal{M}^{2',0}$ and $\mathcal{M}^{3',0}$ are the best two, both yielding the best optimality gap (0.010%), solving the largest number (45) of instances to optimality and consuming the least CPU on the average. Their superiority over the other models is also confirmed by a 95% confidence interval plot of the optimality gap (see Fig. 2). The MIP formulation $\mathcal{M}^{2',0}$ needed 745.35 seconds on average, while $\mathcal{M}^{3',0}$ consumed 768.48 seconds to solve an instance. These values are about three times less than the average CPU time consumed by the next fastest formulation $\mathcal{M}^{1',0}$. On the other hand, the two worst models, in terms of the number of solved instances, turn out to be models $\mathcal{M}^{0',0}$ and $\mathcal{M}'^{0',0}$ for which CPLEX was only able to solve 38 and

[2] https://tinyurl.com/yb6l6vmz.

Table 2
Comparison of models - integrality requirement on variables $z_{m,i,n,j}$ relaxed.

	$\mathcal{M}^{0',0}$	$\mathcal{M}^{0',1}$	$\mathcal{M}^{1',0}$	$\mathcal{M}^{1',1}$	$\mathcal{M}^{2',0}$	$\mathcal{M}^{2',1}$	$\mathcal{M}^{3',0}$	$\mathcal{M}^{3',1}$	$\mathcal{M}'^{0',0}$	$\mathcal{M}'^{0',1}$	$\mathcal{M}'^{1',1}$
# instances	38	50	50	44	50	49	50	47	41	50	50
# optimal	21	29	38	28	45	34	45	34	29	29	34
gap	0.143	0.213	0.020	0.030	0.010	0.037	0.010	0.022	0.029	0.193	0.033
CPU time	3546.10	3449.20	2069.87	3102.73	745.35	2489.73	768.48	2406.56	2325.72	3275.19	2801.96
# nodes	403590.47	919624.10	21718.80	3423.85	6909.22	3721.92	7254.68	4107.19	2115087.02	1522937.64	5134.08

Table 3
Comparison of models on each instance class - integrality requirement on variables $z_{m,i,n,j}$ relaxed.

$\|N\| \times \|I\|$	$\mathcal{M}^{0',1}$			$\mathcal{M}^{1',0}$			$\mathcal{M}^{2',0}$			$\mathcal{M}^{3',0}$			$\mathcal{M}'^{0',1}$			$\mathcal{M}'^{1',1}$		
	value	time	gap	value	time	gap	value	time	gap	value	time	gap	value	time	gap	value	time	gap
8x4	5120.8	2.92	0.000	5120.8	1.15	0.000	5120.8	0.22	0.000	5120.8	0.23	0.000	5120.8	1.69	0.000	5120.8	3.08	0.000
9x4	5978.2	5.82	0.000	5978.2	1.53	0.000	5978.2	0.20	0.000	5978.2	0.38	0.000	5978.2	3.56	0.000	5978.2	4.70	0.000
10x4	6319.8	28.28	0.000	6319.8	2.76	0.000	6319.8	0.34	0.000	6319.8	0.53	0.000	6319.8	15.58	0.000	6319.8	11.99	0.000
10x5	6427.8	297.31	0.000	6427.8	8.73	0.000	6427.8	0.77	0.000	6427.8	1.18	0.000	6427.8	111.90	0.000	6427.8	97.44	0.000
11x5	7555.6	1600.02	0.000	7555.6	14.52	0.000	7555.6	1.40	0.000	7555.6	1.94	0.000	7555.6	572.83	0.000	7555.6	673.42	0.000
12x5	7972.8	5838.02	0.109	7970.2	61.64	0.000	7970.2	2.75	0.000	7970.2	3.58	0.000	7978.8	5791.21	0.107	7970.2	749.89	0.000
12x6	10452.4	5119.59	0.093	10449.8	413.18	0.000	10449.8	12.10	0.000	10449.8	13.75	0.000	10474.8	4655.11	0.056	10452.4	4879.05	0.015
15x6	13819.6	7200.00	0.500	13756.4	5878.42	0.001	13756.4	61.56	0.000	13756.4	128.25	0.000	13849.4	7200.00	0.452	13842.6	7200.00	0.040
15x7	14786.2	7200.00	0.524	14705.8	7200.00	0.028	14688.8	174.13	0.000	14688.8	334.93	0.000	14761.8	7200.00	0.446	14836.0	7200.00	0.061
20x10	29869.4	7200.00	0.902	29904.0	7200.00	0.174	29602.4	7200.00	0.101	29641.4	7200.00	0.101	29638.2	7200.00	0.873	33157.2	7200.00	0.216

41 instances, respectively. In addition, we observe that all models $\mathcal{M}^{0',1}$, $\mathcal{M}^{1',0}$, $\mathcal{M}^{2',0}$, $\mathcal{M}^{3',0}$, $\mathcal{M}'^{0',1}$, and $\mathcal{M}'^{1',1}$ are capable of optimally solving instances with up to 11 origins/destinations and 5 inbound/outbound doors. However, only models $\mathcal{M}^{2',0}$ and $\mathcal{M}^{3',0}$ succeed in optimally solving each instance with up to 15 origins/destinations and 7 inbound/outbound doors. On the largest class of instances, model $\mathcal{M}^{2',0}$ exhibits slightly better performance in terms of solution value than $\mathcal{M}^{3',0}$.

To further assess the performance of models $\mathcal{M}^{0',1}$, $\mathcal{M}^{1',0}$, $\mathcal{M}^{2',0}$, $\mathcal{M}^{3',0}$, $\mathcal{M}'^{0',1}$, and $\mathcal{M}'^{1',1}$ which enable CPLEX to provide a solution for all instances considered, we use performance profiles as suggested in [47]. For each method two performance profiles are generated: one with respect to the best upper bounds found and the another with respect to CPU times consumed. We denote the best upper bound by $U_{\mathcal{M}}$ and denote the CPU time consumed in solving an instance by $T_{\mathcal{M}}$. Then, to compare $U_{\mathcal{M}}$ or $T_{\mathcal{M}}$ for different models, we compute the ratio $R^M_{\mathcal{M}} = M_{\mathcal{M}} / \min_{\mathcal{M}' \in \overline{\mathcal{M}}}\{M_{\mathcal{M}'}\}$, where $M_{\mathcal{M}}$ stands for $U_{\mathcal{M}}$ or $T_{\mathcal{M}}$ and $\overline{\mathcal{M}}$ is the set of models to be compared. Therefore, the performance profile of model $\mathcal{M}$ with respect to metric $R^M_{\mathcal{M}}$ measured over each instance s in a set S is simply the graph of the cumulative distribution function, defined as:

$$F^M_{\mathcal{M}}(r) = |\{s \in S \,|\, R^M_{\mathcal{M}} \le r\}| / |S|.$$

In the graph, $R^M_{\mathcal{M}}$ values are given on the x-axis, while $F^M_{\mathcal{M}}$ values are given on y-axis.

From the performance profiles presented in Figs. 3 and 4 we may conclude that models $\mathcal{M}^{2',0}$ and $\mathcal{M}^{3',0}$ clearly dominate all the others. The average optimally gaps presented in Table 2 were indicative of this advantage, but this is now confirmed by the up-

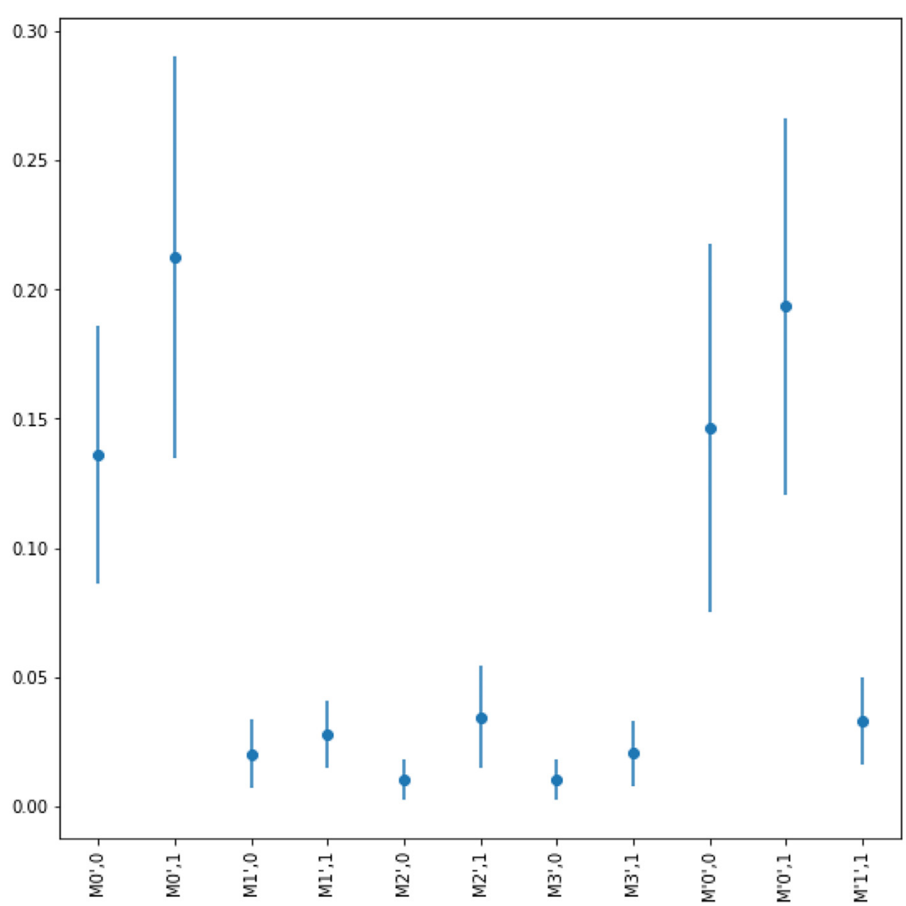

Fig. 2. 95% confidence interval plot of the optimality gap-integrality requirement on variables $z_{m,i,n,j}$ relaxed.

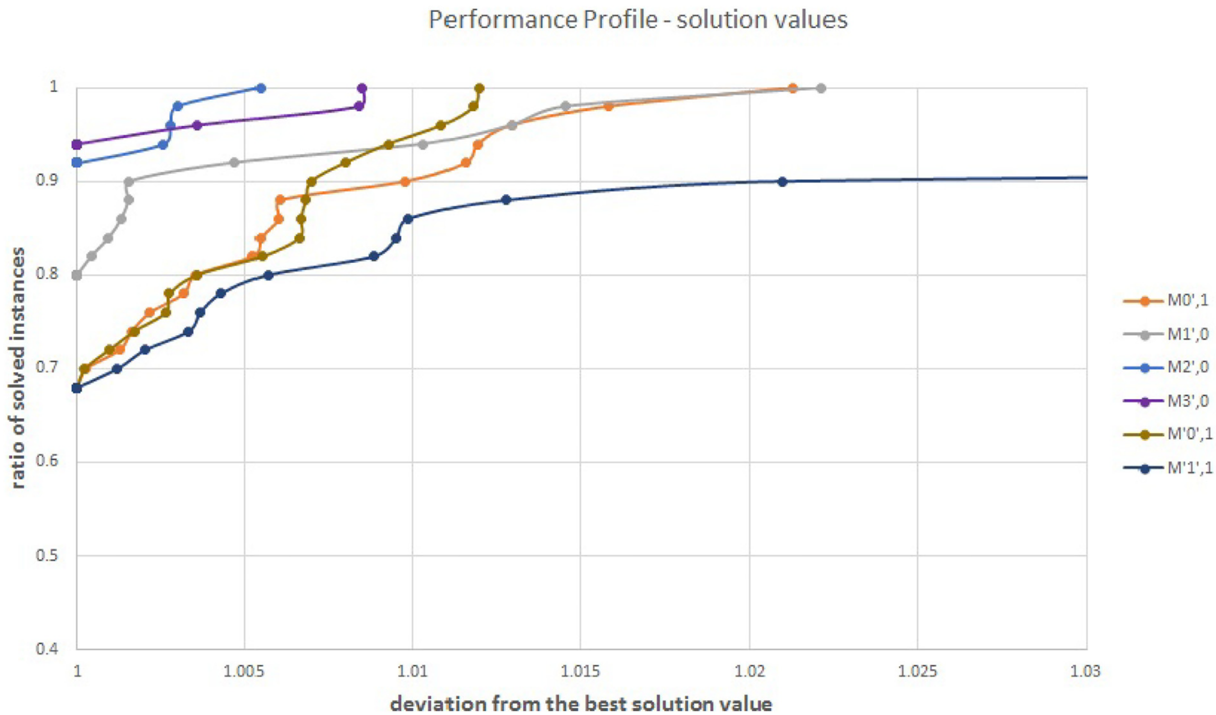

Fig. 3. Performance profile-solution values: integrality requirement on variables $z_{m,\, i,\, n,\, j}$ relaxed.

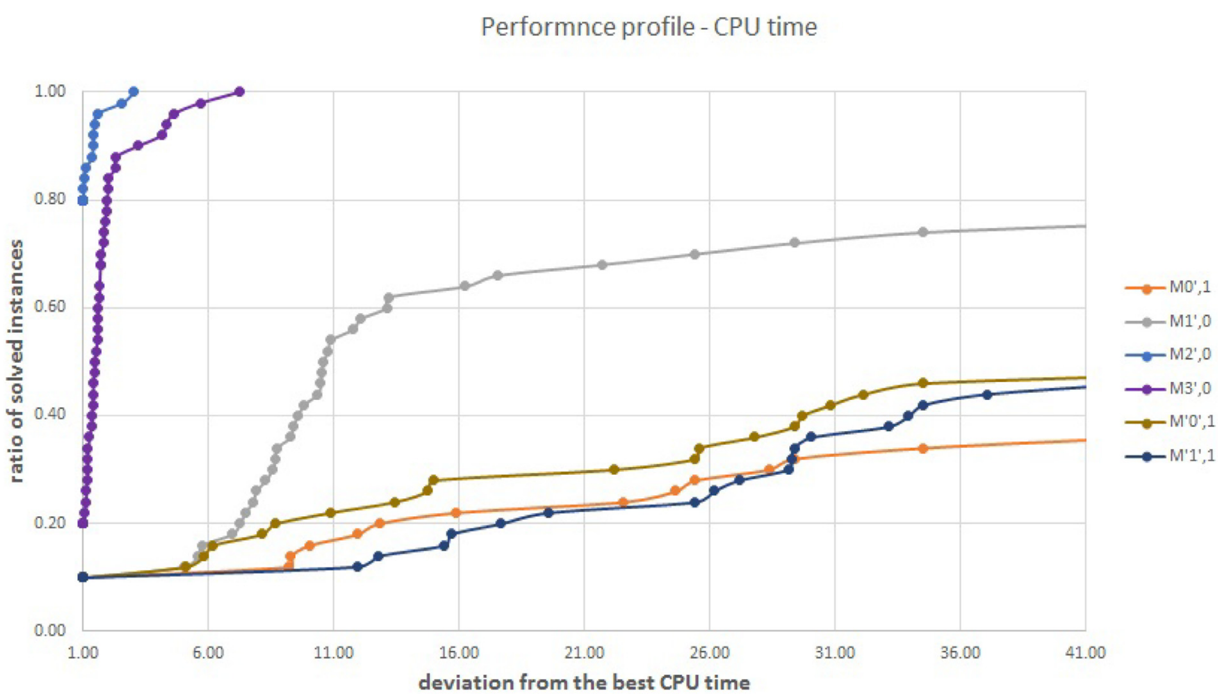

Fig. 4. Performance profile-CPU times: integrality requirement on variables $z_{m,\, i,\, n,\, j}$ relaxed.

per bound and CPU time performance profiles, where we see the graphs of $\mathcal{M}^{2',0}$ and $\mathcal{M}^{3',0}$ on top of the others. If we compare the upper bound performance profiles of $\mathcal{M}^{2',0}$ and $\mathcal{M}^{3',0}$ we see that they cross once in the interval [1, 1.005]. Namely, the upper bound performance profile of $\mathcal{M}^{3',0}$ dominates that of $\mathcal{M}^{2',0}$ in the interval [1, 1.0025], which means that $\mathcal{M}^{3',0}$ finds an upper bound within 0.25% of the best upper bound for more instances than $\mathcal{M}^{2',0}$. Starting from the crossing point, the upper bound performance profile of $\mathcal{M}^{2',0}$ starts to dominate that of $\mathcal{M}^{3',0}$. In addition, we observe that the largest deviation from the best solution value attained by model $\mathcal{M}^{2',0}$ is about 0.25% less than the largest deviation from the best solution value attained by model $\mathcal{M}^{3',0}$. However, the difference between models $\mathcal{M}^{2',0}$ and $\mathcal{M}^{3',0}$ is not statistically significant, in terms of the optimality gap, as can be observed from a 95% confidence interval plot of the optimality gap (see Fig. 2). On the other hand, if we compare CPU times in the performance profiles of $\mathcal{M}^{2',0}$ and $\mathcal{M}^{3',0}$, we observe that the model $\mathcal{M}^{2',0}$ clearly outperforms the model $\mathcal{M}^{3',0}$. The superior-

Table 4
Comparison of models - integrality requirement on variables $z_{m,\, i,\, n,\, j}$ imposed.

	$\mathcal{M}^{0,0}$	$\mathcal{M}^{0,1}$	$\mathcal{M}^{1,0}$	$\mathcal{M}^{1,1}$	$\mathcal{M}^{2,0}$	$\mathcal{M}^{2,1}$	$\mathcal{M}^{3,0}$	$\mathcal{M}^{3,1}$	$\mathcal{M}'^{0,0}$	$\mathcal{M}'^{0,1}$	$\mathcal{M}'^{1,1}$
# instances	46	47	50	47	50	49	50	49	41	48	50
# optimal	21	28	45	39	45	39	45	38	29	27	34
gap	0.268	0.239	0.015	0.016	0.011	0.024	0.012	0.026	0.158	0.248	0.026
CPU time	4324.69	3544.65	1036.82	1818.23	841.08	1891.40	1177.04	2053.82	2284.94	3525.66	2693.76

Table 5
Comparison of models on each instance class - integrality requirement on variables $z_{m,\, i,\, n,\, j}$ imposed.

$\lvert N\rvert \times \lvert I\rvert$	$\mathcal{M}^{0,1}$			$\mathcal{M}^{2,0}$			$\mathcal{M}^{3,0}$			$\mathcal{M}'^{1,1}$		
	value	time	gap	value	time	gap	value	time	gap	value	time	gap
8x4	5120.8	0.70	0.000	5120.8	0.15	0.000	5120.8	0.35	0.000	5120.8	2.49	0.000
9x4	5978.2	1.25	0.000	5978.2	0.21	0.000	5978.2	0.54	0.000	5978.2	4.64	0.000
10x4	6319.8	1.67	0.000	6319.8	0.38	0.000	6319.8	0.93	0.000	6319.8	16.38	0.000
10x5	6427.8	6.31	0.000	6427.8	1.00	0.000	6427.8	3.17	0.000	6427.8	98.49	0.000
11x5	7555.6	6.04	0.000	7555.6	1.64	0.000	7555.6	4.24	0.000	7555.6	210.63	0.000
12x5	7970.2	14.40	0.000	7970.2	3.67	0.000	7970.2	14.32	0.000	6280.4	898.74	0.000
12x6	10449.8	87.18	0.000	10449.8	26.09	0.000	10449.8	80.48	0.000	10449.8	4106.28	0.004
15x6	13756.4	648.34	0.000	13756.4	132.93	0.000	13756.4	622.78	0.000	13867.6	7200.00	0.035
15x7	14688.8	2402.35	0.000	14688.8	1044.73	0.000	14688.8	3843.62	0.000	14965.0	7200.00	0.065
20x10	30165.4	7200.00	0.145	29828.2	7200.00	0.105	30004.2	7200.00	0.124	31067.2	7200.00	0.155

ity of $\mathcal{M}^{2',0}$ over $\mathcal{M}^{3',0}$ in terms of CPU time consumed is established by a Wilkoxon signed rank test [48] which yields a p-value < 0.0001 (i.e., $p = 5.3e^{-6}$). In view of these observations we may say that the model $\mathcal{M}^{2',0}$ is better than any other model compared, especially if a high quality solution is sought in a short time.

5.2. Comparison of models - integrality requirement imposed on variables $z_{m,\, i,\, n,\, j}$

Similar to our analysis of Tables 2 and 3, in Tables 4 and 5 we again compare the preceding models but now with the integrality requirement imposed on the variables $z_{m,\, i,\, n,\, j}$. The detailed results may be found in Appendix B.

The results presented in Table 4 show that only 4 of 11 models enable CPLEX to provide a solution for each instance in the data set. These four models are: $\mathcal{M}^{1,0}$, $\mathcal{M}^{2,0}$, $\mathcal{M}^{3,0}$ and $\mathcal{M}'^{1,1}$. Of these, model $\mathcal{M}'^{1,1}$ enabled 34 instances to be solved to optimality, while the remaining three enabled 45 instances to be solved. More precisely, just on the class containing the largest instances, models $\mathcal{M}^{1,0}$, $\mathcal{M}^{2,0}$, $\mathcal{M}^{3,0}$ failed to find optimal solutions and the best performance in terms of solution quality is exhibited by model $\mathcal{M}^{2,0}$ (see Table 5). Further, if we compare the average optimality gap attained by using these 4 models, we see that the least average optimality gap is provided by $\mathcal{M}^{2,0}$ (0.011%), while model $\mathcal{M}'^{1,1}$ yields the largest average optimality gap (0.024%). From the 95% confidence interval plot of the optimality gaps in Fig. 5, we observe that there is no significant difference among these four models. Comparing the average CPU time consumed to solve an instance, model $\mathcal{M}^{2,0}$ yields the least average CPU time consumed (841.08) which is significantly less than the average CPU time consumed when using model $\mathcal{M}^{1,0}$ (1036.82), the second best of the models by this criterion. To further verify that $\mathcal{M}^{2,0}$ is best in terms of solution quality and solution time performance, in Figs. 6 and 7 we draw the upper bound and CPU time performance profiles of models $\mathcal{M}^{1,0}$, $\mathcal{M}^{2,0}$, $\mathcal{M}^{3,0}$ and $\mathcal{M}'^{1,1}$ using the approach described in the preceding section. These figures show that the graphs representing the upper bound and CPU time performance profiles of model $\mathcal{M}^{2,0}$ lie above the others. The superiority of model $\mathcal{M}^{2,0}$ over models $\mathcal{M}^{1,0}$ and $\mathcal{M}^{3,0}$, the closest competitors in terms of CPU time consumption is confirmed by the Wilcoxon signed rank test which yields p-values of $5.48e^{-8}$ and $5.18e^{-9}$ by comparing $\mathcal{M}^{2,0}$ and $\mathcal{M}^{1,0}$, and $\mathcal{M}^{2,0}$ and $\mathcal{M}^{3,0}$, respectively. On the other hand, we recall that, in terms of the number of solved instances, the models $\mathcal{M}^{0,0}$ and $\mathcal{M}'^{0,0}$ were the two worst, enabling CPLEX to solve only 38 and 41 instances, respectively.

The comparison results in the tables above lead to some interesting observations. We see that after relaxing the integrality restrictions on the variables $z_{m,\, i,\, n,\, j}$, model $\mathcal{M}^{1',0}$ is less efficient(causes CPLEX to perform less efficiently) than the corresponding model $\mathcal{M}^{1,0}$. However, model $\mathcal{M}^{2',0}$ is better than its corresponding model $\mathcal{M}^{2,0}$ regarding both solution quality and CPU time consumed. These observations lead to the conclusion that is difficult to say in the case of certain models whether it is better to relax the integrality requirement for some variables or not. Interestingly, however, as observe that the standard linear MIP formulation $\mathcal{M}^{0,0}$ is the weakest. It consumes a substantial amount of CPU time even for the simple instances, and additionally consumes a lot of memory for some instances. The associated MIP formulation $\mathcal{M}^{0',0}$ behaves the same way in terms of memory consumption but is slightly faster in terms of running times. In sum, we conclude that the model $\mathcal{M}^{2',0}$ is the best among those considered in this paper, both with and without relaxing the integrality requirement. We emphasize once again that to the best of our knowledge, the "winning" models $\mathcal{M}^{2',0}$ and $\mathcal{M}^{2,0}$ are considered here for the first time.

The LP relaxations of models $\mathcal{M}^{0,0}$, $\mathcal{M}^{0,1}$, $\mathcal{M}^{0',0}$, $\mathcal{M}^{0',1}$ are the weakest, yielding an LP relaxation value of zero on all instances. The average LP relaxation values of the remaining models as well as the average CPU times needed to obtain these values, over entire set of instances, are given in Table 6. As we can see the model $\mathcal{M}^{2,0}$ exhibits the best compromise between LP relaxation value and CPU time consumption. This may explain why models $\mathcal{M}^{2',0}$ and $\mathcal{M}^{2,0}$ are the best. In addition, the results reported in Table 6 suggest that the behaviour of the models detected in this paper may be very similar to the behaviour when some other MIP solver is used. The models $\mathcal{M}^{0,0}$, $\mathcal{M}^{0,1}$, $\mathcal{M}^{0',0}$, $\mathcal{M}^{0',1}$ would be most likely the worst, while the models $\mathcal{M}^{2',0}$, $\mathcal{M}^{2,0}$, $\mathcal{M}^{3,0}$, and $\mathcal{M}^{3',0}$ and $\mathcal{M}^{1,0}$ would be most likely among the best.

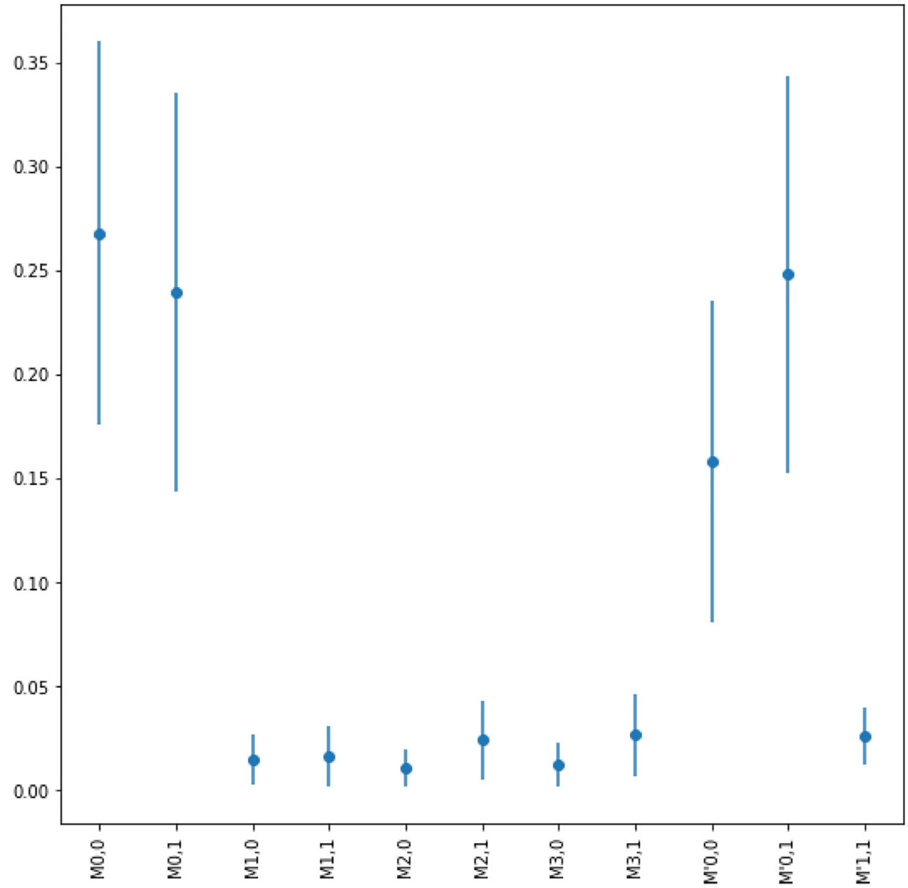

Fig. 5. 95% confidence interval plot of the optimality gap-integrality requirement on variables $z_{m,\,i,\,n,\,j}$ imposed.

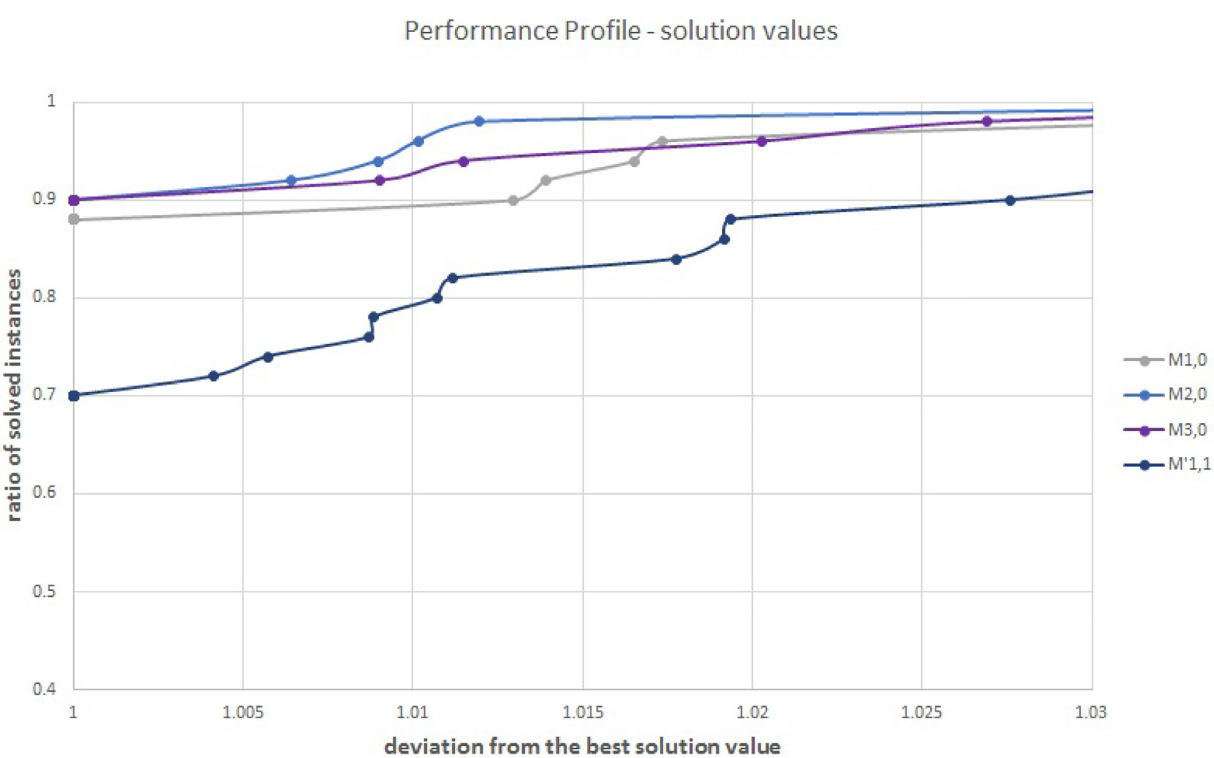

Fig. 6. Performance profile-solution values: integrality requirement imposed on variables $z_{m,\,i,\,n,\,j}$.

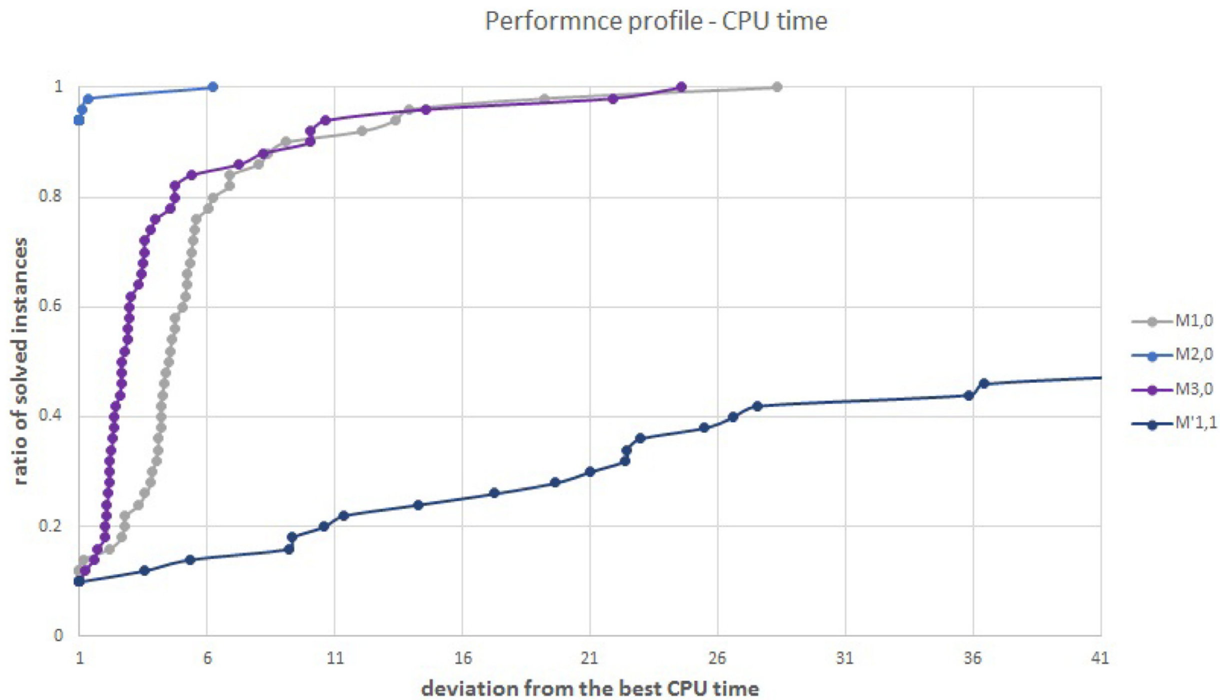

Fig. 7. Performance profile-CPU times: integrality requirement imposed on variables $z_{m,\,i,\,n,\,j}$.

Table 6
Comparison of LP relaxations.

	$\mathcal{M}^{1,0}$	$\mathcal{M}^{1,1}$	$\mathcal{M}^{2,0}$	$\mathcal{M}^{2,1}$	$\mathcal{M}^{3,0}$	$\mathcal{M}^{3,1}$	$\mathcal{M}'^{1,1}$
LP value	9627	10006	9627	10028	9627	10028	10004
CPU time	0.67	13.02	0.21	7.37	0.33	7.52	7.88

6. Conclusion

Our study of the Cross Docking Assignment Problem (CDAP) starts from the standard quadratic formulation of the problem and derives 11 nonstandard linear mixed integer programming (MIP) models for the CDAP. Eight of the 11 proposed MIP models are considered in this paper for the first time. We prove the equivalence of all these models, with an integrality requirement imposed on the z variables, in the sense of admitting the same feasible and optimal solutions. We also establish results about the integrality properties of these models. These results further imply the equivalence of the models that have relaxed integrality requirement on the z variables.

To detect the best model among these 11, an exhaustive empirical study has been performed on benchmark instances from the literature, applying the CPLEX MIP software to compare the formulations in terms of the number of instances they enable to be solved to optimality, upper bounds they provide, and the CPU time consumed. The results reveal that the best model is one of the eight MIP formulations proposed for the first time in this paper.

However, the challenge remains to identify an effective solution algorithm and model formulation for handling large scale instances whose solution remains elusive. A possible future research direction is to propose a hybrid approach that combines the best model from those identified in this paper with an existing or newly proposed heuristic algorithm.

Models considered in this study are applicable to pure dock-to-dock cross-docks with fixed mode dock-doors in a pre-distribution environment without arrival and departure restrictions. Hence, a possible future research direction is to consider a less restrictive model that also takes into account availability of trucks, stochasticity of arrivals, uncertainty in contents of trucks, and state of digitization required.

Future work may also include adapting the models we have presented to handle other cross-dock shapes and to carry out associated theoretical and empirical analyses.

Acknowledgement

We are indebted to Monique Guignard for kindly providing us with benchmark instances. This work has been supported by ELSAT project, which is co-financed by the European Union with the European Regional Development Fund, the French state and the Hauts de France Region Council as well as by the Gaspard Monge Programme for Optimization and Operational Research (PGMO) in the framework of BENMIP project.

Appendix

In Appendices A and B we provide detailed results on entire data sets for all models studied in this paper. The sign '-' in the tables imply that CPLEX could not provide a feasible solution within the imposed time limit. In Appendix C we perform Wilkoxon signed rank statistical tests for all of the models both for solution quality and runtime.

Appendix A. Results of MIP models for CDAP: Integrality requirement on variables $z_{m,\,i,\,n,\,j}$ relaxed

A.1. Detailed results of MIP models $\mathcal{M}'^{0,0}$, $\mathcal{M}'^{0,1}$, $\mathcal{M}'^{1,0}$, $\mathcal{M}'^{1,1}$, $\mathcal{M}'^{2,0}$ and $\mathcal{M}'^{2,1}$ for each instance and each class of instances

Table A1
MIP Models $\mathcal{M}'^{0,0}$, $\mathcal{M}'^{0,1}$, $\mathcal{M}'^{1,0}$, $\mathcal{M}'^{1,1}$, $\mathcal{M}'^{2,0}$ and $\mathcal{M}'^{2,1}$.

Instances	$\mathcal{M}^{0',0}$			$\mathcal{M}^{0',1}$			$\mathcal{M}^{1',0}$			$\mathcal{M}^{1',1}$			$\mathcal{M}^{2',0}$			$\mathcal{M}^{2',1}$		
	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap
8x4S30	5063	9.380	0.000	5063	5.390	0.000	5063	1.422	0.000	5063	6.718	0.000	5063	0.188	0.000	5063	1.516	0.000
8x4S20	5086	7.094	0.000	5086	3.610	0.000	5086	0.922	0.000	5086	5.704	0.000	5086	0.156	0.000	5086	1.125	0.000
8x4S15	5112	1.310	0.000	5112	2.540	0.000	5112	1.109	0.000	5112	8.671	0.000	5112	0.234	0.000	5112	2.063	0.000
8x4S10	5169	1.440	0.000	5169	2.047	0.000	5169	1.157	0.000	5169	10.469	0.000	5169	0.219	0.000	5169	3.390	0.000
8x4S5	5174	1.200	0.000	5174	1.016	0.000	5174	1.140	0.000	5174	14.297	0.000	5174	0.281	0.000	5174	2.563	0.000
8x4S{30,20, 15,10,5}	**5120.8**	**4.085**	**0.000**	**5120.8**	**2.921**	**0.000**	**5120.8**	**1.150**	**0.000**	**5120.8**	**9.172**	**0.000**	**5120.8**	**0.216**	**0.000**	**5120.8**	**2.131**	**0.000**
9x4S30	5904	60.650	0.000	5904	12.312	0.000	5904	1.282	0.000	5904	10.828	0.000	5904	0.230	0.000	5904	2.547	0.000
9x4S20	5937	14.310	0.000	5937	8.078	0.000	5937	1.406	0.000	5937	14.375	0.000	5937	0.120	0.000	5937	2.500	0.000
9x4S15	5976	3.890	0.000	5976	3.454	0.000	5976	1.300	0.000	5976	15.219	0.000	5976	0.140	0.000	5976	3.484	0.000
9x4S10	6027	3.760	0.000	6027	3.218	0.000	6027	1.359	0.000	6027	18.016	0.000	6027	0.300	0.000	6027	3.547	0.000
9x4S5	6047	1.920	0.000	6047	2.031	0.000	6047	1.875	0.000	6047	19.812	0.000	6047	0.230	0.000	6047	9.485	0.000
9x4S{30,20, 15,10,5}	**5978.2**	**16.906**	**0.000**	**5978.2**	**5.819**	**0.000**	**5978.2**	**1.444**	**0.000**	**5978.2**	**15.650**	**0.000**	**5978.2**	**0.204**	**0.000**	**5978.2**	**4.313**	**0.000**
10x4S30	6193	630.450	0.000	6193	35.562	0.000	6193	2.328	0.000	6193	28.125	0.000	6193	0.300	0.000	6193	3.434	0.000
10x4S20	6267	486.640	0.000	6267	43.078	0.000	6267	2.703	0.000	6267	36.828	0.000	6267	0.310	0.000	6267	4.734	0.000
10x4S15	6296	165.090	0.000	6296	37.188	0.000	6296	2.453	0.000	6296	28.219	0.000	6296	0.250	0.000	6296	4.235	0.000
10x4S10	6325	49.250	0.000	6325	22.156	0.000	6325	2.610	0.000	6325	65.469	0.000	6325	0.330	0.000	6325	5.578	0.000
10x4S5	6518	5.360	0.000	6518	3.422	0.000	6518	3.703	0.000	6518	130.781	0.000	6518	0.530	0.000	6518	12.563	0.000
10x4S{30,20, 15,10,5}	**6319.8**	**267.358**	**0.000**	**6319.8**	**28.281**	**0.000**	**6319.8**	**2.759**	**0.000**	**6319.8**	**57.884**	**0.000**	**6319.8**	**0.344**	**0.000**	**6319.8**	**6.109**	**0.000**
10x5S30	6308	7200.000	0.108	6308	585.235	0.000	6308	8.125	0.000	6308	164.312	0.000	6308	0.500	0.000	6308	21.234	0.000
10x5S20	6342	7200.000	0.057	6342	501.547	0.000	6342	7.781	0.000	6342	127.420	0.000	6342	0.940	0.000	6342	30.579	0.000
10x5S15	6397	6265.420	0.000	6397	162.531	0.000	6397	8.406	0.000	6397	289.969	0.000	6397	0.780	0.000	6397	47.547	0.000
10x5S10	6476	2341.390	0.000	6476	200.765	0.000	6476	8.250	0.000	6476	632.828	0.000	6476	0.780	0.000	6476	69.812	0.000
10x5S5	6616	70.670	0.000	6616	36.485	0.000	6616	11.063	0.000	6616	2236.328	0.000	6616	0.840	0.000	6616	88.797	0.000
10x5S{30,20, 15,10,5}	**6427.8**	**4615.496**	**0.033**	**6427.8**	**297.313**	**0.000**	**6427.8**	**8.725**	**0.000**	**6427.8**	**690.171**	**0.000**	**6427.8**	**0.768**	**0.000**	**6427.8**	**51.594**	**0.000**
11x5S30	7443	7200.000	0.281	7420	4581.484	0.000	7420	10.220	0.000	7420	436.484	0.000	7420	0.970	0.000	7420	59.078	0.000
11x5S20	7475	7200.000	0.205	7439	2218.235	0.000	7439	8.047	0.000	7439	279.266	0.000	7439	0.860	0.000	7439	30.797	0.000
11x5S15	7543	7200.000	0.196	7535	699.515	0.000	7535	14.233	0.000	7535	585.766	0.000	7535	1.640	0.000	7535	62.984	0.000
11x5S10	7572	7200.000	0.028	7572	341.891	0.000	7572	17.375	0.000	7572	966.000	0.000	7572	1.660	0.000	7572	204.937	0.000
11x5S5	7812	1533.420	0.000	7812	158.970	0.000	7812	22.734	0.000	7812	7.200.000	0.039	7812	1.880	0.000	7812	995.375	0.000
11x5S{30,20, 15,10,5}	**7569.0**	**6066.684**	**0.142**	**7555.6**	**1600.019**	**0.000**	**7555.6**	**14.522**	**0.000**	**7555.6**	**1893.503**	**0.008**	**7555.6**	**1.402**	**0.000**	**7555.6**	**270.634**	**0.000**
12x5S30	8009	7200.000	0.408	7923	7200.000	0.205	7923	150.469	0.000	7923	4174.328	0.000	7923	3.220	0.000	7923	859.704	0.000
12x5S20	7991	7200.000	0.344	7939	7200.000	0.090	7939	48.609	0.000	7939	1154.922	0.000	7939	2.240	0.000	7939	108.110	0.000
12x5S15	7990	7200.000	0.212	7939	7200.000	0.123	7939	29.891	0.000	7939	626.469	0.000	7939	2.280	0.000	7939	78.328	0.000
12x5S10	8003	7200.000	0.226	7991	7200.000	0.126	7978	47.875	0.000	7978	1038.156	0.000	7978	2.730	0.000	7978	115.250	0.000
12x5S5	8072	6253.406	0.000	8072	390,000	0.000	8072	31.344	0.000	8110	7200.000	0.018	8072	3.300	0.000	8072	406.765	0.000
12x5S{30,20, 15,10,5}	**8013.0**	**7010.681**	**0.238**	**7972.8**	**5838.016**	**0.109**	**7970.2**	**61.638**	**0.000**	**7977.8**	**2838.775**	**0.004**	**7970.2**	**2.754**	**0.000**	**7970.2**	**313.631**	**0.000**
12x6S30	10,228	7200.000	0.416	10,228	7200.000	0.315	10,228	518.766	0.000	10,303	7200.000	0.027	10,228	26.950	0.000	10,228	2350.593	0.000
12x6S20	10,323	7200.000	0.390	10,325	7200.000	0.150	10,312	419.500	0.000	10,377	7200.000	0.031	10,312	9.450	0.000	10,312	2745.188	0.000
12x6S15	10,462	7200.000	0.267	10,362	4331.140	0.000	10,362	313.610	0.000	10,388	7200.000	0.030	10,362	7.050	0.000	10,362	1220.735	0.000
12x6S10	10,534	7200.000	0.286	10,456	6334.672	0.000	10,456	458.340	0.000	10,579	7200.000	0.049	10,456	12.050	0.000	10,456	4438.219	0.000
12x6S5	10,891	699.047	0.000	10,891	532.125	0.000	10,891	461.250	0.000	11,091	7200.000	0.093	10,891	4.980	0.000	10,907	7.200.000	0.030
12x6S{30,20, 15,10,5}	**10487.6**	**5899.809**	**0.272**	**10452.4**	**5119.587**	**0.093**	**10449.8**	**434.293**	**0.000**	**10547.6**	**7200.000**	**0.046**	**10449.8**	**12.096**	**0.000**	**10453.0**	**3590.947**	**0.006**
15x6S30	13,848	7200.000	0.670	13,638	7200.000	0.500	13,567	3701.510	0.000	13,705	7200.000	0.037	13,567	26.390	0.000	13,656	7.200.000	0.027
15x6S20	13,802	7200.000	0.592	13,750	7200.000	0.513	13,720	7200.000	0.002	13,871	7200.000	0.050	13,720	68.500	0.000	13,799	7.200.000	0.033
15x6S15	14,003	7200.000	0.745	13,814	7200.000	0.441	13,765	7200.000	0.005	13,991	7200.000	0.054	13,765	96.470	0.000	13,956	7.200.000	0.040
15x6S10	-	-	-	13,803	7200.000	0.441	13,803	6,834,000	0.000	14,006	7200.000	0.049	13,803	24.390	0.000	13,923	7.200.000	0.329
15x6S5	-	-	-	14,093	7200.000	0.604	13,927	4,457,000	0.000	14,452	7200.000	0.083	13,927	92.030	0.000	14,085	7.200.000	0.036
15x6S{30,20, 15,10,5}	**-**	**-**	**-**	**13819.6**	**7200.000**	**0.500**	**13756.4**	**5878.424**	**0.001**	**14005.0**	**7200.000**	**0.054**	**13756.4**	**61.556**	**0.000**	**13883.8**	**7200.000**	**0.093**
15x7S30	-	-	-	14,488	7200.000	0.480	14,415	7200.000	0.022	14,713	7200.000	0.071	14,409	70.950	0.000	14,491	7.200.000	0.041
15x7S20	-	-	-	14,656	7200.000	0.499	14,533	7200.000	0.023	14,973	7200.000	0.083	14,514	62.940	0.000	14,760	7.200.000	0.041
15x7S15	-	-	-	14,745	7200.000	0.572	14,680	7200.000	0.030	14,829	7200.000	0.074	14,657	283.130	0.000	14,915	7.200.000	0.066
15x7S10	-	-	-	14,814	7200.000	0.438	14,824	7200.000	0.034	15,746	7200.000	0.126	14,810	208.470	0.000	14,958	7.200.000	0.058
15x7S5	-	-	-	15,228	7200.000	0.630	15,077	7200.000	0.031	-	-	-	15,054	245.170	0.000	16,184	7.200.000	0.137
15x7S{30,20, 15,10,5}	**-**	**-**	**-**	**14786.2**	**7200.000**	**0.524**	**14705.8**	**7200.000**	**0.028**	**-**	**-**	**-**	**14688.8**	**174.132**	**0.000**	**15061.6**	**7200.000**	**0.069**
20x10S30	-	-	-	29,158	7200.000	0.917	29,158	7200.000	0.153	33,038	7200.000	0.221	28,943	7.200.000	0.091	34,546	7.200.000	0.254
20x10S20	-	-	-	29,695	7200.000	0.870	29,657	7200.000	0.168	34,362	7200.000	0.247	29,314	7.200.000	0.094	35,563	7.200.000	0.270
20x10S15	-	-	-	29,594	7200.000	0.905	29,719	7200.000	0.171	-	-	-	29,416	7.200.000	0.095	34,497	7.200.000	0.245
20x10S10	-	-	-	30,319	7200.000	0.926	29,827	7200.000	0.172	-	-	-	29,776	7.200.000	0.110	-	-	-
20x10S5	-	-	-	30,581	7200.000	0.892	31,159	7200.000	0.205	-	-	-	30,563	7.200.000	0.114	34,211	7.200.000	0.231
20x10S{30,20, 15,10,5}	**-**	**-**	**-**	**29869.4**	**7200.000**	**0.902**	**29904.0**	**7200.000**	**0.174**	**-**	**-**	**-**	**29602.4**	**7200.000**	**0.101**	**-**	**-**	**-**
Average	**-**	**-**	**-**	**10830,26**	**3449.196**	**0.213**	**10818.84**	**2080.296**	**0.020**	**-**	**-**	**-**	**10786.98**	**745.347**	**0.010**	**-**	**-**	**-**

A.2. Detailed results of MIP models $\mathcal{M}^{3',0}$, $\mathcal{M}^{3',1}$, $\mathcal{M}'^{0',0}$, $\mathcal{M}'^{0',1}$ and $\mathcal{M}'^{1',1}$ for each instance and each class of instances

Table A2
MIPs Models $\mathcal{M}^{3',0}$, $\mathcal{M}^{3',1}$, $\mathcal{M}'^{0',0}$, $\mathcal{M}'^{0',1}$ and $\mathcal{M}'^{1',1}$.

Instances	$\mathcal{M}^{3',0}$			$\mathcal{M}^{3',1}$			$\mathcal{M}'^{0',0}$			$\mathcal{M}'^{0',1}$			$\mathcal{M}'^{1',1}$		
	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap
8x4S30	5063	0.250	0.000	5063	2.297	0.000	5063	0.688	0.000	5063	2.843	0.000	5063	2.266	0.000
8x4S20	5086	0.187	0.000	5086	2.390	0.000	5086	0.218	0.000	5086	2.359	0.000	5086	2.047	0.000
8x4S15	5112	0.219	0.000	5112	3.375	0.000	5112	0.157	0.000	5112	1.391	0.000	5112	2.828	0.000
8x4S10	5169	0.250	0.000	5169	5.454	0.000	5169	0.156	0.000	5169	1.297	0.000	5169	4.188	0.000
8x4S5	5174	0.250	0.000	5174	4.750	0.000	5174	0.110	0.000	5174	0.563	0.000	5174	4.078	0.000
8x4S{30,20,15,10,5}	**5120.8**	**0.231**	**0.000**	**5120.8**	**3.653**	**0.000**	**5120.8**	**0.266**	**0.000**	**5120.8**	**1.691**	**0.000**	**5120.8**	**3.081**	**0.000**
9x4S30	5904	0.422	0.000	5904	3.687	0.000	5904	0.875	0.000	5904	7.094	0.000	5904	3.547	0.000
9x4S20	5937	0.375	0.000	5937	3.328	0.000	5937	0.344	0.000	5937	3.859	0.000	5937	3.609	0.000
9x4S15	5976	0.282	0.000	5976	4.516	0.000	5976	0.297	0.000	5976	3.891	0.000	5976	4.641	0.000
9x4S10	6027	0.391	0.000	6027	10.500	0.000	6027	0.265	0.000	6027	1.671	0.000	6027	5.281	0.000
9x4S5	6047	0.422	0.000	6047	8.672	0.000	6047	0.219	0.000	6047	1.282	0.000	6047	6.437	0.000
9x4S{30,20,15,10,5}	**5978.2**	**0.378**	**0.000**	**5978.2**	**6.141**	**0.000**	**5978.2**	**0.400**	**0.000**	**5978.2**	**3.559**	**0.000**	**5978.2**	**4.703**	**0.000**
10x4S30	6193	0.469	0.000	6193	2.844	0.000	6193	3.969	0.000	6193	27.078	0.000	6193	4.703	0.000
10x4S20	6267	0.531	0.000	6267	4.720	0.000	6267	2.125	0.000	6267	21.937	0.000	6267	8.423	0.000
10x4S15	6296	0.422	0.000	6296	7.390	0.000	6296	1.797	0.000	6296	15.390	0.000	6296	7.282	0.000
10x4S10	6325	0.453	0.000	6325	9.547	0.000	6325	1.421	0.000	6325	9.797	0.000	6325	11.203	0.000
10x4S5	6518	0.781	0.000	6518	23.375	0.000	6518	0.344	0.000	6518	3.688	0.000	6518	28.343	0.000
10x4S{30,20,15,10,5}	**6319.8**	**0.531**	**0.000**	**6319.8**	**9.575**	**0.000**	**6319.8**	**1.931**	**0.000**	**6319.8**	**15.578**	**0.000**	**6319.8**	**11.991**	**0.000**
10x5S30	6308	0.843	0.000	6308	44.578	0.000	6308	22.125	0.000	6308	299.891	0.000	6308	23.438	0.000
10x5S20	6342	0.937	0.000	6342	39.250	0.000	6342	26.063	0.000	6342	64.266	0.000	6342	40.156	0.000
10x5S15	6397	1.094	0.000	6397	54.500	0.000	6397	17.515	0.000	6397	91.000	0.000	6397	66.703	0.000
10x5S10	6476	1.406	0.000	6476	104.344	0.000	6476	6.985	0.000	6476	93.093	0.000	6476	105.453	0.000
10x5S5	6616	1.625	0.000	6616	348.344	0.000	6616	1.078	0.000	6616	11.265	0.000	6616	251.454	0.000
10x5S{30,20,15,10,5}	**6427.8**	**1.181**	**0.000**	**6427.8**	**118.203**	**0.000**	**6427.8**	**14.753**	**0.000**	**6427.8**	**111.903**	**0.000**	**6427.8**	**97.441**	**0.000**
11x5S30	7420	1.422	0.000	7420	75.922	0.000	7420	1055.984	0.000	7420	1683.297	0.000	7420	47.031	0.000
11x5S20	7439	1.344	0.000	7439	102.344	0.000	7439	281.672	0.000	7439	742.485	0.000	7439	42.547	0.000
11x5S15	7535	2.031	0.000	7535	199.906	0.000	7535	37.734	0.000	7535	354.220	0.000	7535	93.218	0.000
11x5S10	7572	1.922	0.000	7572	362.500	0.000	7572	25.610	0.000	7572	42.453	0.000	7572	261.047	0.000
11x5S5	7812	3.000	0.000	7812	1229.484	0.000	7812	7.312	0.000	7812	41.672	0.000	7812	2923.250	0.000
11x5S{30,20,15,10,5}	**7555.6**	**1.944**	**0.000**	**7555.6**	**394.031**	**0.000**	**7555.6**	**281.662**	**0.000**	**7555.6**	**572.825**	**0.000**	**7555.6**	**673.419**	**0.000**
12x5S30	7923	5.984	0.000	7923	597.656	0.000	7965	7200.000	0.139	7944	7200.000	0.127	7923	1137.219	0.000
12x5S20	7939	3.094	0.000	7939	779.641	0.000	7939	2186.516	0.000	7961	7200.000	0.127	7939	335.266	0.000
12x5S15	7939	2.547	0.000	7939	233.328	0.000	7939	3488.593	0.000	7939	7200.000	0.128	7939	151.875	0.000
12x5S10	7978	2.985	0.000	7978	114.125	0.000	-	-	-	7978	7200.000	0.152	7978	261.656	0.000
12x5S5	8072	3.266	0.000	8072	384.953	0.000	8072	1774.141	0.000	8072	156.062	0.000	8072	1863.422	0.000
12x5S{30,20,15,10,5}	**7970.2**	**3.575**	**0.000**	**7970.2**	**421.941**	**0.000**	**-**	**-**	**-**	**7978.8**	**5791.212**	**0.107**	**7970.2**	**749.888**	**0.000**
12x6S30	10,228	8.922	0.000	10,228	2756.407	0.000	-	-	-	10,296	7200.000	0.112	10,228	3984.922	0.000
12x6S20	10,312	6.704	0.000	10,312	3082.359	0.000	-	-	-	10,369	7200.000	0.170	10,312	3178.234	0.000
12x6S15	10,362	8.328	0.000	10,362	3290.953	0.000	-	-	-	10,362	3208.406	0.000	10,362	4246.547	0.000
12x6S10	10,456	8.875	0.000	10,456	5610.969	0.000	-	-	-	10,456	4531.797	0.000	10,456	5785.547	0.000
12x6S5	10,891	35.922	0.000	10,982	7200.000	0.051	10,891	10,000	0.000	10,891	1135.360	0.000	10,904	7200.000	0.073
12x6S{30,20,15,10,5}	**10449.8**	**13.750**	**0.000**	**10468.0**	**4388.138**	**0.010**	**-**	**-**	**-**	**10474.8**	**4655.113**	**0.056**	**10452.4**	**4879.050**	**0.015**
15x6S30	13,567	149.906	0.000	13,679	7200.000	0.025	13,776	7200.000	0.523	13,567	7200.000	0.478	13,687	7200.000	0.038
15x6S20	13,720	112.750	0.000	13,750	7200.000	0.022	13,922	7200.000	0.468	13,869	7200.000	0.469	13,779	7200.000	0.037
15x6S15	13,765	158.266	0.000	13,805	7200.000	0.024	13,920	7200.000	0.404	13,856	7200.000	0.421	13,793	7200.000	0.035
15x6S10	13,803	112.562	0.000	13,843	7200.000	0.020	-	-	-	13,931	7200.000	0.414	13,849	7200.000	0.035
15x6S5	13,927	107.781	0.000	14,258	7200.000	0.058	13,927	7200.000	0.060	14,024	7200.000	0.477	14,105	7200.000	0.053
15x6S{30,20,15,10,5}	**13756.4**	**128.253**	**0.000**	**13867.0**	**7200.000**	**0.030**	**-**	**-**	**-**	**13849.4**	**7200.000**	**0.452**	**13842.6**	**7200.000**	**0.040**
15x7S30	14,409	306.203	0.000	14,634	7200.000	0.060	-	-	-	14,579	7200.000	0.481	14,491	7200.000	0.046
15x7S20	14,514	259.125	0.000	14,724	7200.000	0.051	-	-	-	14,630	7200.000	0.496	14,652	7200.000	0.051
15x7S15	14,657	303.813	0.000	14,862	7200.000	0.053	14,848	7200.000	0.550	14,682	7200.000	0.398	14,711	7200.000	0.052
15x7S10	14,810	313.516	0.000	15,141	7200.000	0.076	-	-	-	14,810	7200.000	0.408	14,956	7200.000	0.065
15x7S5	15,054	492.000	0.000	15,708	7200.000	0.104	15,128	7200.000	0.492	15,108	7200.000	0.449	15,370	7200.000	0.089
15x7S{30,20,15,10,5}	**14688.8**	**334.931**	**0.000**	**15013.8**	**7200.000**	**0.069**	**-**	**-**	**-**	**14761.8**	**7200.000**	**0.446**	**14836.0**	**7200.000**	**0.061**
20x10S30	29,028	7200.000	0.103	34,089	7200.000	0.244	29,086	7200.000	0.826	28,786	7200.000	0.861	31,927	7200.000	0.194
20x10S20	29,232	7200.000	0.096	34,421	7200.000	0.246	29,710	7200.000	0.821	29,581	7200.000	0.881	33,059	7200.000	0.217
20x10S15	29,666	7200.000	0.103	-	-	-	29,753	7200.000	0.814	29,444	7200.000	0.864	31,783	7200.000	0.183
20x10S10	29,687	7200.000	0.089	-	-	-	29,916	7200.000	0.805	29,889	7200.000	0.870	34,723	7200.000	0.250
20x10S5	30,594	7200.000	0.114	-	-	-	30,484	7200.000	0.828	30,491	7200.000	0.890	34,294	7200.000	0.237
20x10S{30,20,15,10,5}	**29641.4**	**7200.000**	**0.101**	**-**	**-**	**-**	**29789.8**	**7200.000**	**0.819**	**29638.2**	**7200.000**	**0.873**	**33157.2**	**7200.000**	**0.216**
Average	**10790.88**	**768.478**	**0.010**	**-**	**-**	**-**	-	**-**	**-**	**10810.52**	**3275.188**	**0.193**	**11166.060**	**2801.957**	**0.033**

Appendix B. Results of MIP models for CDAP: Integrality requirement on variables $z_{m,i,n,j}$ imposed

B.1. Detailed results of MIP models $\mathcal{M}^{0,0}, \mathcal{M}^{0,1}, \mathcal{M}^{1,0}, \mathcal{M}^{1,1}, \mathcal{M}^{2,0}$ for each instance and each class of instances

Table B1
Linear Models $\mathcal{M}^{0,0}, \mathcal{M}^{0,1}, \mathcal{M}^{1,0}, \mathcal{M}^{1,1}$ and $\mathcal{M}^{2,0}$.

Instances	$\mathcal{M}^{0,0}$			$\mathcal{M}^{0,1}$			$\mathcal{M}^{1,0}$			$\mathcal{M}^{1,1}$			$\mathcal{M}^{2,0}$		
	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap
8x4S30	5063	8.140	0.000	5063	4.590	0.000	5063	0.760	0.000	5063	4.688	0.000	5063	0.160	0.000
8x4S20	5086	7.040	0.000	5086	1.968	0.000	5086	0.560	0.000	5086	2.156	0.000	5086	0.140	0.000
8x4S15	5112	1.281	0.000	5112	1.812	0.000	5112	0.610	0.000	5112	3.094	0.000	5112	0.140	0.000
8x4S10	5169	1.320	0.000	5169	2.078	0.000	5169	0.720	0.000	5169	4.250	0.000	5169	0.140	0.000
8x4S5	5174	1.150	0.000	5174	1.204	0.000	5174	0.830	0.000	5174	3.690	0.000	5174	0.160	0.000
8x4S{30,20,15,10,5}	**5120.8**	**3.786**	**0.000**	**5120.8**	**2.330**	**0.000**	**5120.8**	**0.696**	**0.000**	**5120.8**	**3.576**	**0.000**	**5120.8**	**0.148**	**0.000**
9x4S30	5904	60.970	0.000	5904	47.046	0.000	5904	1.170	0.000	5904	11.719	0.000	5904	0.280	0.000
9x4S20	5937	22.450	0.000	5937	15.172	0.000	5937	1.310	0.000	5937	5.953	0.000	5937	0.190	0.000
9x4S15	5976	3.890	0.000	5976	11.719	0.000	5976	1.450	0.000	5976	5.375	0.000	5976	0.160	0.000
9x4S10	6027	4.360	0.000	6027	3.563	0.000	6027	1.280	0.000	6027	9.812	0.000	6027	0.230	0.000
9x4S5	6047	2.560	0.000	6047	2.296	0.000	6047	1.050	0.000	6047	9.406	0.000	6047	0.190	0.000
9x4S{30,20,15,10,5}	**5978.2**	**18.846**	**0.000**	**5978.2**	**15.959**	**0.000**	**5978.2**	**1.252**	**0.000**	**5978.2**	**8.453**	**0.000**	**5978.2**	**0.210**	**0.000**
10x4S30	6193	486.840	0.000	6193	1785.390	0.000	6193	1.300	0.000	6193	14.266	0.000	6193	0.280	0.000
10x4S20	6267	2226.250	0.000	6267	102.330	0.000	6267	1.480	0.000	6267	20.266	0.000	6267	0.360	0.000
10x4S15	6296	200.760	0.000	6296	112.234	0.000	6296	1.520	0.000	6296	22.328	0.000	6296	0.280	0.000
10x4S10	6325	46.970	0.000	6325	313.469	0.000	6325	1.720	0.000	6325	32.718	0.000	6325	0.380	0.000
10x4S5	6518	5.110	0.000	6518	5.000	0.000	6518	2.340	0.000	6518	47.391	0.000	6518	0.580	0.000
10x4S{30,20,15,10,5}	**6319.8**	**593.186**	**0.000**	**6319.8**	**463.685**	**0.000**	**6319.8**	**1.672**	**0.000**	**6319.8**	**27.394**	**0.000**	**6319.8**	**0.376**	**0.000**
10x5S30	6308	7200.000	0.144	6308	2342.703	0.000	6308	3.770	0.000	6308	48.406	0.000	6308	0.750	0.000
10x5S20	6342	5290.090	0.000	6342	7200.000	0.053	6342	3.770	0.000	6342	66.375	0.000	6342	0.720	0.000
10x5S15	6397	4078.950	0.000	6397	1430.656	0.000	6397	7.140	0.000	6397	113.766	0.000	6397	0.890	0.000
10x5S10	6476	921.290	0.000	6476	1692.734	0.000	6476	9.380	0.000	6476	95.156	0.000	6476	1.550	0.000
10x5S5	6616	140.640	0.000	6616	69.610	0.000	6616	7.50s	0.000	6616	77.344	0.000	6616	1.090	0.000
10x5S{30,20,15,10,5}	**6427.8**	**3526.194**	**0.029**	**6427.8**	**2547.141**	**0.011**	**6427.8**	**6.015**	**0.000**	**6427.8**	**80.209**	**0.000**	**6427.8**	**1.000**	**0.000**
11x5S30	7468	7200.000	0.286	7420	2960.140	0.000	7420	5.380	0.000	7420	84.797	0.000	7420	1.130	0.000
11x5S20	7439	7200.000	0.215	7439	919.157	0.000	7439	5.660	0.000	7439	45.000	0.000	7439	0.910	0.000
11x5S15	7543	7200.000	0.141	7542	7200.000	0.094	7535	7.220	0.000	7535	156.671	0.000	7535	1.780	0.000
11x5S10	7572	5300.109	0.000	7572	5090.563	0.000	7572	5.470	0.000	7572	197.234	0.000	7572	1.420	0.000
11x5S5	7812	125.375	0.000	7812	366.187	0.000	7812	6.450	0.000	7812	276.960	0.000	7812	2.980	0.000
11x5S{30,20,15,10,5}	**7566.8**	**5405.097**	**0.128**	**7557**	**3307.209**	**0.019**	**7555.6**	**6.036**	**0.000**	**7555.6**	**152.132**	**0.000**	**7555.6**	**1.644**	**0.000**
12x5S30	8017	7200.000	0.371	7964	7200.000	0.351	7923	15.440	0.000	7923	320.860	0.000	7923	4.640	0.000
12x5S20	7992	7200.000	0.259	7939	7200.000	0.263	7939	16.360	0.000	7939	266.890	0.000	7939	3.910	0.000
12x5S15	7999	7200.000	0.249	7939	5336.187	0.000	7939	12.910	0.000	7939	228.734	0.000	7939	3.390	0.000
12x5S10	8008	7200.000	0.262	7978	1576.938	0.000	7978	12.560	0.000	7978	376.781	0.000	7978	3.000	0.000
12x5S5	8072	7200.000	0.144	8072	452.546	0.000	8072	14.730	0.000	8072	320.937	0.000	8072	3.410	0.000
12x5S{30,20,15,10,5}	**8017.6**	**7200.000**	**0.257**	**7978.4**	**4353.134**	**0.123**	**7970.2**	**14.400**	**0.000**	**7970.2**	**302.840**	**0.000**	**7970.2**	**3.670**	**0.000**
12x6S30	10,276	7200.000	0.394	10,357	7200.000	0.457	10,228	126.490	0.000	10,228	1081.235	0.000	10,228	9.470	0.000
12x6S20	10,396	7200.000	0.419	10,452	7200.000	0.409	10,312	42.950	0.000	10,312	650.234	0.000	10,312	37.750	0.000
12x6S15	10,420	7200.000	0.293	10,413	7200.000	0.384	10,362	62.230	0.000	10,362	87.313	0.000	10,362	5.170	0.000
12x6S10	10,480	7200.000	0.243	10,456	4273.516	0.000	10,456	107.250	0.000	10,456	1135.890	0.000	10,456	5.590	0.000
12x6S5	10,894	7200.000	0.213	10,891	877.531	0.000	10,891	96.980	0.000	10,891	1393.453	0.000	10,891	72.480	0.000
12x6S{30,20,15,10,5}	**10493.2**	**7200.000**	**0.313**	**10513.8**	**5350.209**	**0.250**	**10449.8**	**87.180**	**0.000**	**10449.8**	**869.625**	**0.000**	**10449.8**	**26.092**	**0.000**
15x6S30	13,850	7200.000	0.619	13,722	7200.000	0.618	13,567	389.730	0.000	13,567	3451.828	0.000	13,567	27.980	0.000
15x6S20	14,013	7200.000	0.628	13,916	7200.000	0.569	13,720	966.050	0.000	13,720	5432.985	0.000	13,720	270.550	0.000
15x6S15	-	-	-	13,951	7200.000	0.566	13,765	405.730	0.000	13,765	6511.375	0.000	13,765	147.780	0.000
15x6S10	-	-	-	14,053	7200.000	0.648	13,803	582.450	0.000	13,803	5239.437	0.000	13,803	20.560	0.000
15x6S5	14,136	7200.000	0.631	14,129	7200.000	0.595	13,927	897.750	0.000	14,042	7200.000	0.025	13,927	197.800	0.000
15x6S{30,20,15,10,5}	**-**	**-**	**-**	**13954.2**	**7200.000**	**0.599**	**13756.4**	**648.342**	**0.000**	**13779.4**	**5567.125**	**0.005**	**13756.4**	**132.934**	**0.000**
15x7S30	-	-	-	14,657	7200.000	0.792	14,409	2136.470	0.000	14,469	7200.000	0.023	14,409	504.840	0.000
15x7S20	-	-	-	14,837	7200.000	0.655	14,514	1795.580	0.000	14,723	7200.000	0.043	14,514	680.580	0.000
15x7S15	14,906	7200.000	0.596	-	-	-	14,657	2251.700	0.000	14,797	7200.000	0.045	14,657	417.660	0.000
15x7S10	15,087	7200.000	0.617	-	-	-	14,810	3789.730	0.000	14,929	7200.000	0.045	14,810	1352.470	0.000
15x7S5	15,355	7200.000	0.697	-	-	-	15,054	2038.280	0.000	15,460	7200.000	0.080	15,054	2268.090	0.000
15x7S{30,20,15,10,5}	**-**	**-**	**-**	**-**	**-**	**-**	**14688.8**	**2402.352**	**0.000**	**14875.6**	**7200.000**	**0.047**	**14688.8**	**1044.728**	**0.000**
20x10S30	30,756	7200.000	0.954	30,522	7200.000	0.952	29,458	7200.000	0.124	34,666	7200.000	0.257	29,081	7200.000	0.089
20x10S20	30,928	7200.000	0.970	30,769	7200.000	0.938	29,754	7200.000	0.130	34,162	7200.000	0.240	29,609	7200.000	0.099
20x10S15	30,844	7200.000	0.979	31,278	7200.000	0.980	30,156	7200.000	0.145	-	-	-	29,967	7200.000	0.130
20x10S10	31,388	7200.000	1.000	31,557	7200.000	0.953	30,039	7200.000	0.142	-	-	-	29,880	7200.000	0.099
20x10S5	32,442	7200.000	1.000	33,062	7200.000	0.981	31,420	7200.000	0.186	-	-	-	30,604	7200.000	0.110
20x10S{30,20,15,10,5}	**31271.6**	**7200.000**	**0.980**	**31437.6**	**7200.000**	**0.961**	**30165.4**	**7200.000**	**0.145**	**-**	**-**	**-**	**29828.2**	**7200.000**	**0.105**
Average	**-**	**-**	**-**	**-**	**-**	**-**	**10843.28**	**1057.831**	**0.015**	**-**	**-**	**-**	**10809.56**	**841.080**	**0.011**

B.2. Detailed results of MIP models $\mathcal{M}^{2,1}$, $\mathcal{M}^{3,0}$, $\mathcal{M}^{3,1}$, $\mathcal{M}'^{0,0}\mathcal{M}'^{0,1}$ and$\mathcal{M}'^{1,1}$ for each instance and each class of instances

Table B2
Our Linear Models $\mathcal{M}^{2,1}$, $\mathcal{M}^{3,0}$, $\mathcal{M}^{3,1}$, $\mathcal{M}'^{0,0}\mathcal{M}'^{0,1}$ and$\mathcal{M}'^{1,1}$.

Instances	$\mathcal{M}^{2,1}$			$\mathcal{M}^{3,0}$			$\mathcal{M}^{3,1}$			$\mathcal{M}'^{0,0}$			$\mathcal{M}'^{0,1}$			$\mathcal{M}'^{1,1}$		
	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap	Solution	Time	Gap
8x4S30	5063	1.281	0.000	5063	0.340	0.000	5063	1.250	0.000	5063	1.016	0.000	5063	7.391	0.000	5063	1.485	0.000
8x4S20	5086	0.718	0.000	5086	0.360	0.000	5086	0.735	0.000	5086	0.313	0.000	5086	6.953	0.000	5086	1.594	0.000
8x4S15	5112	1.047	0.000	5112	0.340	0.000	5112	1.172	0.000	5112	0.250	0.000	5112	1.953	0.000	5112	2.750	0.000
8x4S10	5169	1.781	0.000	5169	0.370	0.000	5169	1.672	0.000	5169	0.234	0.000	5169	1.172	0.000	5169	2.937	0.000
8x4S5	5174	1.110	0.000	5174	0.360	0.000	5174	1.328	0.000	5174	0.172	0.000	5174	0.672	0.000	5174	3.672	0.000
8x4S{30,20,15,10,5}	**5120.8**	**1.187**	**0.000**	**5120.8**	**0.354**	**0.000**	**5120.8**	**1.231**	**0.000**	**5120.8**	**0.397**	**0.000**	**5120.8**	**3.628**	**0.000**	**5120.8**	**2.488**	**0.000**
9x4S30	5904	2.094	0.000	5904	0.343	0.000	5904	4.282	0.000	5904	1.047	0.000	5904	23.922	0.000	5904	2.578	0.000
9x4S20	5937	3.546	0.000	5937	0.656	0.000	5937	2.562	0.000	5937	0.390	0.000	5937	10.375	0.000	5937	4.250	0.000
9x4S15	5976	1.954	0.000	5976	0.469	0.000	5976	2.516	0.000	5976	0.328	0.000	5976	9.031	0.000	5976	4.406	0.000
9x4S10	6027	3.125	0.000	6027	0.656	0.000	6027	4.078	0.000	6027	0.359	0.000	6027	4.406	0.000	6027	5.157	0.000
9x4S5	6047	2.968	0.000	6047	0.562	0.000	6047	6.140	0.000	6047	0.266	0.000	6047	3.016	0.000	6047	6.812	0.000
9x4S{30,20,15,10,5}	**5978.2**	**2.737**	**0.000**	**5978.2**	**0.537**	**0.000**	**5978.2**	**3.916**	**0.000**	**5978.2**	**0.478**	**0.000**	**5978.2**	**10.150**	**0.000**	**5978.2**	**4.641**	**0.000**
10x4S30	6193	3.985	0.000	6193	0.578	0.000	6193	3.750	0.000	6193	4.531	0.000	6193	233.594	0.000	6193	7.125	0.000
10x4S20	6267	5.875	0.000	6267	0.719	0.000	6267	4.032	0.000	6267	2.609	0.000	6267	120.312	0.000	6267	20.656	0.000
10x4S15	6296	3.859	0.000	6296	0.985	0.000	6296	4.968	0.000	6296	2.063	0.000	6296	53.484	0.000	6296	15.016	0.000
10x4S10	6325	4.516	0.000	6325	0.594	0.000	6325	6.360	0.000	6325	1.735	0.000	6325	30.516	0.000	6325	19.562	0.000
10x4S5	6518	6.515	0.000	6518	1.750	0.000	6518	8.437	0.000	6518	0.438	0.000	6518	6.250	0.000	6518	19.516	0.000
10x4S{30,20,15,10,5}	**6319.8**	**4.950**	**0.000**	**6319.8**	**0.925**	**0.000**	**6319.8**	**5.509**	**0.000**	**6319.8**	**2.275**	**0.000**	**6319.8**	**88.831**	**0.000**	**6319.8**	**16.375**	**0.000**
10x5S30	6308	15.969	0.000	6308	1.750	0.000	6308	36.813	0.000	6308	24.516	0.000	6308	7200.000	0.052	6308	72.766	0.000
10x5S20	6342	16.375	0.000	6342	3.279	0.000	6342	43.234	0.000	6342	28.375	0.000	6342	7200.000	0.045	6342	72.125	0.000
10x5S15	6397	17.672	0.000	6397	2.344	0.000	6397	57.720	0.000	6397	19.062	0.000	6397	1833.047	0.000	6397	115.031	0.000
10x5S10	6476	62.906	0.000	6476	3.328	0.000	6476	104.593	0.000	6476	7.578	0.000	6476	52.797	0.000	6476	127.422	0.000
10x5S5	6616	47.250	0.000	6616	5.156	0.000	6616	72.703	0.000	6616	1.359	0.000	6616	56.468	0.000	6616	105.125	0.000
10x5S{30,20,15,10,5}	**6427.8**	**32.034**	**0.000**	**6427.8**	**3.171**	**0.000**	**6427.8**	**63.013**	**0.000**	**6427.8**	**16.178**	**0.000**	**6427.8**	**3268.462**	**0.019**	**6427.8**	**98.494**	**0.000**
11x5S30	7420	26.593	0.000	7420	2.312	0.000	7420	47.047	0.000	7420	1144.422	0.000	7428	7200.000	0.056	7420	128.969	0.000
11x5S20	7439	22.204	0.000	7439	3.234	0.000	7439	76.672	0.000	7439	308.235	0.000	7439	770.875	0.000	7439	126.187	0.000
11x5S15	7535	60.594	0.000	7535	3.875	0.000	7535	102.719	0.000	7535	42.140	0.000	7535	376.390	0.000	7535	186.641	0.000
11x5S10	7572	108.859	0.000	7572	4.687	0.000	7572	143.516	0.000	7572	28.172	0.000	7572	154.610	0.000	7572	319.390	0.000
11x5S5	7812	222.980	0.000	7812	7.094	0.000	7812	221.234	0.000	7812	7.610	0.000	7812	185.375	0.000	7812	291.953	0.000
11x5S{30,20,15,10,5}	**7555.6**	**88.246**	**0.000**	**7555.6**	**4.240**	**0.000**	**7555.6**	**118.238**	**0.000**	**7555.6**	**306.116**	**0.000**	**7557.2**	**1737.450**	**0.011**	**7555.6**	**210.628**	**0.000**
12x5S30	7923	113.765	0.000	7923	38.047	0.000	7923	163.375	0.000	7971	7200.000	0.135	-	-	-	7923	1516.469	0.000
12x5S20	7939	149.703	0.000	7939	7.922	0.000	7939	172.484	0.000	7939	2088.500	0.000	7939	2902.703	0.000	7939	874.234	0.000
12x5S15	7939	163.297	0.000	7939	9.797	0.000	7939	217.344	0.000	7939	3534.954	0.000	7939	1371.703	0.000	7939	559.703	0.000
12x5S10	7978	200.296	0.000	7978	6.875	0.000	7978	221.078	0.000	-	-	-	7978	1454.594	0.000	7978	777.641	0.000
12x5S5	8072	229.891	0.000	8072	8.953	0.000	8072	331.297	0.000	8072	20.328	0.000	8072	5735.812	0.000	8072	765.656	0.000
12x5S{30,20,15,10,5}	**7970.2**	**171.390**	**0.000**	**7970.2**	**14.319**	**0.000**	**7970.2**	**221.116**	**0.000**	**-**	**-**	**-**	**-**	**-**	**-**	**7970.2**	**898.741**	**0.000**
12x6S30	10,228	260.750	0.000	10,228	95.250	0.000	10,228	605.078	0.000	-	-	-	10,276	7200.000	0.336	10,228	5770.203	0.000
12x6S20	10,312	416.578	0.000	10,312	64.609	0.000	10,312	719.297	0.000	-	-	-	10,388	7200.000	0.405	10,312	7200.000	0.020
12x6S15	10,362	348.047	0.000	10,362	37.578	0.000	10,362	759.938	0.000	-	-	-	10,582	7200.000	0.392	10,362	3408.453	0.000
12x6S10	10,456	609.422	0.000	10,456	81.469	0.000	10,456	1115.046	0.000	-	-	-	10,500	7200.000	0.314	10,456	2948.844	0.000
12x6S5	10,891	1045.265	0.000	10,891	123.484	0.000	10,891	1368.704	0.000	10,891	11.593	0.000	10,891	2624	0.000	10,891	1203.891	0.000
12x6S{30,20,15,10,5}	**10449.8**	**536.012**	**0.000**	**10449.8**	**80.478**	**0.000**	**10449.8**	**913.613**	**0.000**	**10891**	**11.593**	**0.000**	**10527.4**	**6285**	**0.289**	**10449.8**	**4106.278**	**0.004**
15x6S30	13,567	1516.781	0.000	13,567	687.437	0.000	13,567	3323.718	0.000	13,776	7200.000	0.524	13,809	7200.000	0.624	13,685	7200.000	0.034
15x6S20	13,720	5090.156	0.000	13,720	818.453	0.000	13,805	7200.000	0.017	13,922	7200.000	0.468	13,815	7200.000	0.504	13,873	7200.000	0.036
15x6S15	13,765	5129.641	0.000	13,765	406.375	0.000	13,765	7005.172	0.000	13,911	7200.000	0.407	13,861	7200.000	0.545	13,844	7200.000	0.031
15x6S10	13,803	4759.063	0.000	13,803	449.500	0.000	13,803	4475.172	0.000	-	-	-	-	-	-	13,860	7200.000	0.027
15x6S5	13,940	7200.000	0.009	13,927	752.141	0.000	13,983	7200.000	0.024	13,960	7200.000	0.314	14,331	7200.000	0.625	14,076	7200.000	0.045
15x6S{30,20,15,10,5}	**13759**	**4739.128**	**0.002**	**13756.4**	**622.781**	**0.000**	**13784.6**	**5840.812**	**0.008**	**-**	**-**	**-**	**-**	**-**	**-**	**13867.6**	**7200.000**	**0.035**
15x7S30	14,409	7200.000	0.004	14,409	2711.282	0.000	14,510	7200.000	0.024	-	-	-	14,724	7200.000	0.653	14,665	7200.000	0.055
15x7S20	14,679	7200.000	0.029	14,514	3227.375	0.000	14,583	7200.000	0.027	-	-	-	15,001	7200.000	0.712	14,792	7200.000	0.059
15x7S15	14,742	7200.000	0.036	14,657	4199.359	0.000	14,798	7200.000	0.050	14,855	7200.000	0.508	15,056	7200.000	0.644	14,941	7200.000	0.067
15x7S10	15,113	7200.000	0.062	14,810	4628.078	0.000	14,923	7200.000	0.046	14,843	7200.000	0.446	14,867	7200.000	0.581	14,941	7200.000	0.059
15x7S5	15,443	7200.000	0.068	15,054	4452.016	0.000	14,381	7200.000	0.072	15,190	7200.000	0.365	15,278	7200.000	0.665	15,486	7200.000	0.083
15x7S{30,20,15,10,5}	**14877.2**	**7200.000**	**0.040**	**14688.8**	**3843.622**	**0.000**	**14639**	**7200.000**	**0.044**	**-**	**-**	**-**	**14985.2**	**7200.000**	**0.651**	**14965**	**7200.000**	**0.065**
20x10S30	34,095	7200.000	0.237	29,415	7200.000	0.119	34,252	7200.000	0.248	29,344	7200.000	0.828	30,125	7200.000	0.937	30,069	7200.000	0.135
20x10S20	34,886	7200.000	0.255	29,941	7200.000	0.125	35,010	7200.000	0.259	29,346	7200.000	0.845	30,242	7200.000	0.942	31,759	7200.000	0.177
20x10S15	34,353	7200.000	0.240	29,934	7200.000	0.115	35,565	7200.000	0.267	29,666	7200.000	0.805	31,278	7200.000	0.949	31,327	7200.000	0.164
20x10S10	34,563	7200.000	0.241	30,322	7200.000	0.120	35,308	7200.000	0.258	29,527	7200.000	0.833	31,209	7200.000	0.951	30,933	7200.000	0.147
20x10S5	-	-	-	30,409	7200.000	0.141	-	-	-	-	-	-	32,681	7200.000	0.975	31,248	7200.000	0.151
20x10S{30,20,15,10,5}	**-**	**-**	**-**	**30004.2**	**7200.000**	**0.124**	**35033.75**	**7200.000**	**0.258**	**29470.75**	**-**	**-**	**31107**	**7200.000**	**0.951**	**31067.2**	**7200.000**	**0.155**
Average	**-**	**-**	**-**	**10827.16**	**1177.042**	**0.012**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**-**	**10972.2**	**2693.764**	**0.023**

Table C1
p-values for all pairs of models in terms of solution value and CPU time - integrality requirement on variables $z_{m,\ i,\ n,\ j}$ imposed.

	$\mathcal{M}^{1,0}$	$\mathcal{M}^{2,0}$	$\mathcal{M}^{3,0}$	$\mathcal{M'}^{1,1}$
$\mathcal{M}^{1,0}$	-	0.07	0.81	1.20e-03
$\mathcal{M}^{2,0}$	0.06	-	0.31	6.10e-05
$\mathcal{M}^{3,0}$	0.81	0.31	-	6.10e-05
$\mathcal{M'}^{1,1}$	1.20e-03	6.10e-05	6.10e-05	-

(a)*p*-values in terms of solution value

	$\mathcal{M}^{1,0}$	$\mathcal{M}^{2,0}$	$\mathcal{M}^{3,0}$	$\mathcal{M'}^{1,1}$
$\mathcal{M}^{1,0}$	-	5.48e-08	0.11	5.18e-09
$\mathcal{M}^{2,0}$	5.48e-08	-	5.18e-09	5.18e-09
$\mathcal{M}^{3,0}$	0.11	5.18e-09	-	5.18e-09
$\mathcal{M'}^{1,1}$	5.18e-09	5.18e-09	5.18e-09	-

(b)*p*-values in terms of CPU time

Table C2
p-values for all pairs of models in terms of solution value and CPU time - integrality requirement on variables $z_{m,\ i,\ n,\ j}$ imposed.

	$\mathcal{M}^{0',1}$	$\mathcal{M}^{1',0}$	$\mathcal{M}^{2',0}$	$\mathcal{M}^{3',0}$	$\mathcal{M'}^{0',1}$	$\mathcal{M'}^{1',1}$
$\mathcal{M}^{0',1}$	-	0.06	4.37e-04	3.2e-03	0.28	0.01
$\mathcal{M}^{1',0}$	0.06	-	2.00e-03	2.00e-03	0.31	4.38e-04
$\mathcal{M}^{2',0}$	4.37e-04	2.00e-03	-	0.81	0.01	4.38e-04
$\mathcal{M}^{3',0}$	3.2e-03	2.00e-03	0.81	-	0.10	4.38e-04
$\mathcal{M'}^{0',1}$	0.25	0.31	0.01	0.10	-	0.10
$\mathcal{M'}^{1',1}$	0.01	4.38e-04	4.38e-04	4.38e-04	0.10	-

(a)*p*-values in terms of solution value

	$\mathcal{M}^{0',1}$	$\mathcal{M}^{1',0}$	$\mathcal{M}^{2',0}$	$\mathcal{M}^{3',0}$	$\mathcal{M'}^{0',1}$	$\mathcal{M'}^{1',1}$
$\mathcal{M}^{0',1}$	-	1.07e-07	5.18e-09	5.18e-09	4.17e-05	4.6e-03
$\mathcal{M}^{1',0}$	1.07e-07	-	5.18e-09	5.18e-09	2.34e-07	7.74e-08
$\mathcal{M}^{2',0}$	5.18e-09	5.18e-09	-	5.3e-06	5.18e-09	5.18e-09
$\mathcal{M}^{3',0}$	5.18e-09	5.18e-09	5.3e-06	-	5.18e-09	5.18e-09
$\mathcal{M'}^{0',1}$	4.17e-05	2.34e-07	5.18e-09	5.18e-09	-	0.3421
$\mathcal{M'}^{2',1}$	4.6e-03	7.74e-08	5.18e-09	5.18e-09	0.3421	-

(b)*p*-values in terms of CPU time

Appendix C. Wilkoxon signed rank statistical tests for all of the models both for solution quality and runtime

In this section we provide results of Wilkoxon signed rank test applied to each pair of models regarding both solution quality and runtime. The corresponding *p*-values are provided in Tables C.11 and C.12. The *p*-value <0.0001 means that there is significant difference between two models in the comparison, otherwise there is no significant difference. The models included in comparison are these that are able to provide a feasible solution for each test instance in the benchmark set.

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