Asymmetric information and the pecking (dis)order[*]

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Abstract

We study the classical problem of raising capital under asymmetric information. Following Myers and Majluf (1984), we consider firms endowed with assets in place and riskier growth opportunities. When asymmetric information is concentrated on assets in place (rather than growth opportunities), equity-like securities are more likely to be optimal. In contrast, when asymmetric information falls on growth options, debt is optimal. Intuitively, this happens because when the asset with greater volatility is less affected by asymmetric information, issuing a security with greater exposure to upside potential (such as equity) can be less dilutive than issuing a security lacking such exposure (such as debt). Our results suggest that equity is more likely to dominate debt for younger firms with larger investment needs, endowed with riskier, more valuable growth opportunities. Thus, our model can explain why high-growth firms may prefer equity over debt, and then switch to debt financing as they mature.

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When outside investors have less information than insiders on the value of the securities issued by a firm, existing shareholders are exposed to value dilution due to mispricing. In a classic paper, Myers and Majluf (1984) suggest that, under these circumstances, firms that have greater exposure to asymmetric information can reduce mispricing by issuing debt rather than equity. The usual rationale behind this intuition, known as the "pecking order theory," is that debt, by virtue of being senior to equity, is less sensitive to private information, limiting dilution (Myers, 1984).

Nachman and Noe (1994) show that the original Myers and Majluf intuition obtains only under special conditions regarding how the insiders' private information affects firm value. Debt emerges as the solution to an optimal security design problem, for any level of capital raised, if and only if the private information held by firm insiders orders firm-value distributions by Conditional Stochastic Dominance (CSD), a condition that is considerably stronger than First Order Stochastic Dominance (FOSD).[1]

We first study the optimal security design problem of Nachman and Noe (1994) in a more general setting, when distributions satisfy FOSD but fail CSD. We find that, when a certain "low-information-cost-in-the-right-tail" condition holds, debt is optimal when the firm needs to raise low levels of capital, but "equity-like" securities — such as convertible debt — emerge as optimal securities when the firm must raise larger amounts of capital. In addition, raising capital by issuing warrants can be optimal for firms with pre-existing debt, a situation not considered in Nachman and Noe (1994).

The intuition for our results is as follows. First, firms with low investment needs can raise required funds by issuing securities backed only by left-tail cash flows, such as debt, limiting exposure to dilution. For larger investment needs, firms need to pledge their right-tail cash flows to new investors. In this case, when the "low-information-cost-in-the-right-tail" condition holds, firms can limit dilution by issuing equity-like securities, such as convertible debt. In addition, if a firm has debt in its capital structure that was issued earlier, the left-tail cash flow has already been pledged, and the firm will raise required capital by issuing only warrants.

Our security design results do not speak directly to the traditional pecking order theory, the choice of debt over equity, which is at the center of much of current theoretical and empirical research.[2] Building on Myers and Majluf (1984), we then study a traditional real-options specification where firms are endowed with a portfolio of assets in place and riskier growth opportunities. Growth options and assets in place are both characterized by lognormal distributions, and firm insiders have private information on the mean of the distributions, while variances are common knowledge.[3] We find that when the asymmetric information is concentrated on the assets in place,

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[1]Intuitively, CSD requires that private information orders the conditional distributions in the right tail by FOSD, for all possible truncations. The Statistics and Economics literature uses the term Hazard Rate Ordering to refer to CSD.

[2]Note that equity is never a solution of the optimal security design problem.

[3]While Myers and Majluf (1984) do not provide an explicit model that supports the pecking order, they clearly invoke lognormality as a critical part of their argument: "Option pricing theory tells us that ΔD will have the same

equity-like securities are more likely to be optimal. When the asymmetric information falls on the growth options, debt is optimal.

Our results depend on the different sensitivities of debt and equity to the value of a firm's assets. Debt gives investors maximum exposure to the downside risk, the "left tail"of the firm-value distribution, with no exposure to a firm's upside potential, its "right tail."In contrast, equity gives investors exposure to both downside risk and upside potential of the firm. Our paper exploits the fact that the relative dilution of debt and equity depends crucially on the "location" of the exposure to asymmetric information in the firm-value distribution. When asymmetric information is more pronounced in the right tail, as implied by the CSD condition in Nachman and Noe (1994), firms limit dilution by issuing a security that maximizes payoff to investors in the left tail, such as debt. In contrast, when asymmetric information is less severe in the right tail firms can limit dilution by issuing equity.

The relative dilution of debt and equity depends on the relative exposure to asymmetric information of the firm's assets and their volatilities. The exposure to asymmetric information in the tails of the firm-value distribution is driven by the exposure to asymmetric information of the asset with greater volatility. Thus, when the asset that has greater risk —the growth opportunity— is less affected by asymmetric information, the low-information-cost-in-the-right-tail condition will be satisfied, making equity potentially less dilutive than debt. In contrast, when the asymmetric information falls on the growth opportunity, equity is more sensitive to asymmetric information, making debt optimal. This implies that, contrary to ordinary intuition, the relative dilution of debt and equity is not driven by the overall level of asymmetric information affecting a firm, but rather by the composition of its assets and their relative exposure to asymmetric information.

Consider the case where firm value, the sum of assets in place and growth options, is log-normally distributed, as Myers and Majluf (1984) suggest, where the single source of asymmetric information is the difference in the means of the distributions. In this case, we show that FOSD implies CSD, and thus debt is optimal, as shown by Nachman and Noe (1994). We depart from this setting by assuming that both assets in place and (riskier) growth opportunities are individually distributed as lognormal random variables, and each have different exposures to asymmetric information. Our model breaks the link between FOSD and CSD, making convertible debt optimal and equity potentially less dilutive than debt.[4]

Our model allows us to identify economically relevant scenarios where Myers and Majluf's pecking order can be reversed, and to characterize such scenarios. We show that, consistent with empirical observations, deviations from the pecking order are more likely to occur in the case of young firms, endowed by risky growth opportunities requiring large capital infusions. Financing

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sign as ΔE, but that its absolute value will always be less" (pages 207–208), and "Our proof that debt dominates equity uses the standard option-pricing assumption that percentage changes in value are lognormally distributed with a constant variance rate known by everyone" (page 209).

[4]The key property that we exploit is that while CSD holds individually for single random variables, that condition may very well not be satisfied by the sum of such random variables.

with new equity issues is also more likely when firms already have debt outstanding in their capital structure. Finally, our analysis implies that deviations from the pecking order may occur more generally in firms endowed with a portfolio of heterogeneous assets if the more volatile assets are also less affected by asymmetric information. Our model produces other cross-sectional implications that we summarize in Section 6.

Our paper belongs to the ongoing research on firm financing under asymmetric information building on the seminal work by Myers and Majluf (1984) and Myers (1984) discussed above. Subsequent research focuses on different aspects of the security design problem. DeMarzo and Duffie (1999) consider the *ex-ante* security design problem faced by a firm before learning its private information, rather than the *interim* security design problem (that is, after becoming informed) studied by Nachman and Noe (1994). DeMarzo (2005) considers both the *ex-ante* and the *interim* security design problems, and examines both the question of whether to keep multiple assets in a single firm (pooling) and the priority structure of the securities issued by the firm (tranching).[5]

Our paper differs from this literature in several ways. First, and most importantly, in our paper we only require FOSD and, thus, our distributions can violate conditions posited in the previously mentioned literature, e.g., the uniform worst case condition of DeMarzo and Duffie (1999), or MLRP in DeMarzo, Kremer, and Skrzypacz (2005). Second, as in Myers and Majluf (1984) and Nachman and Noe (1994), we constrain the firm to raise a fixed amount of capital, which typically leads to pooling rather than separating equilibria. In contrast, in DeMarzo and Duffie (1999) issuers can separate in the interim security issuance stage by using retention as a signal (in the spirit of Leland and Pyle, 1977).

Other closely related papers include Chakraborty and Yılmaz (2011), which shows that if investors have access over time to noisy public information on the firm's private value, the dilution problem can be costlessly avoided by issuing securities having the structure of callable, convertible bonds. Chemmanur and Fulghieri (1997) argue that warrants may be part of the optimal security structure in a signaling game. Chakraborty, Gervais, and Yılmaz (2011) examine the optimal security design problem in the simultaneous presence of informed and uninformed investors that are exposed to the winner's curse in the context of an IPO (as in Rock, 1986). In a vein similar to ours, the paper finds that the optimal security depends on whether the information difference (between the two classes of investors) is located in the left or the right tail of the firm-value distribution.[6]

There are several other papers that challenge Myers and Majluf (1984) and Myers (1984) by extending their framework in various ways.[7] These papers derive a wide range of financing choices,

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[5]DeMarzo, Kremer, and Skrzypacz (2005) and Che and Kim (2010) examine the security design problem in the context of auctions of assets with informed buyers, and provide conditions where the revenue maximizing security for an uninformed issuer may be either debt or equity.

[6]Recent work (Yang and Zeng, 2018; Daley, Green, and Vanasco, 2017; Yang, 2018) looks at the interactions of security design, information acquisition and/or credit ratings. Hebert (2018) establishes optimality of debt in the context of moral hazard. Malenko and Tsoy (2018) study security design when investors are ambiguity averse.

[7]While we focus only on papers that study informational frictions, moral hazard considerations are also important drivers of capital structure choices, i.e., DeMarzo and Fishman (2007), Biais, Mariotti, Plantin, and Rochet (2007).

which allow for signaling with costless separation that can invalidate the pecking order (e.g., Brennan and Kraus, 1987; Noe, 1988; Constantinides and Grundy, 1989).[8] Cooney and Kalay (1993) relax the assumption that projects have a positive net present value. Boot and Thakor (1993), Fulghieri and Lukin (2001), and recently Yang (2018) and Yang and Zeng (2018), relax the assumption that the informational asymmetry between a firm's insiders and outside investors is exogenous, and allow for endogenous information production. Dybvig and Zender (1991) study the effect of optimally designed managerial compensation schemes, Hackbarth (2008) shows that managers with risk perception bias or "overconfidence" have a reverse pecking order preference, and Edmans and Mann (2018) look at the possibility of asset sales for financing purposes. Bond and Zhong (2014) show that undervalued firms may prefer to execute a share repurchase before proceeding with a seasoned equity offer, thus reducing its negative impact on its share price. Strebulaev, Zhu, and Zryumov (2016) consider a dynamic model of the issuance decision, where information asymmetry is reduced over time. In contrast to these papers, but in the spirit of Myers and Majluf (1984), we consider a pooling equilibrium of a static model where the only friction is asymmetric information between insiders and outsiders.

The remainder of this paper is organized as follows. We begin in Section 1 by providing a simple example that illustrates the basic results of our paper and its underlying intuition. Section 2 presents the basic model. Section 3 considers the security design problem, where we provide conditions under which convertible debt and warrants are the optimal securities. Section 4 studies the drivers of the debt-equity choice. Section 5 studies the capital raising game when the firm has pre-existing debt in its capital structure. In Section 6 we discuss the empirical implications of our model. Section 7 concludes. All proofs are in the Appendix.

1 A Simple Example

The core intuition of the pecking order theory is typically illustrated via a pooling equilibrium with two types of firms and a discrete state space. Table 1 presents such a simple numerical example.[9] We consider two types of firms: good type, $\theta = G$, and bad type, $\theta = B$, where a firm's type is private information to its insiders. We assume that the two types of firms are equally likely in the eyes of investors. At the beginning of the period, firms have assets in place and wish to raise capital I to invest in a new growth opportunity. We focus on a pooling equilibrium such that, when raising capital, the two types of firms issue the same security, so that investors do not update their priors on the firms' type when seeing the security issuance decision.

For reasons that will become apparent below, we assume that a firm's end-of-period value, Z_θ,

[8]Admati and Pfleiderer (1994) points out, however, that the conditions for a fully revealing signaling equilibrium identified in these papers are rather restrictive.

[9]The numerical example of this section builds on the discussion in Nachman and Noe (1994), Section 4.3.

for $\theta \in \{G, B\}$, is characterized by a distribution with three possible outcomes $Z_\theta \in \{z_1, z_2, z_3\}$.[10] To fix ideas, we assume that states z_1 and z_2 are relevant for the value of assets in place, while state z_3 is relevant for the value of the growth opportunity. In particular, we assume that the end-of-period value of the assets in place is given by $z_1 = 10$, $z_2 = z_3 = 100$. If the firm invests, firm value becomes $z_1 = 10$, $z_2 = 100$, $z_3 = 300$. Thus, exploitation of the growth opportunity adds value to the firm only in state z_3, increasing the end-of-period firm value in that state from 100 to 300. We set the investment amount to $I = 60$.

The probability of the three possible outcomes of Z_θ depends on private information held by the firm's insiders, and is given by $f_\theta \equiv \{f_{\theta 1}, f_{\theta 2}, f_{\theta 3}\}$ for a firm of type θ. In our examples below, we will assume that $f_G = \{0.2, 0.4, 0.4\}$ and $f_B = \{0.3, 0.4 - x, 0.3 + x\}$, and we will focus in the cases $x = 0.02$ and $x = 0.08$ for the discussion.[11] Note that the presence of the growth opportunity has the effect of changing the distribution of firm value in its right tail, and that the parameter x affects the probability on the high state, z_3, relative to the middle state, z_2, for the type-B firm.

Consider first the case where $x = 0.08$, so that $f_B = \{0.3, 0.32, 0.38\}$. Firm values for the good and bad types are given by $\mathbb{E}[Z_G] = 162$ and $\mathbb{E}[Z_B] = 149$, with a pooled value equal to 155.5. Firms can raise the investment of 60 to finance the growth opportunity by issuing a fraction of equity equal to $\lambda = 0.386 = 60/155.5$. Hence, under equity financing, the initial shareholders of a firm of type-G retain a residual equity value equal to $(1 - 0.386)162 = 99.5$. The firm could also raise the required capital by issuing debt, with face value equal to $K = 76.7$. In this case debt is risky, with payoffs equal to $\{10, 76.7, 76.7\}$, and it will default only in state z_1. The value of the debt security when issued by a type G firm is $D_G = 63.3$, and when issued by a type-B firm is $D_B = 56.7$, with a pooled value of 60, since the two types are equally likely. This implies that under debt financing the shareholders of a type-G firm will retain a residual equity value equal to $\mathbb{E}[Z_G] - D_G = 98.7 < 99.5$, and equity is less dilutive than debt, reversing the pecking order.

The role of the growth opportunity in reversing the pecking order can be seen by considering the following perturbation of the basic example. Now set $x = 0.02$, so that $f_B = \{0.30, 0.38, 0.32\}$. In the new example the growth opportunity is relatively less important for a type-B firm than in the base case. Note that this change does not affect debt financing, because debt is in default only in state z_1. Therefore the change in x only affects equity dilution. In the new case, $\mathbb{E}[Z_B] = 137$, lowering the pooled value to 149.5. Now the firm must issue a larger equity stake, $\lambda = 0.401 = 60/149.5$, and thus existing shareholders' value is now equal to $(1 - 0.401)162 = 97.0 < 98.7$. Thus, equity financing is now more dilutive than debt financing, restoring the pecking order.

The reason for the change in the relative dilution of debt and equity rests on the impact of asymmetric information on the right tail of the firm-value distribution. In the base case, for

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[10]Note that, for binomial distributions, FOSD implies CSD, which, from Nachman and Noe (1994), implies that debt is the optimal security.

[11]Table 1 considers all cases $x \in (0, 0.1)$. We remark that $x \leq 0.1$ is necessary to maintain first-order stochastic dominance.

$x = 0.08$, asymmetric information has a modest impact on the growth opportunity (since $f_{G3} - f_{B3} = 0.02$) relative to the middle of the distribution (since $f_{G2} - f_{B2} = 0.08$), whose impact is determined by the exposure of the assets in place to asymmetric information. Thus, firms of type G can reduce dilution by issuing a security that has greater exposure to the right tail of the firm-value distribution, such as equity, rather than debt, which lacks such exposure. In contrast, in the case of $x = 0.02$, asymmetric information has a more substantial impact on the growth opportunity and, thus, on the right tail relative to the middle of the distribution (since now we have $f_{G3} - f_{B3} = 0.08$ and $f_{G2} - f_{B2} = 0.02$) making equity more mis-priced.

A second key ingredient of our example is that the firm is issuing (sufficiently) risky debt to make dilution a concern. If debt is riskless, or nearly riskless, the pecking order would hold. To illustrate this, we can assume $z_1 = 10$ and set $I = 10$. At the lower level of investment, the firm can issue riskless debt and avoid any dilution altogether. Similarly, for investment needs sufficiently close to $I = 10$, debt has little default risk and the potential mispricing will be small. In contrast, for sufficiently large investment needs the firm will need to issue debt with non-trivial default risk, creating the potential for a reversal of the pecking order.

Finally, note that in the special case in which $f_B \equiv \{0.3, 0.3, 0.4\}$ there is no asymmetric information at all in the right tail (that is, for $z_3 = 300$). In this case, type-G firms would in fact be able to avoid dilution altogether by issuing securities that load only on cash flows in the right tail, such as warrants.

The intuition behind the simple example presented in this section carries over to the more general settings studied below. In Section 2.3 we introduce a condition, which we refer to as "low-information-costs-in-the-right-tail," that corresponds to the case with $x > 0.05$ in the example. This condition is novel in the literature and it is critical to generate reversals of the pecking order. In Section 2.4 we introduce a bi-variate real-options model; we will show that adding a second source of uncertainty to the firm-value distribution will have similar properties as moving from the binomial to the trinomial structure of the simple example. In Section 4.1 we will decompose the firm-value distribution into three regions that correspond to the trinomial structure of our example.

2 The Basic Model

2.1 The Capital Raising Game

We study a one-period model with two dates, $t \in \{0, T\}$. At the beginning of the period, $t = 0$, a firm wishes to raise a certain amount of capital, I, that needs to be invested in the firm immediately.[12] We interpret the capital raised I as representing the amount of capital needed by

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[12]Following Nachman and Noe (1994), we do not explicitly model the reason for this capital requirement. The investment requirement I may reflect, for example, a new investment project that the firm wishes to undertake, as discussed in Section 4. Also note that, in the spirit of Myers and Majluf (1984), we rule out the possibility that firms finance their growth opportunities separately from the assets in place, i.e., by "project financing."

the firm on top of internally available funds, if any.[13] We initially assume that the firm is all equity financed, we will later study the effect of the presence of pre-existing debt in the firm capital structure at $t = 0$. Firm value at the end of the period, $t = T$, is given by a random variable Z_θ. There are two types of firms: "good" firms, $\theta = G$, and "bad" firms, $\theta = B$, which are present in the economy with probabilities p and $1 - p$, respectively. A firm of type θ is characterized by its density function $f_\theta(z)$ and by the corresponding cumulative distribution function $F_\theta(z)$, with $\theta \in \{G, B\}$. Because of limited liability, we assume that Z_θ takes values on the positive real line. For ease of exposition, we will also assume that the density function of Z_θ satisfies $f_\theta(z) > 0$ for all $z \in \mathbb{R}_+$. In addition, we assume type-G firms dominate type-B firms by first-order stochastic dominance.

Definition 1 (FOSD). *The distribution F_G dominates the distribution F_B by (strong) first-order stochastic dominance if $F_G(z) \leq (<)F_B(z)$ for all $z \in \mathbb{R}_{++}$.*

The stronger property of Conditional Stochastic Dominance, CSD, plays a crucial role in the security design problem, as argued in Nachman and Noe (1994).

Definition 2 (CSD). *We will say that the distribution F_G dominates the distribution F_B by conditional stochastic dominance if $F_G(z|z') \leq F_B(z|z')$ for all $z' \in \mathbb{R}_+$ and $z \geq z'$, where*

$$F_\theta(z|z') \equiv \frac{F_\theta(z + z') - F_\theta(z')}{1 - F_\theta(z')}.$$

By setting $z' = 0$, it is easy to see that CSD implies FOSD. Note that CSD can equivalently be defined by requiring that the truncated random variables $[Z_\theta | Z_\theta \geq \bar{z}]$, with distribution functions $(F_\theta(z) - F_\theta(\bar{z}))/(1 - F(\bar{z}))$, satisfy FOSD for all $\bar{z}$.[14] In addition, Nachman and Noe (1994) show that CSD is equivalent to the condition that the ratio $(1 - F_G(z))/(1 - F_B(z))$ is non-decreasing in z for all $z \in \mathbb{R}_+$ (see their Proposition 4). Thus, loosely speaking, CSD implies that the set of payoffs in the right tail of the firm-value distribution are always more likely to occur for a type-G firm relatively to a type-B firm.[15]

Firms raise the amount I by seeking financing in capital markets populated by a large number of competitive, risk-neutral investors. Capital markets are characterized by asymmetric information in that a firm's type, $\theta \in \{G, B\}$ is private information to its insiders. We assume that firms always find it optimal to issue securities and raise I, rather than foregoing the investment opportunity.

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[13]Note that, in the spirit of Myers and Majluf (1984), in our model firms would first use all internally available funds before raising any capital from investors.

[14]We remark that the CSD (hazard-rate) ordering is weaker than the Monotone Likelihood Ratio order, which requires $[Z_G | Z_G \in (\underline{z}, \bar{z})] \overset{\text{fosd}}{\geq} [Z_B | Z_B \in (\underline{z}, \bar{z})]$ for all $\underline{z}$ and $\bar{z}$; see equation (1.B.7) and Theorem 1.C.5 in Shaked and Shanthikumar (2007).

[15]Referring back to the example in Section 1, it is easy to verify that if $x \leq 0.05$ the type$-G$ distribution not only dominates the type$-B$ in the first$-$order sense, but also in the CSD sense. A necessary condition for the distributions in the example to not satisfy CSD is that $x > 0.05$.

We make this assumption to rule out the possibility of separating equilibria where type-B firms raise capital, I, while type-G firms separate by not issuing any security. Finally, the realization of the random variable Z_θ is observable and contractible.

When insiders have private information, firms will typically issue securities at prices that diverge from their symmetric information values. Under these circumstances, better-quality firms will find it desirable to raise capital by issuing securities that reduce the adverse impact of asymmetric information. To fix ideas, let $\mathbb{S}$ be the set of admissible securities that the firm can issue to raise the required capital I. As is common in this literature (see, for example, Nachman and Noe (1994)), we let $\mathbb{S}$ be the set of functions satisfying the following conditions:

$$0 \leq s(z) \leq z, \qquad \text{for all } z \geq 0, \tag{1}$$

$$s(z) \quad \text{is non-decreasing in } z \quad \text{for all } z \geq 0, \tag{2}$$

$$z - s(z) \quad \text{is non-decreasing in } z \quad \text{for all } z \geq 0. \tag{3}$$

Condition (1) ensures limited liability for both the firm and investors, while (2) and (3) are monotonicity conditions that ensure absence of risk-less arbitrage.[16] We define $\mathbb{S} \equiv \{s(z) : \mathbb{R}_+ \to \mathbb{R}_+; s(z)$ satisfies (1), (2), and (3)$\}$ as the set of *admissible securities*.

We consider the following capital raising game. The firm moves first, and chooses a security, $s(z)$, from the set of admissible securities $\mathbb{S}$. After observing the security, $s(z)$, issued by the firm, investors update their beliefs on firm type, θ, and form posterior beliefs, $p(s) : \mathbb{S} \to [0,1]$. Given their posterior beliefs on firm type, investors purchase the security issued by the firm at a price, $V(s)$. The value $V(s)$ that investors are willing to pay for a security $s(z)$ is equal to its expected value, conditional on the posterior beliefs, $p(s)$, that is:

$$V(s) = p(s)\mathbb{E}[s(Z_G)] + (1 - p(s))\mathbb{E}[s(Z_B)]. \tag{4}$$

Condition (4) implies that securities are fairly priced, given investors' beliefs. If security s is issued, capital $V(s)$ is raised, and the investment project is undertaken. The payoff to the initial shareholders for a firm of a type θ is given by

$$W(\theta, s, V(s)) \equiv \mathbb{E}[Z_\theta - s(Z_\theta)] + V(s) - I. \tag{5}$$

The firm will choose the security to issue to finance the investment project by maximizing its payoff, (5), subject to the constraint that the security is admissible and that it raises at least the required funds, I. Let $s_\theta(z) \in \mathbb{S}$ be the security issued by a firm of type θ.

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[16]See, for example, the discussion in Innes (1990). As pointed out in Nachman and Noe (1994), condition (2) is critical to obtain debt as an optimal security. In absence of (2), the optimal contract may have a "do or die" component, whereby outside investors obtain all of the firm cash flow when it falls below a certain threshold, and nothing otherwise.

2.2 Equilibrium

Following the literature, we will adopt the notion of Perfect Bayesian Equilibrium, PBE, as the solution concept for the capital raising game.

Definition 3 (**Equilibrium**). *A Perfect Bayesian Equilibrium (PBE) of the capital raising game is a collection $\{s_G^*(z), s_B^*(z), p^*(s), V^*(s)\}$ such that: (i) $s_\theta^*(z)$ maximizes $W(\theta, s, V^*(s))$ subject to the constraint that $s \in \mathbb{S}$ and $V^*(s) \geq I$, for $\theta \in \{G, B\}$, (ii) securities are fairly priced, that is $V^*(s) = p^*(s)\mathbb{E}[s(Z_G)] + (1 - p^*(s))\mathbb{E}[s(Z_B)]$ for all $s \in \mathbb{S}$, and (iii) posterior beliefs $p^*(s)$ satisfy Bayes rule whenever possible.*

We start with a characterization of the possible equilibria in the capital raising game. The following proposition mimics Proposition 1 of Nachman and Noe (1994) and restricts our attention to pooling equilibria.

Proposition 1. *Let F_θ satisfy strong FOSD. No separating equilibrium exists in the capital raising game. In addition, in a pooling equilibrium with $s_G^* = s_B^* = s^*$, the capital raising game is uninformative, $p(s^*) = p$, and the financing constraint is met with equality*

$$I = p\mathbb{E}[s(Z_G)] + (1 - p)\mathbb{E}[s(Z_B)]. \tag{6}$$

This equilibrium is supported by the out-of-equilibrium belief that if investors observe the firm issuing a security $s' \neq s^*$ they believe that $p(s') = p$ (passive conjectures).

Proposition 1 follows from the fact that, with two types of firms only, a type-B firm always has the incentive to mimic the behavior of a type-G firm (i.e., to issue the same security). Condition (2) and strong FOSD together imply that securities issued by a type-G firm are always priced better by investors than those issued by a type-B firm, and a type-B firm is always better-off by mimicking type-G actions. This also implies that, in equilibrium, the type-G firm is exposed to dilution due to the pooling with a type-B firm, and the corresponding loss of value can be limited by issuing only the securities needed to raise the capital outlay I.

Proposition 1 allows us to simplify the exposition as follows. Since both types of firms pool and issue the same security $s(z)$, and the capital constraint (6) is met as equality, the payoff to the original shareholders of a type-G firm, in equation (5), becomes

$$W(G, s, V(s)) = \mathbb{E}[Z_G] - I - (1 - p)\mathcal{D}_s, \tag{7}$$

where the term

$$\mathcal{D}_s \equiv \mathbb{E}[s(Z_G)] - \mathbb{E}[s(Z_B)] \tag{8}$$

represents the mispricing when security $s \in S$ is used, which is the cause of the dilution suffered by a firm of type G.

Since, from Proposition 1, type-B firms will always pool with type-G firms, firms of type G will find it optimal to issue a security that minimizes dilution $\mathcal{D}_s$, that is

$$\min_{s \in \mathbb{S}} \quad \mathcal{D}_s \tag{9}$$

subject to the financing constraint (6).

2.3 Asymmetric Information in the Right Tail

Nachman and Noe (1994) show that the solution to the optimal security design problem (9) is standard debt for all investment levels I (and, thus, the pecking order obtains) if and only if the distribution F_G dominates F_B by (strong) CSD. The aim of our paper is to study (and characterize) economic environments where private information orders firm-value distributions F_θ by FOSD, but not by CSD, leading to potential violations of the pecking order. We will show below that the properties of optimal security (and failures of the pecking order) hinge critically on the impact of asymmetric information on the right tail of the firm-value distributions F_θ.

The effect of asymmetric information on firm-value distributions can be characterized by a function $H(z)$, defined as

$$H(z) \equiv \frac{F_B(z) - F_G(z)}{1 - F(z)}, \tag{10}$$

where $F(z) = pF_G(z) + (1-p)F_B(z)$ denotes the mixture of the distributions of the good and bad types. Note that $H(z)$ is increasing in z for all $z \in R_+$ if and only if the distribution F_G dominates F_B by CSD.[17]

The function $H(z)$ measures the incremental cost to a type-G firm, relative to a type-B firm, of promising to investors an extra dollar in state z.[18] Thus, the function $H(z)$ determines the cost due to asymmetric information for a firm of type G to pay cash flows in the right tail, and will play a critical role in our analysis (see Section 3).

While FOSD dictates the monotonicity properties of $H(z)$ on the left tail of firm-value distributions, this is not the case for its right tail. In particular (as noted above) the function $H(z)$ is increasing in z, for all $z \in R_+$, if and only if CSD holds. This means that, under CSD, information asymmetries are progressively more severe for higher realizations of the firm-value distribution (and are most severe in its right tail), making debt an optimal security. If, in contrast, information asymmetries are relatively less severe in the right tail of the firm-value distributions, the function $H(z)$ is non-monotonic. In this case, it may indeed be relatively cheaper for firms of type G to pay cash flows at high realizations of z, leading to potential violations of the pecking order.

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[17]For distributions endowed with density functions, monotonicity of $H(z)$ is equivalent to requiring that the hazard rates $h_\theta(z) \equiv f_\theta(z)/(1 - F_\theta(z))$ satisfy $h_G(z) \leq (<)h_B(z)$ for all $z \in \mathbb{R}_+$; see Ross (1983) for further discussion.

[18]For monotonic securities, an extra dollar paid in state z means that investors will be paid an extra dollar also in all states $z\prime > z$.

To characterize the impact of the information cost in the right tail of the firm-value distributions, F_θ, we introduce the following definition, which will play a key role in our analysis.

Definition 4 (h-ICRT). *We will say that distribution F_G has information costs in the right tail of degree h (h-ICRT) over distribution F_B if $\lim_{z\uparrow\infty} H(z) \leq h$.*

We denote NICRT (no-information-costs-in-the-right-tail) as the case where $h = 0$. The relationship between FOSD, CSD and h-ICRT may be seen by noting that for two distributions, $\{F_G, F_B\}$, that satisfy FOSD, there may exist a sufficiently low $h \in \mathbb{R}_+$ such that the h-ICRT property holds, while CSD fails. Thus, intuitively, distributions that satisfy the h-ICRT condition fill part of the space of distributions that satisfy FOSD but do not satisfy the CSD condition. In particular, all distributions that satisfy Definition 4 for $h = 0$ (NICRT) will fail to satisfy the CSD condition.

2.4   A Real Options Model

In this section we introduce an explicit parametric specification of Myers and Majluf (1984) where firms are endowed with both assets in place and growth options. Our aim is to identify plausible economic environments that generate firm-value distributions that satisfy the NICRT condition and, thus, can lead to reversals of the pecking order. This parametric specification will be the base of the numerical examples and comparative statics derivations in Section 4.2. Furthermore, the parametric specification allows us to link some of the conditions, such as NICRT, to potentially measurable firm attributes, such as the volatility of the firm's assets in place and growth options, and their relative exposure to asymmetric information. This will allow us to make predictions on the circumstances where reversals of the pecking order are likely to be observed empirically.

We represent the firm as a collection (a portfolio) of assets, where the end-of-period firm value Z_θ is the combination of two lognormal random variables, X_θ and Y_θ. This lognormal specification closely mirrors the one invoked in the pecking-order argument of Myers and Majluf (1984). By making the investment I at $t = 0$, the firm generates a new "growth option" that can be exercised at the future date T, with resulting firm payoffs at date T:[19]

$$Z_\theta = X_\theta + \max(Y_\theta - I_T, 0). \tag{11}$$

We interpret the random variable X_θ as representing the value of the firm's assets in place at time T, and $\max(Y_\theta - I_T, 0)$ as the payoff from the growth opportunity that the firm obtains from investing I at $t = 0$. To exercise this growth opportunity, the firm must make an additional investment I_T at date T, where T represents the time-to-maturity of the new investment opportunity. The

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[19]An earlier version of the paper also included the specification $Z_\theta = \max(X_\theta, Y_\theta)$, where the firm has the option to exchange two assets, X_θ and Y_θ at the end of the period ("rainbow" or exchange option case), as in Stulz (1982), which is slightly more tractable. Details are available upon request.

additional investment I_T is financed by rasing new capital at T.[20]

As in Myers and Majluf (1984), we model asymmetric information by assuming that the firm insiders have private information on the means of the distributions, while their variances are common knowledge. Letting $\mathbb{E}[X_\theta] = \bar{X}_\theta$ and $\mathbb{E}[Y_\theta] = \bar{Y}_\theta$, we assume $\bar{X}_G \geq \bar{X}_B$ and $\bar{Y}_G \geq \bar{Y}_B$, with at least one strict inequality. These assumptions ensure FOSD, and allow for scenarios in which the NICRT condition holds. To measure the asymmetric information costs associated with the two drivers of firm value, we define $c_x \equiv \bar{X}_G - \bar{X}_B$ and $c_y \equiv \bar{Y}_G - \bar{Y}_B$. The variables c_x and c_y measure the exposure to asymmetric information of the assets in place and the growth opportunity, respectively.

We also assume that the growth option is of the European type.[21] Note that, by setting $I_T = 0$, this specification nests the case of a multi-divisional firm, where $Z_\theta = X_\theta + Y_\theta$, and I represents an investment need at $t = 0$.

The following proposition provides conditions at which CSD and NICRT obtain in this specification.

Proposition 2. *Let X_θ and Y_θ be two lognormal random variables with $\mathbb{E}[X_\theta] = \bar{X}_\theta$, and $\mathbb{E}[Y_\theta] = \bar{Y}_\theta$. Without loss of generality, assume that $\sigma_y > \sigma_x$. Then:*

1. *If $Z_\theta = X_\theta$, that is, Z_θ has a lognormal distribution, then F_G dominates F_B by CSD if and only if $\bar{X}_G > \bar{X}_B$.*

2. *If $c_y \equiv \bar{Y}_G - \bar{Y}_B = 0$, and $c_x \equiv \bar{X}_G - \bar{X}_B > 0$, then the NICRT condition holds.*

Proposition 2 allows us to compare our model with Myers and Majluf (1984). Part (i) of the proposition confirms the conclusions of the original Myers and Majluf (1984) paper: when firm value is characterized by a single lognormal random variable (that is, under Black and Scholes (1973) conditions) both FOSD and CSD hold and debt is the optimal security. Part (ii) of Proposition 2 shows that the introduction of a second source of uncertainty can break the connection between FOSD and CSD that is assured with lognormal distributions. The introduction of a second state variable is akin to moving from the 2-state model to the 3-state model in Section 1. The proposition also shows that, in the limiting case where is no asymmetric information on the high volatility asset Y_θ, our new NICRT condition is always satisfied. These violations of CSD condition lead to the deviations of the pecking order that we describe in Sections 3 and 4.

---

[20]Given our assumption that at date T there are no capital markets imperfections, firms raise the needed investment amount I_T if $Y_\theta > I_T$. Alternatively, we could assume the random variables X_θ and Y_θ are actual cash payments, in which case the firm will certainly exercise the call option $\max(Y_\theta - I_T, 0)$ if it is in-the-money. We note that the parameter I_T does not play a role in our analytical results, but it affects our numerical examples: it moves mass to the right-tail of the payoff distribution.

[21]This eliminates issues related to the optimal exercise of the growth option, which we leave for future research (see, e.g., Morellec and Schürhoff, 2011, for a model with endogenous investment timing).

Proposition 2 shows that simple deviations from the univariate lognormal model of Myers and Majluf (1984) can generate environments where the CSD condition does not hold and, thus, violations of the pecking order may occur. Importantly, Part (ii) of Proposition 2 suggests that second moments of firm-value distributions play a critical role in generating violations of CSD. For Gaussian random variables (such as lognormal distributions) second moments of the distributions characterize tail behavior. This implies that the joint assumptions that Y_θ has higher volatility, $\sigma_y > \sigma_x$, and it suffers no information costs, $c_y = 0$, are sufficient to guarantee that the NICRT condition holds. We conclude by emphasizing that, while the NICRT condition is sufficient to generate non-monotonic $H(z)$ functions, it is by no means necessary, as the numerical solutions in Section 3 will demonstrate. In particular, we will show that violations of CSD, and deviations from the pecking order, may occur when the asset that has relatively lower exposure to asymmetric information also has greater volatility, that is when c_x is large relative to c_y.

3   Optimal Security Design

In this section we solve the general optimal security design problem (9), and we provide conditions under which equity-like securities, such as convertible debt and warrants, emerge as optimal securities. Our analysis closely follow Nachman and Noe (1994), which we extend to the case where the CSD condition does not hold.

Following Nachman and Noe (1994), the optimal security design problem (9) can be expressed as

$$\min_{s \in \mathcal{S}} \int_0^\infty s'(z)(F_B(z) - F_G(z))dz, \tag{12}$$

subject to

$$\int_0^\infty s'(z)(1 - F(z))dz = I. \tag{13}$$

The Lagrangian to the above problem is

$$\mathcal{L}(s', \gamma) = \int_0^\infty s'(z)(F_B(z) - F_G(z) - \gamma(1 - F(z)))dz \tag{14}$$

$$= \int_0^\infty s'(z)(1 - F(z))(H(z) - \gamma)dz. \tag{15}$$

It is easy to verify that linearity of the security design problem implies that a solution s^* must satisfy, for some $\gamma \in \mathbb{R}_+$,

$$(s^*)'(z) = \begin{cases} 1 & \text{if } H(z) < \gamma; \\ [0,1] & \text{if } H(z) = \gamma; \\ 0 & \text{if } H(z) > \gamma. \end{cases} \tag{16}$$

Note that the value of the Lagrangian multiplier γ depends on the tightness of the financing

constraint (13) and, thus, on the level of the required investment, I, with $\partial\gamma/\partial I > 0$. From $H(0) = 0$ and FOSD the optimal security must satisfy $(s^*)'(z) = 1$ in a right neighborhood of $z = 0$. This means that an optimal security will always have a (possibly small) straight-debt component.[22] The importance of this straight-debt component (that is, the face value of the debt) will depend on the size of the investment, I (since it affects the Lagrangian multiplier γ), as well as on the particular functional form for $H(z)$.

The overall shape of the optimal security for a greater value of z depends on the monotonicity properties of the function $H(z)$ (and, thus, on the extent of asymmetric information in the right tail of the firm-value distribution). It is characterized in the following proposition.

Proposition 3. *Consider the security design problem in equations (12)–(13).*

(a) *If the distribution F_G conditionally stochastically dominates F_B, then straight debt is the optimal security (Nachman and Noe, 1994).*

(b) *If the NICRT condition holds, and $H'(z^*) = 0$ for a unique $z^* \in \mathbb{R}_+$, then convertible debt is optimal for all investment levels I.*

(c) *If $\lim_{z\uparrow\infty} H(z) = \bar{h} > 0$ and there exists a unique $z^* \in \mathbb{R}_+$ such that $H'(z^*) = 0$, then there exists an $\bar{I}$ such that for all $I \leq \bar{I}$ straight debt is optimal, whereas for all $I \geq \bar{I}$ convertible debt is optimal.*

Part (a) of Proposition 3 assumes CSD. In this case, monotonicity of the function $H(z)$ implies that there is a z^* below which $(s^*)'(z) = 1$, for all $z \leq z^*$, with $(s^*)'(z) = 0$ otherwise, yielding straight debt as an optimal security. The intuition for the optimality of straight debt follows from Nachman and Noe (1994): firms of better types prefer to have the maximum payout to investors for low realizations of z, that is in the (right) neighborhood of $z = 0$, where the impact of asymmetric information is relatively weaker, and then to limit the payout to investors for high realizations of z, where the impact of asymmetric information is always higher (under CSD). These considerations, together with the requirement that the security is monotonic, lead to the optimality of debt contracts.

Table 2 presents numerical examples, following the parametric specification introduced in Section 2.4. Figure 1 plots the $H(z)$ function in the left panels, and the optimal security in the right panels, each row corresponding to each of the cases in Table 2. In all cases, we assume that $p = 0.5$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $\rho = 0.5$, $T = 5$, and that $I_T = 50$. Case A presents the case where the asymmetric information is concentrated entirely in the high volatility asset, Y, which corresponds to part (a) in Proposition 3. Namely, we set $\bar{X}_G = \bar{X}_B = 100$, $\bar{Y}_G = 250$, $\bar{Y}_B = 150$ and $I = 100$.

---

[22]Note, however, that as Proposition 4 below shows, this property hinges critically on the assumption that the firm has no pre-existing debt.

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In this case, the $H(z)$ function is monotone over its whole domain (see the top left graph in Figure 1). Thus, the optimal security is standard debt with a face value $K = 138.8$.

Part (b) of Proposition 3 provides conditions under which securities with equity-like components, such as convertible debt, are optimal. The key driver of the optimal security choice is the size of the informational costs in the right tail of the payoff distribution, measured by $H(z)$. Under NICRT, we have that, in the limit, $H(z) = 0$ and, thus, that the information costs suffered by a type-G firm becomes progressively smaller as the firm value z increases. Part (b) of Proposition (3) shows that, in this case, type-G firms can reduce their overall dilution by maximizing the payout to investors in the right tail of the distribution, in addition to a neighborhood of $z = 0$. Increasing the payoffs in the right tail, where information costs are now low due to NICRT, allows the firm to correspondingly reduce the (fixed) payout in the middle of the distribution, where the information costs are now relatively high. The optimal security has the shape of a convertible bond, where the bond is convertible into 100% of equity with a lump-sum payment to original shareholders equal to κ, which we will refer to as the "conversion price."

Case B in Table 2 illustrates Part (b) of Proposition 3. Namely, we set $\bar{Y}_G = \bar{Y}_B = 200$, $\bar{X}_G = 150$, $\bar{X}_B = 50$, and $I = 120$. In this case, the $H(z)$ function is "hump-shaped" (see the middle left graph in Figure 1). The NICRT condition holds, since the asymmetric information is concentrated entirely in the low-volatility asset, X. Thus, the optimal security is a convertible debt contract with face value $K = 69.5$ and conversion price $\kappa = 593.4$.[23] As shown in Proposition 2 securities load in the lower-end of the payoffs, due to the usual Myers and Majluf (1984) intuition, but also on upper-end of the payoff distribution, because of the NICRT property introduced in our paper.

In part (c) of Proposition (3), neither CSD nor NICRT hold, since we have both a non-monotone function H and the h-ICRT condition holds for $\bar{h} > 0$.[24] The proposition shows that the size of a project affects the financing choices of a firm: straight debt is optimal for low levels of I, while convertible debt becomes optimal for large levels of the investment I. When investment needs are low, the firm can finance the project by issuing only straight debt, a security that loads only in the left tail of the distribution, where the information costs are the lowest. For greater investment needs, under h-ICRT the firm finds it optimal to maximize its payout to investors in the right tail of the distribution, as in part (b) of the proposition, issuing convertible debt.

---

[23]The values $\{K, \kappa\}$ satisfy $H(K) = H(\kappa) = \gamma$.

[24]We note that the Proposition does not cover the case where $H'(z)$ may have more than one sign change. The proof can be easily altered to consider this case: the optimal security will be similar to a structured product with different tranches/call features.

4 The Pecking (Dis)Order

In this section, we examine a special case of the capital raising game by restricting our attention to two classes of securities, namely debt and equity. We consider this case explicitly because the debt-equity choice problem has attracted so much attention in both the theoretical and empirical corporate finance literature. Note that equity is never a solution to the optimal security design problem (9), as its slope is always in the $(0, 1)$ interval. It is however possible that equity is less dilutive than debt in the cases where convertible securities are optimal, such as cases (b) and (c) in Proposition 3.

In Section 4.1 we identify the key economic drivers of the debt-equity choice, and we relate them to the example of Section 1. In Section 4.2 we study the debt-equity choice within the real-options specification in Section 2.4. In particular, we show that the parameter configurations that make convertible debt optimal in Section 3 mirror the ones that favor equity over debt.

4.1 The Debt-Equity Choice

To identify the key drivers of the relative dilution of debt and equity, note that the dilution costs (8) associated with equity and debt are given by

$$
\begin{aligned}
\mathcal{D}_E &= \lambda\left(\mathbb{E}[Z_G] - \mathbb{E}[Z_B]\right), \quad \text{and} &(17)\\
\mathcal{D}_D &= \mathbb{E}[\min(Z_G, K)] - \mathbb{E}[\min(Z_B, K)], &(18)
\end{aligned}
$$

respectively, where $\lambda = I/\mathbb{E}[Z]$, with $\mathbb{E}[Z] = p\mathbb{E}[Z_G] + (1\text{-}p)\mathbb{E}[Z_B]$ denoting the unconditional value of the firm, and the parameter K represents the (smallest) face value of debt that satisfies the financing constraint

$$
I = p\mathbb{E}[\min(Z_G, K)] + (1 - p)\mathbb{E}[\min(Z_B, K)]. \tag{19}
$$

The dilution of debt relative to equity can then be written as

$$
\mathcal{D}_D - \mathcal{D}_E = \int_0^\infty \left(\min(z, K) - \lambda z\right) c(z)dz, \tag{20}
$$

where $c(z) \equiv f_G(z)\text{-}f_B(z)$. Intuitively, the function $c(z)$ is related to the cost due to asymmetric information that are suffered by a firm of type-G, when pooling with a firm of type-B and issuing a security with a payoff of \$1 if the final firm value is z. Thus, if $c(z) > 0$ we will say that the "information costs" for a type-G are positive, and that these costs are negative if $c(z) < 0$. The function $c(z)$ for our base-case values (see Table 3) is displayed in the top portion of Figure 2, together with the payoffs of the debt, $\min(z, K)$, and equity, λz. Let $\hat{z}$ and $\bar{z}$ be defined by $c(\hat{z}) = 0$ and $\lambda\bar{z} = K$.

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The relative dilution of debt and equity (20), assuming that $\bar{z} > \hat{z}$,[25] can be further decomposed as follows:

$$\mathcal{D}_D - \mathcal{D}_E = -\int_0^{\hat{z}} (\lambda z - \min(z, K))c(z)dz + \int_{\hat{z}}^{\bar{z}} (\min(z, K) - \lambda z)c(z)dz - \int_{\bar{z}}^{\infty} (\lambda z - K)c(z)dz. \quad (21)$$

The decomposition (21) reveals that the preference for a type-G firm of debt versus equity financing depends on the relative importance of three regions that together concur to determine the overall relative dilution of debt and equity. We note that these three regions are reminiscent of the trinomial distribution considered in Section 1.

In the first "low-value" region information costs to a firm of good type are negative, $c(z) \leq 0$, and the payoff to equity is lower than the payoff to debt, $\lambda z < \min(z, K)$, the first term in (21). In this region, debt is less dilutive than equity because it has a higher payout than equity, but these payouts have negative information costs. These effects echo the traditional intuition that type-G firms have a preference to promise investors payouts in states of the world with low realizations of firm value (the left-tail of the firm-value distribution) precisely because these states are relatively less likely to occur to firms of good type.

In the second "intermediate-value" region debt still has higher payouts than equity, but now type-G firms suffer a positive information cost, $c(z) > 0$, given by the second term in (21). In this region dilution costs of equity are lower than those of debt because equity has a lower payoff than debt and information cost are positive. The presence of this region, and its relative importance, can generate a reversal of the pecking order.

The third and last region is a "high-value region," where equity payoff is now greater than debt in states of the world that are more likely to occur to a type-G firm, and thus carry positive information costs, $c(z) > 0$, the third term in (21). In this region, which occurs for high payoff realizations, debt is less dilutive than equity because equity has higher payout than debt and it has a positive information cost for type-G firms. These effects echo again the traditional intuition that type-G firms dislike to promise investors payouts in states of the world with high realizations of firm value (the right-tail of the firm-value distribution) precisely because these states are relatively more likely to occur for type-G firms.

The preference for debt over equity financing (the pecking order) depends on the relative importance of these three regions, i.e., the three terms in (21). In particular, equity dominates debt when the advantages of equity financing originating from the intermediate region dominate the disadvantages determined by the low-value and high-value regions. Note that the relative importance of these three regions depends crucially on the term $c(z)$ and, thus, on how information asymmetries affect the firm-value distributions (i.e., the location of the information asymmetry in the domain of the firm-value distribution).

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[25]In the case where $\bar{z} \leq \hat{z}$ it is straightforward to verify that debt dominates equity. In all the numerical examples we have studied we have $\bar{z} > \hat{z}$.

Intuitively, equity can be less dilutive than debt when asymmetric information is relatively more pronounced in the center of the firm-value distribution, generating large values of $c(z)$ in the intermediate-value region, while it has a relatively smaller impact on either the left tail (the low-value region) or the right tail (the high-value region) of the firm-value distribution. While (strong) FOSD (always) induces a preference for debt financing, through its effect on the first term in (21), the importance of the high-value region depends on the impact of asymmetric information on the right tail of the distribution. If the third term in (21) is sufficiently small, or zero, which can happen when the NICRT condition holds, a reversal of the pecking order may occur.

4.2 Cross-Sectional Predictions

We study the debt-equity choice problem of Myers and Majluf (1984) within the parametric specification introduced in Section 2.4. Because it is well known that real-options models of this nature do not admit closed-form solutions, following existing literature (see, for example, Childs, Mauer, and Ott, 2005; Gamba and Triantis, 2008, among many others) we study the problem numerically.

Our numerical solutions are centered on the base case reported in Table 3. In this base case, asymmetric information is more severe on assets in place, where $\bar{X}_G = 125$ and $\bar{X}_B = 75$, rather than the growth opportunity, where $\bar{Y}_G = 205$ and $\bar{Y}_B = 195$. In addition, we assume that assets in place have lower return volatility than the growth opportunities, as in Berk, Green, and Naik (2004), and we set $\sigma_x = 0.3$, $\sigma_y = 0.6$, $T = 15$ and $\rho = 0$. We let both types be equally likely, $p = 0.5$. We highlight that under these parameters Z_G does not dominate Z_B in the conditional stochastic dominance sense. In this base case specification, we set the initial investment amount to be $I = 100$, and the investment at exercise of the growth option to be $I_T = 50$.

The lower panel of Figure 2 plots the distributions, $f_\theta(z)$, of Z_θ, as well as the unconditional distribution $f(z)$ for our base case values. By direct inspection, it is easy to verify that the distribution of firm value Z_θ closely resembles a lognormal distribution, with the important difference that the asymmetric information loads in the middle of the distribution, and to a lesser extent in its right tail.[26]

Note first that the efficient outcome is for both types of firms to finance the project, since for a type-G firm $\mathbb{E}[Z_G] - I = 307.9 - 100 = 207.9 > 125 = \bar{X}_G$, and for a type-$B$ firm $\mathbb{E}[Z_B] - I = 248.2 - 100 = 148.2 > 75 = \bar{X}_B$. If the investment opportunity is taken, the value of the firm is $p\mathbb{E}[Z_G] + (1-p)\mathbb{E}[Z_B] = 278.1$, where the values for the two types is given by $\mathbb{E}[Z_G] = 307.9$ and $\mathbb{E}[Z_B] = 248.2$. It is easy to verify that issuing equity will require that the equity holders give up a stake of $\lambda = 0.360 = 100/278.1$. In order to finance the project with debt, the firm needs to promise bondholders a face value at maturity of $K = 218.4$. The value of the debt for a type-G and type-B firm is $\mathbb{E}[\min(Z_G, K)] = 111.9$ and $\mathbb{E}[\min(Z_B, K)] = 88.1$, respectively. The dilution costs of equity are $\mathcal{D}_E = 0.36 \times (307.9 - 248.2) = 21.5$, whereas those of debt are $\mathcal{D}_D = 111.9 - 88.1 = 23.7$, with

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[26]Recall that if Z_θ is log-normal (e.g. if $Z_\theta = X_\theta$), then CSD holds and debt is the optimal security.

a relative dilution $\mathcal{D}_D/\mathcal{D}_E = 23.7/21.5 = 1.10$. Thus, the type-$G$ firm is exposed to lower dilution by raising capital with equity rather than debt.[27]

The bottom portion of Table 3 and Figures 3–5 examine the impact of changes of some of the key parameters in the base case on the relative dilution of debt and equity. These examples highlight one of the new insights of our paper: the relative dilution of debt and equity financing is not driven by the overall level of asymmetric information affecting a firm but by the relative exposure to private information of each assets in a firm's portfolio, their volatilities, and the firm's investment requirements. These example provide new and testable predictions on the cross-sectional variation of firms' capital raising choices.

4.2.1 Asymmetric Information and Dilution

The first set of examples focus on the effect of changes of the exposure to asymmetric information of the assets in place relative to the growth opportunity. In order to do so, we let $c_x = \bar{X}_G - \bar{X}_B$ denote the asymmetric information on the assets in place, and we let $c_y = \bar{Y}_G - \bar{Y}_B$ denote the asymmetric information on the growth options. The base case has $c_x = 50$ and $c_y = 10$, so that the information asymmetry is more significant for the assets in place than for the growth option.

In the first two examples in the lower panel of Table 3, we change the exposure to asymmetric information in the assets in place to $c_x = 0$ (letting $\bar{X}_G = \bar{X}_B = 100$), and to $c_x = 100$ (letting $\bar{X}_G = 150$ and $\bar{X}_B = 50$). When there is no information asymmetry about the assets in place, we find that debt is significantly less dilutive than equity: $\mathcal{D}_D = 0.7$ versus $\mathcal{D}_E = 3.5$. This is in line with our previous results, since when $c_x = 0$ the CSD condition holds, and debt is optimal.

In the next two examples in the lower panel of Table 3, we change the exposure to asymmetric information in the growth options to $c_y = 0$ (letting $\bar{Y}_G = \bar{Y}_B = 200$), and to $c_y = 50$ (letting $\bar{Y}_G = 225$ and $\bar{Y}_B = 175$). The first case corresponds to a parametrization that satisfies the NICRT condition from Section 2.3: lowering the asymmetric information in the right-tail of the firm value distribution increases the relative dilution of debt (from 1.10 to 1.27). When we increase the exposure to asymmetric information in the right tail, setting $c_y = 50$, the dilution of debt relative to equity drops to 0.76, making debt the preferred financing vehicle.

More generally, the impact of the relative exposure to asymmetric information of assets in place and growth opportunities on debt and equity dilution is further studied in the top graph in Figure 3. The pictures displays indifference lines where $\mathcal{D}_D = \mathcal{D}_E$, as a function of the exposure to asymmetric information of the assets in place, c_x, and the growth opportunity, c_y, for three levels of the volatility of the growth opportunity, $\sigma_y \in \{0.6, 0.7, 0.8\}$. In the region above the lines, we

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[27]It is worthwhile to remark that the investment choices are individually rational when using either debt or equity. To see this, note that in the case of equity financing the residual equity value for a type-G firm is equal to $(1 - 0.36) \times 307.9 = 197.1 > 125 = \bar{X}_G$, and for a type-$B$ firm it is equal to $(1 - 0.36) \times 248.2 = 158.9 > 75 = \bar{X}_B$. In the case of debt financing, the residual equity value for a type-G firm is equal to $307.9 - 119.9 = 188 > 125 = \bar{X}_G$, and for a type-$B$ firm it is equal to $248.2 - 88.1 = 160.1 > 75 = \bar{X}_B$.

have that $\mathcal{D}_D > \mathcal{D}_E$ and, hence, equity is less dilutive than debt and the reverse pecking order obtains. In the region below the lines, we have that $\mathcal{D}_D < \mathcal{D}_E$ and, hence, equity is more dilutive than debt, and the usual pecking order obtains. These graphs reveal that equity is more likely to be less dilutive than debt when the exposure to asymmetric information on the less volatile assets in place, c_x, is large and when the exposure to asymmetric information of the more volatile growth opportunities, c_y, is small. In addition, the parameter region where equity dominates debt becomes larger when the volatility of the growth opportunity increases.

The bottom graph of Figure 3 examines the effect of a firm's asset composition on dilution. The graphs display the average values of assets in place and of the growth option $(\bar{X}, \bar{Y})$, where $\bar{X} = p\bar{X}_G + (1-p)\bar{X}_B$ and $\bar{Y} = p\bar{Y}_G + (1-p)\bar{Y}_B$, for which the dilution costs of equity and debt are the same $(\mathcal{D}_E = \mathcal{D}_D)$ for different level of asymmetric information on asset $c_x \in \{10, 25, 40\}$. For pairs of $(\bar{X}, \bar{Y})$ below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt when the growth opportunities represent a larger component of firm value. In addition, the parameter region where equity dominates debt becomes larger when the exposure to asymmetric information of assets in place, c_x, increases.

These comparative static results on the choice of debt versus equity mirror the security design results showing that right-tail exposure to asymmetric information tends to make debt optimal, whereas lack of such exposure makes equity-like securities the optimal financing instrument.

4.2.2 Asset Volatility and Investment Requirements

We consider now the effect of the volatility parameters, σ_x and σ_y, on the debt-equity choice. In the next two examples in the lower panel of Table 3, we document that an increase of the volatility of the assets in place to $\sigma_x = 0.40$ has the effect of reducing the dilution of debt relative to equity from $D_D/D_E = 1.10$ to 1.01, while an increase of the volatility of the growth opportunity to $\sigma_y = 0.80$ has the opposite effect of increasing the relative dilution of debt and equity to 1.53. The increase in volatility of both the assets in place and the growth opportunity makes the right-tail more important in both cases, with opposing effects on the relative dilution of debt and equity.

More generally, the top graph of Figure 4 plots the pairs of volatilities, (σ_x, σ_y), such that the dilution costs of equity and debt are the same (i.e., $\mathcal{D}_E = \mathcal{D}_D$) for three levels of the investment cost $I \in \{100, 110, 120\}$. For pairs of volatilities, (σ_x, σ_y), below the lines debt is optimal, whereas equity is optimal above the lines. These graphs reveal that equity is more likely to be less dilutive than debt when the volatility of assets in place is low, and when the volatility of growth opportunities is large. In addition, the parameter region where equity dominates debt becomes larger when the firm's investment need, I, increases. The bottom panel in Figure 4 performs a similar exercise changing both time to maturity and the initial investment, I, for different values of the assets in place, $\bar{X}$. Rather intuitively, higher time to maturity puts more mass on the right-tail of the payoff

distribution, favoring equity. More assets in place, and/or lower investment requirements, I, on the other hand, lower debt's dilution relative to equity.

These examples show that equity is less dilutive than debt when the volatility of the growth opportunities is sufficiently large relative to the volatility of the assets in place, and the asymmetric information is concentrated in the assets in place. We note that the critical role played by the relative volatilities of growth opportunities and assets in place was already discussed in Proposition 2, and our results of the optimality of equity mimic those in Proposition 3.

In the next to last set of examples in Table 3, we focus on the impact of the firm's investment requirements, I and I_T on dilution. When we change the investment amount from $I = 100$ to $I = 80$ we find that the relative dilution of debt is 0.95, which is lower than under our base case parameters. The opposite occurs when we raise investment to $I = 120$, mirroring the results from part (c) of Proposition 3. A decrease of the future investment requirement, from $I_T = 50$ to $I_T = 0$, reduces the dilution of debt relative to equity to 0.88, restoring the pecking order. In contrast, an increase of the subsequent investment to $I_T = 100$ worsens the relative dilution of debt and equity, which is now equal to 1.18. An increase of the subsequent investment requirements I_T, increases the "exercise price" of the growth option and it has the same effect as an increase of the volatility σ_y (it essentially shifts mass to the right-tail, where in our base case informational costs are low).

More generally, the impact of investment requirements are further explored he top graph of Figure 5. The graph reveals that equity financing is more likely to be less dilutive than debt when the firm has greater investment needs either at the time of the initial investment, I, or at the time the growth option is exercised, I_T. These observations imply that capital needs of the firm will have an independent effect on the financing decisions: specifically, equity is more likely to be less dilutive than debt when firms have greater investment requirements.

In summary, the examples in Tables 2 and 3, as well as Figures 3–5, reveal a very consistent pattern: violations of the pecking order are likely to emerge in younger firms, endowed with valuable growth opportunities that represent a greater proportion of firm value and require larger investments. In addition, equity is more likely to be less dilutive than debt when such growth opportunities are riskier and have lower exposure to asymmetric information than the assets in place.

5   The Effect of Pre-Existing Debt

In our basic model, we assume that at the beginning of the period, $t = 0$, the firm is all equity financed. In this section we allow for the possibility that firms have pre-existing debt in the capital structure, a situation that was not studied in Nachman and Noe (1994). The presence of debt in the initial capital structure may, for example, be the outcome of previous security issuance, which we do not model explicitly.

At the beginning of the period, $t = 0$, the firm has already issued straight debt with face value

$K_0 \geq 0$ which is due at the end of the period, T. In accordance to anti-dilutive "me-first" rules that may be included in the debt covenants, we assume that this pre-existing debt is senior to all new debt that the firm may issue at $t = 0$. We maintain the assumption that the firm always finds it optimal to raise external capital, I, through either an equity or a debt offer.[28] We assume throughout this section that the random variable $Z_G \mathbf{1}_{Z_G \geq K_0}$ dominates $Z_B \mathbf{1}_{Z_B \geq K_0}$ in the FOSD sense.[29]

We first extend the results in Section 3 on general securities, and then revisit the debt-equity choice.

5.1 Security design

The security design game is modified as follows. The firm raises the desired capital I by issuing a security $s \in \mathbb{S}$ which is junior to the existing debt, K_0. Thus, the set $\mathbb{S}$ satisfies (2)-(3), with the added constraints $s(z) = 0$ for all $z < K_0$, and

$$0 \leq s(z) \leq z - K_0, \quad \text{for all} \ \ z \geq K_0.$$

The presence of pre-existing debt changes the structure of information costs in a non-trivial way, because cash flows in the left tail of the distribution can no longer be pledged to new investors. Interestingly, pre-existing debt makes equity-like securities, such as warrants, relatively more attractive, as shown in the following proposition.

Proposition 4. *Consider the optimal security design problem when the firm has a senior debt security with face value K_0 outstanding. Assume that the NICRT condition holds, and that there exists a unique z^* such that $H'(z^*) = 0$.*

(a) *If $H'(K_0) > 0$, then there exists $\hat{I}$ such that: (i) warrants are optimal for $I < \hat{I}$, and (ii) convertible debt is optimal for $I \geq \hat{I}$.*

(b) *If $H'(K_0) < 0$, then the optimal securities are warrants.*

Proposition 4 shows that levered equity, such as warrants, can arise as an optimal financing instrument.[30] Intuitively, warrants are optimal securities because pre-existing debt has absorbed the information benefits in the left tail of the distribution (that is, in a right neighborhood of $z = 0$,

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[28]This assumption allows us to ignore a possible debt overhang problem in the sense of Myers (1977).

[29]This is without loss of generality: the case where $Z_B \mathbf{1}_{Z_B \geq K_0}$ FOSD $Z_G \mathbf{1}_{Z_G \geq K_0}$ follows as in the text, changing the labels G and B.

[30]Our results are reminiscent of the optimality of unit IPOs in Chakraborty, Gervais, and Yılmaz (2011), see pages 336–337 and Proposition 2 in their paper. In that paper, the use of warrants in unit IPOs is optimal when asymmetric information (between buyers) is mostly concentrated in "bad states," thus reducing the winners' curse problem. In contrast to Chakraborty, Gervais, and Yılmaz (2011), who compare equity to other option-like securities, we show how warrants are the optimal financing instruments among all monotone securities, minimizing insiders' dilution.

as discussed above). In particular, when K_0 is moderate, so that $H'(K_0) > 0$, NICRT implies that the optimal security design is one that always loads in the right tail, where information costs are now the lowest (since the left tail is already committed to pre-existing bondholders). In addition, when the financing needs are low, the firm is able to raise the required capital by issuing only warrants. When the financing needs are high, the firm raises the additional capital by also issuing (junior) debt, that is, by using convertible debt. When K_0 is large, so that $H'(K_0) < 0$, the firm will always find it optimal to issue only warrants (since the firm now faces decreasing information costs). We note that warrants can emerge as optimal securities when the firm has pre-existing debt in its capital structure, even when the asymmetric information environment is such that straight debt would be optimal in the absence of pre-existing debt.

Case C in Table 2 studies the effect of pre-existing debt on the security design problem discussed in Proposition 4. We modify Case B by assuming that the firm has debt outstanding with $K_0 = 100$, and that the initial investment is 70 (see the lower left graph in Figure 1). The value of the pre-existing debt is 79.3, while total firm value (debt plus equity) is equal to 259.2. The fact that the cash flows below $K_0 = 100$ have been pledged makes the optimal security design be a warrant with an exercise price of $\kappa = 502.5$, as in the case (i) of part (a) of Proposition 4.

If we change the initial investment from 70 to 120, as in the previous Case B in Table 2, the optimal security will again be convertible debt, where the face value of the new (junior) debt is $K = 135.7$, and the conversion price becomes $\kappa = 317.2$, as in case (ii) of part (a) of Proposition 4. It can be shown that, given the parameters values of Case C, warrants will always be optimal if the pre-existing debt has a face value of $K_0 \geq 199$, as in part (b) of Proposition 4.

5.2 Pre-Existing Debt and the Pecking Order

We assume that the firm can raise the necessary capital either by sale of junior debt with face value K, or by sale of a fraction λ of total (levered) equity of the firm to outside investors. Following an argument similar to the one in Section 4, the relative dilution of debt versus equity is now given by:

$$\mathcal{D}_D - \mathcal{D}_E = \int_{K_0}^{\infty} \left[(1 - \lambda) \max(z - K_0, 0) - \max(z - (K_0 + K), 0) \right] c(z) dz. \tag{22}$$

The main difference of (22) relative to the corresponding expression (20) is the fact that all payoffs below K_0 are now allocated to the pre-existing senior debt. This implies that only the probability mass located in the interval $[K_0, \infty)$ is relevant for the determination of the relative dilution costs of debt and equity and, thus, for the choice of financing of the new project. Recall from (21) that the two regions located at the left and the right tails of the probability distribution favor debt financing, while the intermediate region favors equity financing. This observation implies that the presence of pre-existing debt in a firm's capital structure, by reducing the importance of the left tail region, makes equity more likely to be the less dilutive source of financing (all else equal), and

therefore for the pecking order to be reversed.

Returning to our numerical exercise from Section 4.2, the last two rows in Table 3 show how the presence of pre-existing debt with face value $K_0 = 20$ in our base-case parameter constellation has the effect of increasing the relative dilution of debt to equity to 1.28, raising the advantage to equity relative to debt financing. This effect is further reinforced at greater levels of pre-existing debt: for $K_0 = 40$ the relative dilution of debt to equity becomes 1.47.

The bottom graph of Figure 5 examines the impact of pre-existing debt on the form of financing. The graph reveals that, for a given level of assets in place, $\bar{X}$, equity financing is less dilutive than debt when the firm has a greater amount of pre-existing debt, K_0. In addition, the graph suggests that firms are likely to switch from equity to debt financing as they accumulate assets in place, that is as $\bar{X}$ becomes larger. At the same time, firms that finance asset acquisitions through debt financing are likely to switch to equity financing as they increase the amount of debt in their capital structure, K_0.

6   Empirical Implications

The validity of the pecking order theory has been challenged by several empirical studies. For example, Frank and Goyal (2003) and Fama and French (2005) document that small, high-growth firms, a class of firms which is presumably more exposed to asymmetric information, typically rely heavily on financing through outside equity, rather than debt. Leary and Roberts (2010) conclude that "the pecking order is never able to accurately classify more than half of the observed financing decisions."[31] In an extensive study, Fama and French (2005) conclude that "asymmetric information problems are not the sole (or perhaps even an important) determinant of capital structures."

We argue that this conclusion is not warranted: deviations from the pecking order may occur even when asymmetric information is the only friction in the capital markets. This means that observed violations of the pecking order do not necessarily imply that asymmetric information is at best a second-order driver of the capital structure of firms. Rather, such deviations may just mean that the conditions underlying the pecking order are not met.[32]

A contribution of our paper is to identify economically relevant scenarios where the Myers and Majluf's pecking order can be reversed, and to characterize such scenarios. The results displayed

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[31]Leary and Roberts (2010) also note that most of the empirical evidence is inconclusive, and write: "Shyam-Sunder and Myers (1999) conclude that the pecking order is a good descriptor of broad financing patterns; Frank and Goyal (2003) conclude the opposite. Lemmon and Zender (2010) conclude that a 'modified' pecking order—which takes into account financial distress costs—is a good descriptor of financing behavior; Fama and French (2005) conclude the opposite. Frank and Goyal (2003) conclude that the pecking order better describes the behavior of large firms, as opposed to small firms; Fama and French (2005) conclude the opposite. Finally, Bharath, Pasquariello, and Wu (2010) argue that firms facing low information asymmetry account for the bulk of the pecking order's failings; Jung, Kim, and Stulz (1996) conclude the opposite."

[32]Note that we do not suggest that asymmetric information is the sole driver of the firm capital structure. For example, Holderness (2018) provides new evidence suggesting that agency costs can play an important role in the capital structure choice.

in Table 3, as well as Figures 3–5 reveal very consistent patterns, which we summarize below. Violations of the pecking order are likely to be observed for firms with the following properties:

(i) *Firms with larger investments needs.* Violations of the pecking order are more likely to occur when investment needs are greater. When external financing needs are smaller, firms are more likely to be able to meet such needs by issuing small amounts of debt: a bond with low default risk has a relatively smaller potential for mispricing, making it less dilutive than equity. In contrast, large debt issues (that are required by greater investment needs) are more likely to be characterized by high default risk, exposing issuing firms to the potential of greater mispricing and, thus, leading to reversals of the pecking order.

(ii) *Younger firms with valuable investment opportunities.* Our model suggests that younger firms that have large investment needs are ideal candidates for violations of the pecking order.[33] Such firms are more likely to be endowed with both assets in place and risky growth opportunities, where the assets in place are relatively more affected by information asymmetries. Greater information asymmetry on a firm's assets in place relative to its growth opportunities may emerge in cases where a firm is exposed to substantial "learning-by-doing," as in Berk, Green, and Naik (2004). In this case, firms may have accumulated more private information from operating its assets in place, relative to the still undeveloped growth opportunities, where critical information has yet to be revealed.[34] If the new growth opportunities have greater volatility than assets in place, our model shows that deviations of the pecking order may obtain.

(iii) *Firms endowed with a portfolio of heterogeneous assets.* Deviations from the pecking order may occur more generally in firms endowed with a portfolio of heterogeneous assets (or subdivisions), if the more volatile assets are also less affected asymmetric information.[35] Assets in a firm's portfolio may include both assets in place, growth opportunities, or any combination thereof. Exposure to asymmetric information in the right tail of the firm-value distribution (where equity is more valuable) is determined by the asset that has greater volatility (the "right-tail" effect uncovered in our paper). This means that violations of the pecking order do not depend, as commonly suggested, on the "absolute" level of asymmetric information that affects a firm but, rather, on the relative exposure of each asset to private

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[33]A similar prediction is in Yang and Zeng (2018), where equity financing can be valuable for entrepreneurial firms to provide investors with incentives to produce information, in the spirit of Boot and Thakor (1993) and Fulghieri and Lukin (2001).

[34]An example of such situation is provided by a pharmaceutical company whose assets are formed by fully developed drugs as well as new drugs where substantial additional R&D is necessary to obtain a commercially exploitable product. The new R&D will privately reveal to the company valuable information to assess the true commercial value of the drug, thus increasing the extent of asymmetric information with outside investors with respect to the initial patent stage.

[35]Exposure to asymmetric information has been measured by several empirical proxies in the existing literature (see Leary and Roberts (2010) for an in-depth discussion).

information. The critical role of volatility in affecting the relative dilution of debt and equity (and the pecking order) is a new testable implication that is unique to our model.

(iv) *Young firms and the pecking order.* Violations of the pecking order are more likely to occur when the duration of the growth opportunity is longer, which is presumably more likely to be the case of younger firms. Greater time to expiration of the growth opportunities allows the right-tail effect to have a greater impact. In contrast, more mature firms are more likely to raise new capital, when needed, using debt securities. The relationship between violation of the pecking order and time to maturity (duration) of a firm's growth opportunities is a new empirical implication of our model.

(v) *Firms with pre-existing debt.* Our paper suggests that firms that already have debt in their capital structure are more likely to use equity (or equity-like securities such as warrants and convertible debt) in follow-up security offerings. The presence of debt in the capital structure limits the ability of the firm to issue additional debt, increasing the potential for the next debt issue to have high default risk and, thus, to be mis-priced. This means that if firms issue securities in sequential tranches (for example, because of fixed transaction costs), a debt issue is more likely to be followed by an equity issue, and vice-versa. These considerations suggest that asymmetric information may in fact lead to "mean reversion" in leverage levels, as is often documented in the empirical literature on capital structure (see Frank and Goyal, 2003; Fama and French, 2005; Leary and Roberts, 2005). This means that asymmetric information models may be observationally equivalent to dynamic trade-off models where firms adjust over time to a certain target capital structure.

In summary, our paper can help explain the stylized fact that small and young firms with large financing needs and valuable growth opportunities (i.e. high-growth firms) often prefer equity over debt financing, even in circumstances where asymmetric information is potentially severe. Our paper can also help explaining the commonly observed financing life-cycle whereby young growth firms are initially financed by equity, and then switch to debt financing as they mature. Finally, our paper generates new potentially testable empirical predictions linking violations of the pecking order to a firm's asset structure, and the relative exposure to asymmetric information and volatility of each asset.

7   Conclusion and Future Research

In this paper, we revisit the pecking order of Myers and Majluf (1984) and Myers (1984). We show that when insiders are relatively better informed on the assets in place of their firm, rather than on its (riskier) growth opportunities, equity financing can be less dilutive than debt financing, reversing the pecking order. We find that equity is more likely to dominate debt for younger firms

that have larger investment needs and with riskier, more valuable growth, opportunities. Thus, our model can explain why high-growth firms may prefer equity over debt, and then switch to debt financing as they mature.

More generally, our results apply also to firms endowed with portfolios of heterogeneous assets, such as multidivisional firms, each with different exposure to asymmetric information. Deviations from the pecking order can occur when the asset with relatively lower volatility is more affected by asymmetric information. A firm's preference for debt versus equity financing is not driven by its overall level of asymmetric information but, rather, by the composition of its assets and by the location of the asymmetric information across assets. In a similar vein, our model generates implications for the choice of financing in mergers: the composition of the target and acquiring firms assets will determine the least dilutive financing instrument.

An additional area of applications of our model is the design of structured products, such as Collateralized Debt Obligations, CDOs. In these cases, the CDO arranger must both select mix of assets forming asset pool and the structure of the tranches of securities backed by such pools of assets. Our model suggest a novel way to link the exposures to asymmetric information of the assets in the pool, their individual volatility, and the design of the seniority structure of the tranches.

Our results in Section 5.2 suggest that, in a dynamic model of securities offering, firms that have recently issued debt are more likely (all else equal) to use equity the following round of financing, with equity and debt offerings alternating each other over time. This means that a pure asymmetric information friction may in fact lead to a "mean reversion" in leverage levels, as is often documented in the empirical literature on capital structure (see Leary and Roberts, 2005). These predictions are novel within models based on informational frictions. We leave an explicit analysis of these important issues to future research.

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Appendix

Proof of Proposition 1. In a separating equilibrium $\{s_G^*, s_B^*\}$ where we have that $s_G^* \neq s_B^*$, $p(s_G^*) = 1$, and $p(s_B^*) = 0$, which implies that $V^*(s_\theta^*) = \mathbb{E}[s_\theta^*(Z_\theta)]$ and that $W(\theta, s_\theta^*, V^*(s_\theta^*)) = \mathbb{E}[Z_\theta] - I$. This implies that $W(B, s_G^*, V(s_G^*)) - W(B, s_B^*, V(s_B^*)) = V(s_G^*) - \mathbb{E}[s_G^*(Z_B)] = \mathbb{E}[s_G^*(Z_G)] - \mathbb{E}[s_G^*(Z_B)] > 0$ by FOSD. Thus, the pair $\{s_G^*, s_B^*\}$ cannot be an equilibrium. Furthermore, if in a candidate pooling equilibrium where the security s^* is offered by both types of firms, we have that $V^*(s^*) > I$, consider the scaled down contract γs^* for $\gamma \in (0,1)$. Then, there is at least one value of $\gamma \in (0,1)$ such that $p(\gamma s^*) = p$, by passive beliefs, $V^*(\gamma s^*) \geq I$ and $W(G, \gamma s^*, V^*(\gamma s^*)) = \mathbb{E}[Z_G] - \gamma(\mathbb{E}[s^*(Z_G)] - V(s^*(Z_G))) - I > \mathbb{E}[Z_G] - (\mathbb{E}[s^*(Z_G)] - V(s^*(Z_G))) - I = W(G, s^*, V^*(s^*))$, a contradiction. Thus, any pooling equilibrium must satisfy the budget constraint with equality, $V(s) = I$. $\blacksquare$

Proof of Proposition 2. It is more convenient to use the means of the underlying normal random variables for this proof. This involves a simple notation change from the body of the text: for any random variable Z_θ such that $\text{var}(\log(Z_\theta)) = \sigma_z$ and $\bar{Z}_\theta \equiv \mathbb{E}[Z_\theta]$, the mean of the normal random variable $\hat{Z}$ such that $\mathbb{E}[e^{\hat{Z}}] = \bar{Z}_\theta$ is $\mu_\theta = \log(\bar{Z}_\theta) - 0.5\sigma_\theta^2$.

In order to prove the first statement, we argue that the distribution of the good type dominates the distribution of the bad type in the likelihood ratio sense, namely $f_G(z)/f_B(z)$ is monotonically non-decreasing for all $z \in \mathbb{R}_+$. From basic principles we have:

$$
\begin{aligned}
\frac{f_G(z)}{f_B(z)} &= \frac{\frac{1}{z\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_G}{\sigma}\right)^2}}{\frac{1}{z\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_B}{\sigma}\right)^2}} \\
&= e^{-\frac{1}{2}\left(\frac{\log(z)-\mu_G}{\sigma}\right)^2 + \frac{1}{2}\left(\frac{\log(z)-\mu_B}{\sigma}\right)^2} \\
&= e^{-\frac{1}{2}\frac{(\mu_G^2-\mu_B^2)}{\sigma^2} + \log(z)\left(\frac{\mu_G-\mu_B}{\sigma^2}\right)} \\
&= e^{-\frac{1}{2}\frac{(\mu_G^2-\mu_B^2)}{\sigma^2}} z^{\left(\frac{\mu_G-\mu_B}{\sigma^2}\right)};
\end{aligned}
$$

which is monotonically increasing in z when $\mu_G > \mu_B$, as we set to prove. Since the likelihood ratio order implies conditional stochastic dominance (Shaked and Shanthikumar, 2007), this concludes the proof.

In order to prove the second statement, we start with case in which $Z_\theta = X_\theta + Y_\theta$. Let $F_m(z)$ denote the distribution function of a lognormal random variable with log-mean μ_{yG} and log-variance σ_y^2. Since $1 - F(z) = p(1 - F_B(z)) + (1-p)(1 - F_G(z))$, we have that

$$
\lim_{z\uparrow\infty} \frac{1 - F(z)}{1 - F_m(z)} = \lim_{z\uparrow\infty} p\frac{1 - F_B(z)}{1 - F_m(z)} + (1-p)\frac{1 - F_G(z)}{1 - F_m(z)}. \tag{23}
$$

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Using Theorem 1 from Asmussen and Rojas-Nandayapa (2008), we have that

$$\lim_{z \uparrow \infty} \frac{1 - F_G(z)}{1 - F_m(z)} = 1, \tag{24}$$

and that

$$\lim_{z \uparrow \infty} \frac{1 - F_B(z)}{1 - F_m(z)} = \begin{cases} 1 & \text{if} \quad \mu_{yB} = \mu_{yG}, \\ 0 & \text{if} \quad \mu_{yB} < \mu_{yG}. \end{cases} \tag{25}$$

Further note that

$$H(z) = \left(\frac{1 - F_G(z)}{1 - F_m(z)} \right) \left(\frac{1 - F(z)}{1 - F_m(z)} \right)^{-1} - \left(\frac{1 - F_B(z)}{1 - F_m(z)} \right) \left(\frac{1 - F(z)}{1 - F_m(z)} \right)^{-1}. \tag{26}$$

Using this last expression together with (23)-(25), we conclude that

$$\lim_{z \uparrow \infty} H(z) = \begin{cases} 0 & \text{if} \quad \mu_{yB} = \mu_{yG}, \\ (1 - p)^{-1} & \text{if} \quad \mu_{yB} < \mu_{yG}. \end{cases} \tag{27}$$

This completes the proof of the case in which $Z_\theta = X_\theta + Y_\theta$.

In order to see the general case in (11), note that

$$\mathbb{P}(X_\theta + \max(Y_\theta - I_T, 0) > z) > \mathbb{P}(X_\theta + Y_\theta > z + I_T) \tag{28}$$

and

$$\mathbb{P}(X_\theta + \max(Y_\theta - I_T, 0) > z) < \mathbb{P}(X_\theta + Y_\theta > z). \tag{29}$$

These two inequalities serve as a bound for the limit of the function $H(z)$ for the random variable $X_\theta + \max(Y_\theta - I_T, 0)$. The two bounds fall within the scope of the proof of the case in which $Z_\theta = X_\theta + Y_\theta$, and therefore have the same limits. This completes the proof. ∎

Proof of Proposition 3. Since H is increasing in (a), there is a single crossing point z such that $H(z) = \gamma$, for any $\gamma \in \mathbb{R}_+$. First note that, from the Lagrangian in (15), the objective function is linear in the choice variable $s'(z)$. Thus, only corner solutions are optimal. When $H(z) < \gamma$ the Lagrangian is minimized making $s'(z)$ be equal to its upper bound, $s'(z) = 1$, whereas for $H(z) > \gamma$, the minimization calls for setting $s'(z)$ to its lower bound, $s'(z) = 0$. The claim in (a) follows immediately (see Theorem 8 in Nachman and Noe (1994)). Assuming NICRT, and that $H'(z^*) = 0$ at most once, it is immediate that there are two unique crossing points for $H(z^*) = \gamma$, for any $\gamma \in \mathbb{R}_+$. The claim in (b) is immediate using the same argument as in case (a). In order to prove (c), we note there is a one-to-one mapping from the investment level I and the Lagrange multiplier γ. Since $H'(0) > 0$, for a sufficiently low $\bar{I}$ all investment levels $I \leq \bar{I}$ are associated with $s'(z) = 0$ for all $z \leq z^*$, since the condition $H(z^*) = \gamma$ will have at most one single solution for z^*, so that straight debt is optimal. This will be true up to the level $\bar{I}$ that is possible to finance

pledging all residual cash flows above z^*, namely $\bar{I} = \mathbb{E}[\max(Z - z^*, 0)]$. For $I > \bar{I}$, we have that the condition $H(z) = \gamma > 0$ defines two crossings, and the optimal securities are convertible bonds, as in (b). ∎

Proof of Proposition 4. The proof is analogous to that of Proposition 3. The first-order conditions require $s'(z)$ to be either one (or zero) at points for which $H(z) < \gamma$ (or $H(z) > \gamma$). Under the conditions in (b), and the initial assumptions, there is only one crossing, and all mass of the security is concentrated in the right tail. This occurs for low values of γ, or equivalently of the investment I. The claim in (a) mirrors case (b) from Proposition 3. ∎

Table 1: A simple example

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 1. The payoff of the firm is given by a trinomial random variable $Z \in \{z_1, z_2, z_3\}$. The growth opportunity requires an investment of $I = 60$, and generates an extra cash flow of 200 in the high state. The payoff and the state probabilities are summarized below.

Payoffs	z_1	z_2	z_3
Assets in place	10	100	100
Growth opportunity	0	0	200
Total payoff	10	100	300

Distributions	z_1	z_2	z_3
Good-type, f_G	0.2	0.4	0.4
Bad-type, f_B	0.3	$0.4 - x$	$0.3 + x$

The column labelled "Pooled value" below computes the expected value of the firm, $\mathbb{E}[Z]$, where each type is assumed equally likely. The variable x can take values in $[0, 0.10]$, to guarantee that the distribution f_G first-order stochastically dominates f_B. The variable λ denotes the fraction of equity the firm needs to issue to finance the investment of $I = 60$. The column labelled $\mathcal{D}_E$ denotes the dilution costs of equity, namely $\lambda(\mathbb{E}[Z_G] - \mathbb{E}[Z_B])$. For all values of x, the firm can also finance the project with a debt security with a face value $K = 76.7$, for which the dilution costs, $\mathcal{D}_D \equiv \mathbb{E}[\min(Z_G, K)] - \mathbb{E}[\min(Z_B, K)]$, are 6.7 (last column).

x	$\mathbb{E}[Z_G]$	$\mathbb{E}[Z_B]$	Pooled value	λ	$\mathcal{D}_E$	$\mathcal{D}_D$
0.00	162	133	147.5	0.407	11.8	6.7
0.01	162	135	148.5	0.404	10.9	6.7
0.02	162	137	149.5	0.401	10.0	6.7
0.03	162	139	150.5	0.399	9.2	6.7
0.04	162	141	151.5	0.396	8.3	6.7
0.05	162	143	152.5	0.393	7.5	6.7
0.06	162	145	153.5	0.391	6.6	6.7
0.07	162	147	154.5	0.388	5.8	6.7
0.08	162	149	155.5	0.386	5.0	6.7
0.09	162	151	156.5	0.383	4.2	6.7
0.10	162	153	157.5	0.381	3.4	6.7

Table 2: Optimal security design problem

The table presents the parameter values and equilibrium outcomes of the security design problem discussed in Section 3. The payoff of the firm for type θ is given by $Z_\theta = X_\theta + \max(Y_\theta - I_T, 0)$, where both X_θ and Y_θ are lognormal, with $\mathbb{E}[X_\theta] = X_\theta$, $\mathbb{E}[Y_\theta] = Y_\theta$. We further denote $\mathrm{var}(\log(X_\theta)) = \sigma_x^2 T$, $\mathrm{var}(\log(Y_\theta)) = \sigma_y^2 T$, and $\mathrm{cov}(\log(X_\theta), \log(Y_\theta)) = \rho \sigma_x \sigma_y T$. The labels "Straight debt," "Convertibles," and "Warrants" refer to the functions $s(z) = \min(K, z)$, $s(z) = \min(K, z) + \max(z - \kappa, 0)$, and $s(z) = \max(z - \kappa, 0)$ respectively.

	Symbol	Case A	Case B	Case C
Primitives				
Value of assets in place type G	$\bar{X}_G$	100	150	150
Value of assets in place type B	$\bar{X}_B$	100	50	50
Value of new assets type G	$\bar{Y}_G$	250	200	200
Value of new assets type B	$\bar{Y}_B$	150	200	200
Time to maturity	T	5	5	5
Volatility of assets in place	σ_x	0.3	0.3	0.3
Volatility of new assets	σ_y	0.6	0.6	0.6
Probability of the good type	p	0.5	0.5	0.5
Correlation between assets	ρ	0.5	0.5	0.5
Pre-existing debt face value	K_0	0	0	100
Initial investment	I	100	120	70
Investment at exercise	I_T	50	50	50
Equilibrium outcomes				
Optimal security	$s(z)$	Straight debt	Convertibles	Warrants
Face value	K	138.8	69.5	–
Conversion trigger/exercise price	κ	–	593.4	502.5

Table 3: Optimal debt-equity choice

The table presents the parameter values and equilibrium outcomes of the capital raising problem discussed in Section 4. The payoff of the firm for type θ is given by $Z_\theta = X_\theta + \max(Y_\theta - I_T, 0)$, where both X_θ and Y_θ are lognormal, with $\mathbb{E}[X_\theta] = X_\theta$, $\mathbb{E}[Y_\theta] = Y_\theta$. We further denote $\mathrm{var}(\log(X_\theta)) = \sigma_x^2 T$, $\mathrm{var}(\log(Y_\theta)) = \sigma_y^2 T$, and $\mathrm{cov}(\log(X_\theta), \log(Y_\theta)) = \rho \sigma_x \sigma_y T$.

	Symbol	Value
Base case		
Value of assets in place for the good type	$\bar{X}_G$	125
Value of assets in place for the bad type	$\bar{X}_B$	75
Value of new assets for the good type	$\bar{Y}_G$	205
Value of new assets for the bad type	$\bar{Y}_B$	195
Good type firm value	$\mathbb{E}[Z_G]$	307.9
Bad type firm value	$\mathbb{E}[Z_B]$	248.2
Time to maturity	T	15
Volatility of assets in place	σ_x	0.30
Volatility of new assets	σ_y	0.60
Probability of the good type	p	0.50
Correlation between assets	ρ	0
Investment amount	I	100
Investment at maturity	I_T	50
Equilibrium outcomes		
Value of firm post-investment	$\mathbb{E}[Z]$	278.1
Equity fraction issued	λ	0.360
Face value of debt	K	218.4
Credit spread	$r_D = (K/D)^{1/T} - 1$	5.3%
Dilution costs of debt	$\mathcal{D}_D = \mathbb{E}[\min(Z_G, K)] - \mathbb{E}[\min(Z_B, K)]$	23.7
Dilution costs of equity	$\mathcal{D}_E = \lambda(\mathbb{E}[Z_G] - \mathbb{E}[Z_B])$	21.5
Relative dilution	$\mathcal{D}_D/\mathcal{D}_E$	1.10

Comparative statics

| New parameter(s)	Equity share λ	Face value K	Spread r_D	Debt dil. $\mathcal{D}_D$	Equity dil. $\mathcal{D}_E$	Relative dil. $\mathcal{D}_D/\mathcal{D}_E$
$\bar{X}_G = \bar{X}_B = 100$	0.360	213.9	5.2%	0.7	3.5	0.21
$\bar{X}_G = 150, \bar{X}_B = 50$	0.360	233.4	5.8%	49.7	39.5	1.26
$\bar{Y}_G = \bar{Y}_B = 200$	0.360	218.3	5.3%	22.9	18.0	1.27
$\bar{Y}_G = 225, \bar{Y}_B = 175$	0.359	219.4	5.4%	26.9	35.3	0.76
$\sigma_x = 0.4$	0.360	290.6	7.4%	21.6	21.5	1.01
$\sigma_y = 0.8$	0.344	316.6	8.0%	31.6	20.6	1.53
$I = 80$	0.289	141.8	3.9%	16.5	17.3	0.95
$I = 120$	0.434	333.3	7.0%	31.1	25.9	1.19
$I_T = 0$	0.333	169.8	3.6%	17.6	20.0	0.88
$I_T = 100$	0.375	247.9	6.2%	26.5	22.3	1.19
$K_0 = 20$	0.386	303.3	7.7%	29.5	23.0	1.28
$K_0 = 40$	0.410	406.1	9.8%	34.5	23.4	1.47
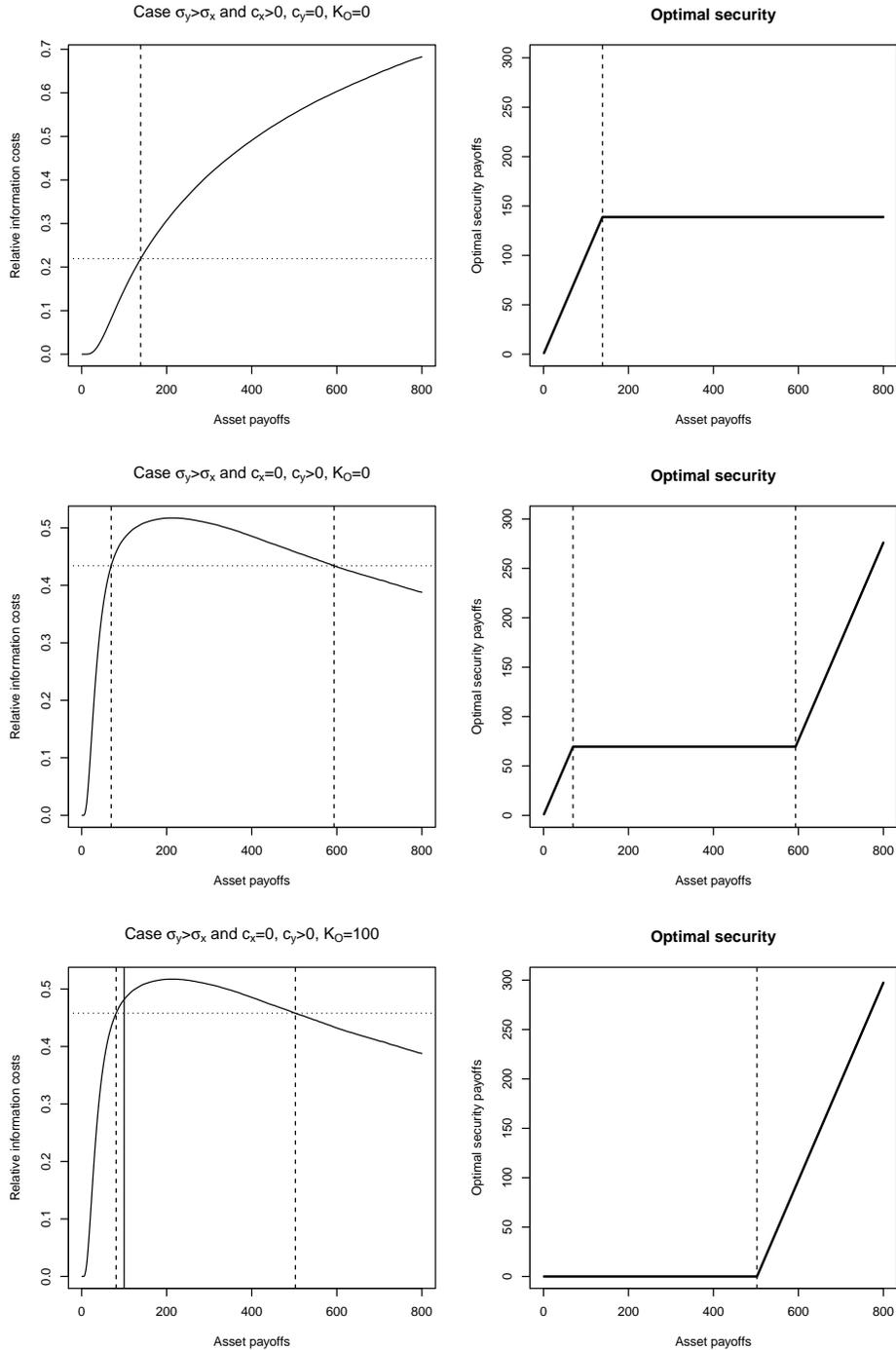
Figure 1: The left panels plot the function $H(z) = (F_B(z) - F_G(z))/(1 - F(z))$, whereas the right panels plot the optimal securities. The parameter values correspond to the cases listed in Table 2: Case A is depicted in the top two graphs, Case B corresponds to the middle figure, and Case C to the bottom plots. The vertical dashed lines mark the points z for which $H(z) = \gamma$, where γ is given by the dotted horizontal line in the left panels. The vertical solid line in the bottom left graph shows the value of existing debt in Case C, namely $K_0 = 100$.

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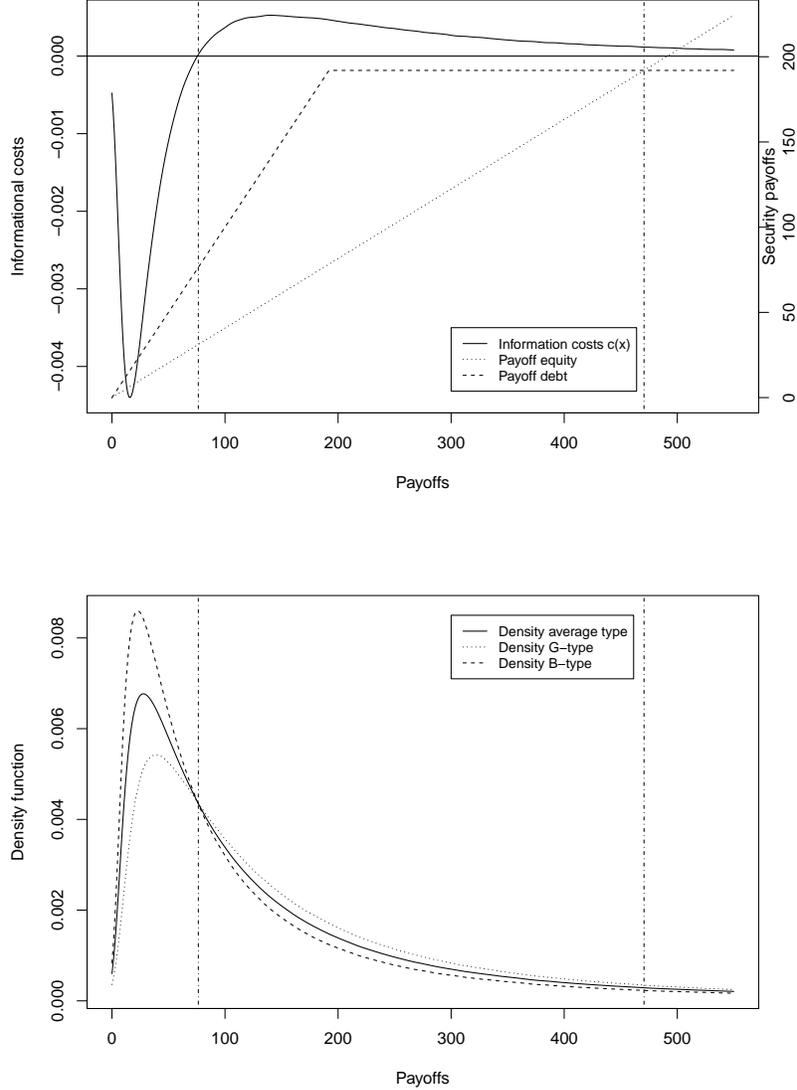
Figure 2: The top graph plots on the x-axis the payoffs from the firm at maturity, and on the y-axis it plots as a solid line the difference in the densities of the good and bad type firms, $f_G(z) - f_B(z)$ (y-axis labels on the left), and as dashed/dotted lines the payoffs from debt and equity (y-axis labels on the right). The vertical dashed lines are: (a) the point $\hat{z}$ for which $f_G(\hat{z}) = f_B(\hat{z})$, (b) the point $\bar{z}$ for which $K = \lambda\bar{z}$. The bottom graph plots the densities of the good and bad types (dashed/dotted lines), as well as the joint density (integrated over types). The payoff of the firm for type θ is given by $Z_\theta = X_\theta + \max(Y_\theta - I_T, 0)$, where both X_θ and Y_θ are lognormal. The parameter values used in the figures are $\bar{X}_G = 125$, $\bar{X}_B = 75$, $\bar{Y}_G = 205$, $\bar{Y}_B = 195$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $\rho = 0$, $T = 15$, $p = 0$, $I = 100$, $I_T = 50$.
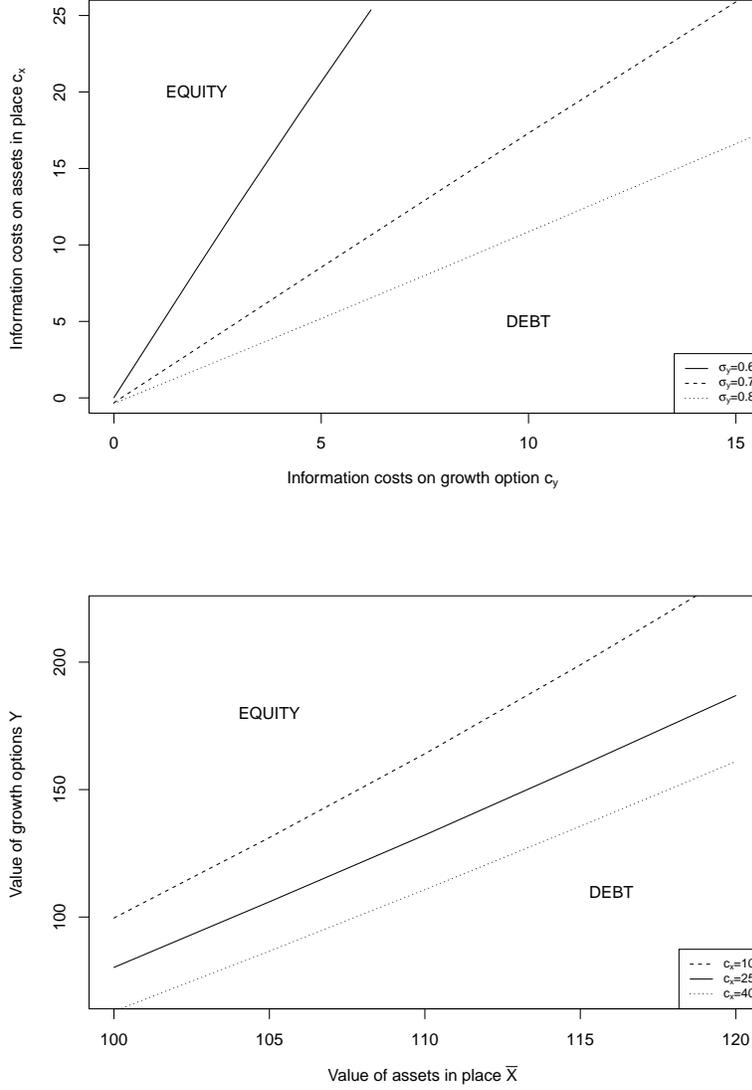
Figure 3: The top graph plots the set of points (c_y, c_x) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $\bar{X} = 100$, $\bar{Y} = 200$, $\sigma_x = 0.3$, $I = 120$, $I_T = 50$, $T = 10$, $\rho = 0$ and $p = 0.5$. We set $\bar{X}_G = \bar{X} + c_x/2$ and $\bar{X}_B = \bar{X} - c_x/2$, and similarly $\bar{Y}_G = \bar{Y} + c_y/2$ and $\bar{Y}_B = \bar{Y} - c_y/2$. The solid line corresponds to the case where $\sigma_y = 0.6$, whereas the other two lines correspond to $\sigma_y = 0.7$ and $\sigma_y = 0.8$. Debt is optimal for pairs of (c_y, c_x) below the lines, whereas equity is optimal above the lines. The bottom graph plots the set of points $(\bar{X}, \bar{Y})$ for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $c_x = 25$, $c_y = 0$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $I = 110$, $T = 15$, $I_T = 50$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case $c_x = 25$, whereas the other two lines correspond to $c_x = 10$ and $c_x = 40$. Debt is optimal for pairs of $(\bar{X}, \bar{Y})$ below the lines, whereas equity is optimal above the lines.
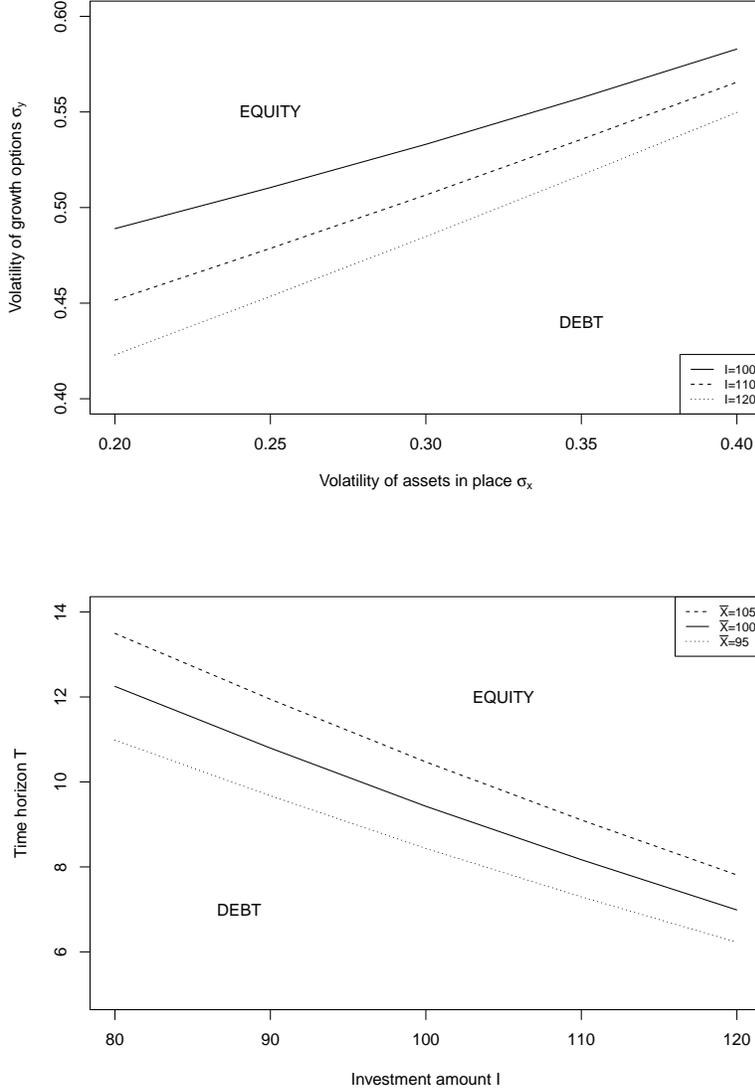
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Figure 4: The top graph plots the set of points (σ_x, σ_y) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $c_x = 25$, $c_y = 0$, $\bar{X} = 100$, $\bar{Y} = 150$, $T = 15$, $I_T = 50$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case $I = 110$, whereas the other two lines correspond to $I = 100$ and $I = 120$. Debt is optimal for pairs of (σ_x, σ_y) below the lines, whereas equity is optimal above the lines. The bottom graph plots the set of points (I, T) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $\bar{Y} = 200$, $\sigma_x = 0.3$, $I_T = 50$, $T = 10$, $c_x = 25$, $c_y = 0$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case where $\bar{X} = 100$, whereas the other two lines correspond to $\bar{X} = 105$ and $\bar{X} = 95$. Debt is optimal for pairs of (I, T) below the lines, whereas equity is optimal above the lines.
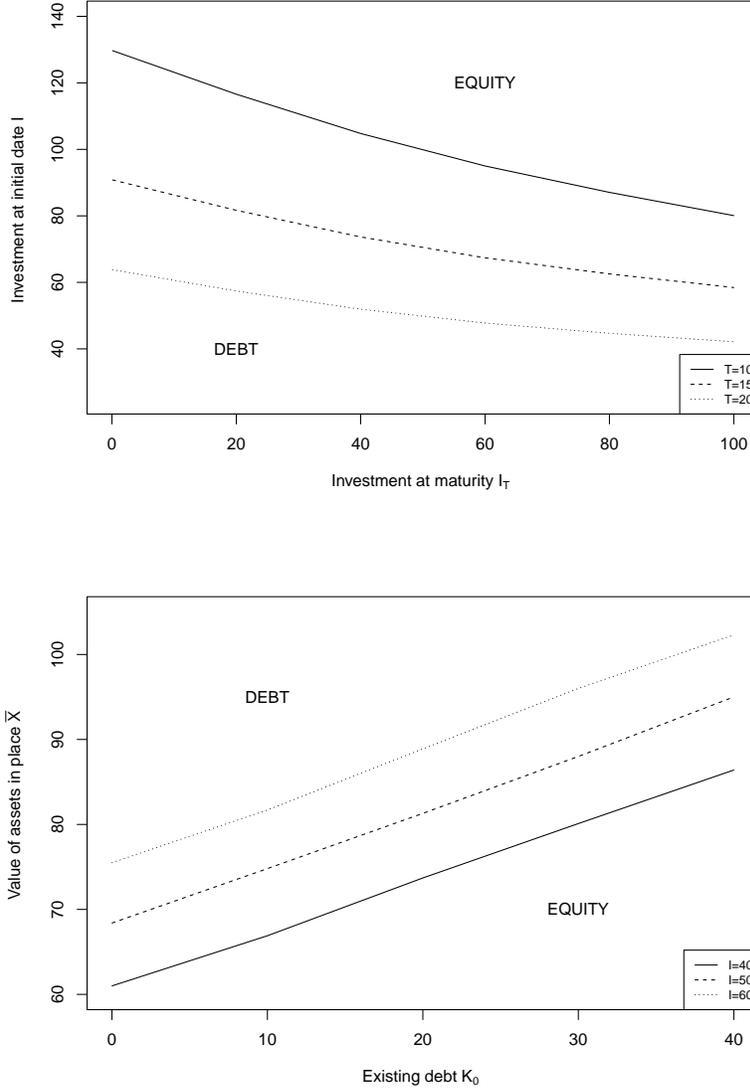
Figure 5: The top graph plots the set of points (I_T, I) for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $c_x = 25$, $c_y = 0$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $Y_G = Y_B = 175$, $\bar{X} = 100$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case $T = 10$, whereas the other two lines correspond to $T = 15$ and $T = 20$. Debt is optimal for pairs of (I_T, I) below the lines, whereas equity is optimal above the lines. The bottom graph plots the set of points $(K_0, \bar{X})$ for which the dilution costs of equity and debt are the same, i.e. $\mathcal{D}_E = \mathcal{D}_D$. We consider the following parameter values: $c_x = 25$, $c_y = 0$, $\sigma_x = 0.3$, $\sigma_y = 0.6$, $\bar{Y} = 175$, $I_T = 0$, $T = 10$, $\rho = 0$ and $p = 0.5$. The solid line corresponds to the case $I = 40$, whereas the other two lines correspond to $I = 50$ and $I = 60$. Equity is optimal for pairs of $(K_0, \bar{X})$ below the lines, whereas debt is optimal above the lines.

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