Noise and aggregation of information in large markets[*]

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Abstract

We study the relation between noise (liquidity traders, endowment shocks) and the aggregation of information in financial markets with large number of agents. We show that as long as noise increases with the number of agents, the limiting equilibrium is well–defined and leads to non–trivial information acquisition, even when per–capita noise tends to zero. In such equilibrium risk sharing and price revelation play different roles than in the standard limiting economy in which per–capita noise is finite. We apply our model to study information sales by a monopolist, and information acquisition in multi–asset markets, showing that it leads to qualitatively different results with respect to those in the existing literature. Our conditions on noise are shown to be necessary and sufficient to have limiting economies with perfectly competitive behavior consistent with endogenous information acquisition.

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1 Introduction

Ever since the seminal work of Grossman (1976), the informational content of prices has been a subject of significant interest in economics as well as in different areas of applied research.[1] As emphasized in Black (1986), *noise* plays a crucial role in preventing prices of traded assets from fully revealing agents' private information, thereby overcoming the Grossman and Stiglitz (1980) paradox. In the literature the source of noise varies from model to model: it can be the result of noise traders' demand (Kyle, 1985), it could arise due to endowment shocks that the agents receive prior to trading (Spiegel and Subrahmanyam, 1992), or due to supply shocks to the amount of tradable shares (Hellwig, 1980).

The purpose of this paper is to take a closer look at the role that noise plays in the acquisition, revelation and aggregation of private information in large competitive markets. We decouple the concepts of a large number of agents and finite per–capita noise within the standard CARA–normal framework, allowing for endogenous information acquisition of costly signals. Our main contribution is the study of large economies where the amount of noise is negligible on a per–capita basis. We refer to this class of economies as "diversifiable noise" models. We compare this model to the standard one, characterized by non–negligible per–capita noise, which we label "systematic noise" model, as well as to a competitive economy with risk–neutral market makers.[2]

We first show that the model with diversifiable noise has a well–defined limiting competitive equilibrium in which asset prices strictly partially reveal information, just as in the standard model. Intuitively, as we increase the number of agents, noise and the endogenous number of informed traders grow at the same rate, which prevents prices from fully revealing asset payoffs, maintaining the incentives of traders to gather information. Whereas in the standard model a positive fraction of the population of traders becomes informed, with diversifiable noise the number of informed agents in the limiting economy is always a negligible fraction of the total investor population. As a result, the risk–sharing capacity of the informed population, and, as

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[1]In Finance, rational expectations models have been used to study markets for information (Admati and Pfleiderer, 1986, 1990; Naik, 1997a), derivatives (Brennan and Cao, 1996; Cao, 1999), insider trading (Ausubel, 1990; Bushman and Indjejikian, 1995), security design problems (Duffie and Rahi, 1995; Demange and Laroque, 1995; Rahi, 1996), and the dynamics of asset prices, volume and portfolio choices (Campbell, Grossman, and Wang, 1993; Wang, 1993; Naik, 1997b; Zhou, 1998), among other topics (see O'Hara, 1995; Brunnermeier, 2001, for two book treatments on rational expectations models in Finance). In Accounting, many issues around disclosure and compensation have been studied within the standard rational expectations paradigm (see, for example, Diamond and Verrecchia (1982), Diamond (1985), Banker and Datar (1989), Bushman and Indjejikian (1993), Kim and Verrecchia (1991), and the references in Verrecchia (2001)).

[2]The systematic noise model was developed by Hellwig (1980) and Verrecchia (1982) in order to capture the idea of information aggregation, on top of the role of prices revealing private information (Grossman, 1976; Grossman and Stiglitz, 1980). The competitive model with a risk–neutral market maker was first discussed in Hirshleifer, Subrahmanyam, and Titman (1994) and Vives (1995).

a consequence, equilibrium asset prices, differ across the two models.

Perhaps the most important reason to study the diversifiable noise model is that it leads to qualitatively different predictions from the standard competitive models in applications. We revisit the information sales problem of Admati and Pfleiderer (1986), who show that a monopolist seller of information should offer signals of finite precision to all investors in the standard model. Cespa (2007) shows that the same result holds with a risk–neutral market maker. In contrast to the previous literature, in economies with diversifiable noise the monopolist seller of information finds it optimal to sell newsletters with the lowest possible precision. Intuitively, the information seller would like to offer the newsletters to as many agents as possible. However, since the amount of noise in the economy is small, the asset price would fully reveal the monopolist's signal if newsletters with bounded precision were sold to a positive fraction of investors. The diversifiable noise model's prediction of very low quality newsletters is consistent with the empirical evidence in Graham and Harvey (1996), Jaffe and Mahoney (1999) and Metrick (1999), who show that newsletters have little informational content.

The economy with diversifiable noise has a similar flavor to one with a risk–neutral market maker – the uninformed population plays a role parallel to the competitive risk–neutral market maker. There are significant differences, however. Although the informed population in the diversifiable noise model is negligible in size, it can still be treated, in the limit, as a continuum of agents. Given the explicit modelling of the uninformed population, the model offers different predictions in terms asset pricing properties (i.e. trading volume, asset risk–premia). One of the important insights of the paper is that simply assuming the existence of a continuum of traders, standard in much of the applied work, may not be at all innocuous, and may yield very different conclusions than those reached starting from a finite–agent economy and then taking limits explicitly. Our results show that it is very different to allow for information acquisition when constructing the limiting equilibrium, vis–à–vis studying the allocation of information once this limit has been taken.

We further extend our analysis to the multi–asset case, in order to analyze the role of different types of noise in the interactions between different asset markets. We focus on a model with two assets, where one asset has systematic noise, while the second one has diversifiable noise. We show that the incentives to gather information on the systematic noise asset are significantly altered once the diversifiable noise asset is introduced. Moreover, the qualitative aspects of the equilibrium differ substantially compared with a model where the second asset has systematic noise. Agents informed on the diversifiable noise asset do not affect the information revealed by the systematic noise asset, since their trading is negligible in size relative to the amount of noise in this asset. As a consequence, we show that complementarities in information acquisition activities across assets critically depend on the amount of noise these assets markets possess.

The economies under consideration have competitive limits even if a strategic solution concept is used. In particular, we show that the share auctions studied in Kyle (1989) have competitive limits compatible with information acquisition *if and only if* noise is large in the sense defined in this paper. Therefore, the paper contributes to the foundations of competitive rational expectations models by providing necessary and sufficient conditions on the amount of noise in the economy that yield competitive limits consistent with endogenous information acquisition.

There is a large literature on information acquisition in financial markets.[3] The early papers by Hellwig (1980), Diamond and Verrecchia (1981) and Verrecchia (1982) are closest to our work, both in terms of the motivation and the model setup. Our diversifiable noise equilibrium follows the setup in Diamond and Verrecchia (1981), endogenizing, in addition, the information acquisition decisions as in Verrecchia (1982). The literature on aggregation of information in auction settings is also related.[4] There are relatively few papers that consider the endogenous acquisition of information in such settings.[5] It is well known from the Grossman and Stiglitz (1980) paradox that for traders to have an incentive to gather information on asset payoffs prices should not fully reveal information.[6] Our paper provides necessary and sufficient conditions on the level of noise in the economy needed to support endogenous acquisition of information in a particular type of large auction environment.

The structure of the paper is as follows. In section 2 we present our main model based on the assumptions of agents' homogeneity, competitive trading, and the existence of an exogenous random aggregate supply shock (or, equivalently, noise traders). Section 3 studies limiting competitive equilibria with endogenous information acquisition, and discusses the main differences between the model with diversifiable noise and other related models in the literature. Section 4 studies information sales and information acquisition in multi–asset markets. Proofs are collected in the Appendix.

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[3]For some recent work see Barlevy and Veronesi (2000), Holden and Subrahmanyam (2002), Mendelson and Tunca (2004), Peress (2004), Morrison (2004), Dierker (2006), Veldkamp and VanNieuwerburgh (2007) and García and Vanden (2009).

[4]See Wilson (1977), Milgrom (1979), and Milgrom (1981) for some of the early references. More recent work in this area includes Pesendorfer and Swinkels (1997), Pesendorfer and Swinkels (2000), Kremer (2002), Hong and Shum (2004), Jackson and Kremer (2004), Reny and Perry (2006) and Cripps and Swinkels (2004).

[5]See Matthews (1984), Hausch and Li (1991), Guzman and Kolstad (1997), Persico (2000), Moresi (2000), Gaier and Katzman (2002), and Jackson (2003) for some examples from the auction literature. Vives (1988) studies the aggregation of information in a Cournot–type product market and also studies information acquisition decisions. Kovalenkov and Vives (2004) study a similar class of limiting economies to those in our paper. While we focus on the differences of the limiting economies, Kovalenkov and Vives (2004) study the convergence rates of models with competitive versus strategic behavior to the limiting equilibrium.

[6]The paper by Jackson (2003) shows that informational efficiency cannot arise in large uniform price single–unit auctions once there is a cost associated with gathering information, and further studies efficiency properties of different types of auctions. In contrast, we focus on the informational properties of equilibrium prices that are consistent with endogenous information acquisition in a particular type of uniform–price auction.

2 The model

We study a symmetric one period economy with N traders. We assume that all agents have CARA preferences with a risk aversion parameter τ. Thus, given a final payoff W_i, each agent i derives the expected utility $\mathbb{E}\left[u(W_i)\right] = \mathbb{E}\left[-\exp(-\tau W_i)\right]$. There are two assets in the economy: a risk–less asset in perfectly elastic supply, and a risky asset with a random final payoff $X \in \mathbb{R}$. All random variables in the economy are defined on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$, and unless stated otherwise, are normally distributed, uncorrelated, and have zero mean. The risky asset payoff X is assumed to have mean μ_x and variance normalized to 1. The risky asset has random aggregate supply Z_N. This variable, which we will refer to loosely as *noise*, is the main driver in preventing private information to be revealed perfectly to other market participants.

One of the key assumptions of the model is that agents *endogenously* decide, in a process described below, whether or not to become informed, i.e. to purchase costly private signal of the form $Y_i = X + \epsilon_i$, where the set of random variables $\{\epsilon_i\}$ is i.i.d. with $\mathrm{var}(\epsilon_i) = \sigma_\epsilon^2$. We let $m_N \leq N$ denote the number of agents that decide to become informed. Without loss of generality we can label the informed agents with the subscripts $1, \ldots, m_N$, and the uninformed with the subscripts $m_N + 1, \ldots, N$. We use θ_i, for $i = 1, \ldots, N$, to denote the trading strategy of agent i, i.e. the number of shares of the risky asset that agent i acquires. With this notation, the final wealth for agent i is given by $W_i = \theta_i(X - P_N)$, where P_N denotes the price of the risky asset in the economy with N agents. We normalize, as is customary in the literature, the agents' initial wealth and the risk–free rate to zero. These assumptions are innocuous since the model contains only one period of trading, there are no borrowing or lending constraints imposed on the agents, and there are no wealth effects with CARA utility. We use $\mathcal{F}_i$ to denote the information set of agent i at the time of trading. We note that $\mathcal{F}_i = \sigma(Y_i, P_N)$ for $i = 1, \ldots, m_N$, and $\mathcal{F}_i = \sigma(P_N)$ for $i = m_N + 1, \ldots, N$, where $\sigma(X)$ denotes the σ–algebra generated by a random variable X.

A rational expectations equilibrium, fixing the number of informed agents m_N, is characterized by a set of trading strategies $\{\theta_i\}_{i=1}^N$ and a price function $P_N : \Omega \to \mathbb{R}$ such that:

(1) Each agent i chooses her trading strategy θ_i so as to maximize her expected utility conditional on her information set $\mathcal{F}_i$, i.e. θ_i solves

$$\max_{\theta_i} \quad \mathbb{E}\left[u(W_i)|\mathcal{F}_i\right]; \qquad i = 1, \ldots, N. \tag{1}$$

(2) Markets clear, i.e.

$$\sum_{i=1}^N \theta_i = Z_N. \tag{2}$$

We remark that agents act as price takers in (1), an assumption which we will revisit at the end of section 3. As is customary in the literature, we conjecture that the equilibrium price is linear in the signals and the aggregate random supply. Given the symmetry of the economy this conjecture implies that prices are described by three parameters a_N, b_N and d_N, namely

$$P_N = a_N + b_N \sum_{i=1}^{m_N} Y_i - d_N Z_N. \tag{3}$$

Our analysis of information acquisition follows the standard approach in the literature (Verrecchia, 1982). At the information gathering stage each agent can acquire a signal Y_i at the cost of $c > 0$. Upon acquisition, an agent's expected utility is given by $\mathbb{E}\left[u(W_i - c)\right]$. Note that this expectation is unconditional, i.e. that it is taken before the signals are realized, and that the agents anticipate the rational expectations equilibrium price given in (3). Due to the symmetry of the model, we determine the Nash equilibrium at the information acquisition stage, characterized by m_N agents becoming informed, by equating the ex–ante expected utilities of informed and uninformed agents. In other words, an equilibrium at the information acquisition stage is characterized by a number of informed agents m_N such that

$$\mathbb{E}\left[u(W_i - c)\right] = \mathbb{E}\left[u(W_j)\right] \tag{4}$$

for all informed agents i and all uninformed agents j. Throughout the paper we ignore integer constraints on m_N, given that we are interested in the large N limit.

The main goal of the paper is to study properties of the economy when the number of agents becomes large. Towards that end, we consider a sequence of economies characterized by the primitives $(\tau, \sigma_\epsilon^2, c)$ and Z_N, and for each economy we determine the number of informed agents m_N that choose to acquire signals, and the associated equilibrium price P_N. We denote such sequence of economies by $\{\mathcal{E}_N\}_{N=1}^\infty$.[7]

Since properties of aggregate *noise* drive our results, we formalize in the next definition the assumptions we make on Z_N.

Definition 1. *We say that a sequence of economies $\mathcal{E}_N$ has* large noise *if there exist $\beta > 0$ and $\sigma_z^2 \in \mathbb{R}_+$ such that*

$$\lim_{N\uparrow\infty} \text{var}(N^{-\beta} Z_N) = \sigma_z^2. \tag{5}$$

If $\beta = 1$, we say that the sequence of economies has systematic noise. *If $\beta \in (0, 1)$, we will use the term* diversifiable noise. *We further define average per–capita aggregate supply of the risky*

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[7]In the next section we discuss convergence properties of the sequence of prices P_N. The reader should bear in mind that all convergence statements are both a.s. as well as in $\mathcal{L}^2$, although, for brevity, we omit these qualifiers in what follows.

asset $\mu_z \in \mathbb{R}$ by

$$\lim_{N\uparrow\infty} \frac{\mathbb{E}[Z_N]}{N} = \mu_z. \tag{6}$$

Among all sequences of economies with unbounded noise, this definition makes a distinction between economies that have finite per–capita noise, i.e. $\beta = 1$, and those with zero per–capita noise, i.e. $\beta \in (0, 1)$.[8]

In order to motivate these choices, let us assume that the aggregate noise is the sum of N i.i.d. random variables. In particular, let $Z_N = \sum_{i=1}^{N} Z_i$, for a set of random variables $\{Z_i\}_{i=1}^{N}$. If the individual shocks $\{Z_i\}_{i=1}^{N}$ are i.i.d. with constant variance, it is simple to verify that (5) holds for $\beta = 1/2$. Aggregate noise in this case grows with the number of agents, but it is negligible on a per–capita basis. On the other hand, if the random variables $\{Z_i\}_{i=1}^{N}$ are sufficiently correlated with each other, as they would be if there were driven by some common factor, one can verify that (5) holds with $\beta = 1$.[9] The names diversifiable and systematic are borrowed from portfolio theory in a natural way. The reader should also bear in mind that the distinction between the diversifiable and systematic noise in Definition 1 is with respect to the second moment of the aggregate supply of the risky asset, and not with respect to the (possibly) non–zero average per–capita aggregate supply of the risky asset μ_z.

Following most of the literature, we have modelled the noise as a black–box mechanism which enters into the model directly only via the equilibrium equation (2), and indirectly through its effect on prices in (3). At this stage one can interpret the noise as either unmodelled aggregate supply shocks (as in Hellwig, 1980), or as noise trading demand (Kyle, 1985).

We highlight the fact that Definition 1 covers other cases than $\beta = 1/2$ and $\beta = 1$, which can arise with different correlation structures among the sequence of random variables.[10] Our choice of model primitives can be further motivated as follows. Namely, consider the construction of noise trader demands in a social network of N agents described by a circle (Ozsoylev, 2005). In particular, start with a set of i.i.d. normally distributed random variables $\hat{Z}_i$, for $i = 1, \ldots, N$. In the spirit of the social networks literature, we let $Z_i = \hat{Z}_i + \sum_{j\neq i} w_{ij} \hat{Z}_j$. This definition captures the idea that noise trader i's demand for the risky asset is correlated with the demand of other agents, where this correlation is governed by the weight matrix w_{ij}.

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[8] The case $\beta > 1$ in (5) would yield economies in which all agents would become informed yet prices are completely uninformative in equilibrium. Due to these unrealistic features we exclude $\beta > 1$ case from further consideration. In section 3 we briefly discuss the case where (5) is satisfied for $\beta = 0$.

[9] Verrecchia (1982) motivates the systematic noise model by assuming the individual shocks $\{Z_i\}_{i=1}^{N}$ are i.i.d. with variance that grows linearly in N, which reduces to a limiting economy satisfying (5) for $\beta = 1$.

[10] One simple way to encompass the full $\beta \in (0, 1]$ spectrum is as follows. Start with a set of i.i.d. normally distributed random variables $\hat{Z}_i$, for $i = 0, \ldots, N$. Let $Z_i = w_N \hat{Z}_0 + v_N \hat{Z}_i$, where w_N and v_N are weights that depend on the number of agents N. In this specification, an individual agent's endowment shock contains both a systematic component, common for all agents, as well as an agent-specific component. Clearly, we have that $\text{var}(\sum_{i=1}^{N} Z_i) = N^2 w_N \sigma^2 + N v_N \sigma^2$. Different sequences w_N and v_N generate, therefore, different β primitives.

To keep the discussion parsimonious, we let $w_{ij} = w(|i - j|)$, for some decreasing function $w : \mathbb{R}_+ \rightarrow \mathbb{R}_+$. In such a setup the correlation between traders' demands decreases with the distance of one agent from another.[11] We consider next some special cases. In particular, let $w(n) = 1/n^k$, where n is the distance between the two traders. When $k > 1$, the correlation between traders' demands decreases quickly with the distance between traders. In this case, it is straightforward to verify that Definition 1 holds for $\beta = 1/2$, the case we refer to as diversifiable noise model. On the other hand, If $k \in (0, 1)$, the correlation between traders' demands do not fall off as quickly as the distance between traders increases. One can show that such parameters correspond to $\beta \in (1/2, 1)$ in Definition 1. Finally, when all noise trades are perfectly correlated, i.e. $k = 0$, then the systematic noise ($\beta = 1$) model obtains.

It is not our goal here to determine which of the types of noise is more plausible. It is clear from the previous discussion that model corresponding to any $\beta \in (0, 1]$ can be generated under reasonable assumptions. In particular, if we think that the noise stems from idiosyncratic shocks the diversifiable noise construction appears more appropriate, whereas if the shocks have a systemic component systematic noise would be a normal choice. Barber, Odean, and Zhu (2009) argue for the existence of systematic components, but it is natural to expect significant idiosyncratic behavior along the cross–section.

In order to distinguish between the systematic and diversifiable noise models, we use the notation $\mathcal{E}_N(\beta)$, $P_N(\beta)$ and $m_N(\beta)$ to denote the sequence of economies, prices and numbers of informed agents. We let $P_\beta \equiv \lim_{N\uparrow\infty} P_N(\beta)$ denote the limiting prices. Further, we say that a sequence of economies $\mathcal{E}_N(\beta)$ has a *partially revealing REE* if $\text{var}(X|P_\beta) \in (0, 1)$, i.e. if prices neither fully reveal information nor are they completely uninformative.

3 Endogenous information acquisition and large noise

In this section we study the properties of the economies $\mathcal{E}_N(\beta)$ as N becomes large. As we increase the size of the economy N, the number of informed agents is expected to grow: noise increases in N, which hides the informed traders' demands, indirectly increasing their ex-ante incentives to acquire information. The following Proposition formalizes this intuition.

Proposition 1. *Consider a sequence of economies $\mathcal{E}_N(\beta)$, where $\beta \in (0, 1]$. If $C\sigma_\epsilon^2 < 1$, where $C \equiv e^{2\tau c} - 1$, then in the limit as $N \uparrow \infty$ prices are partially revealing. In particular, the price function is given by*

$$P_\beta = a_\beta + b_\beta X - d_\beta Z_\beta; \tag{7}$$

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[11] The actual labels in the circle that make this happen are cumbersome to discuss. The analytic framework we discuss does capture the essence of a simple circle model of social interactions in a parsimonious way.

for some $a_\beta, b_\beta, d_\beta > 0$, *where* $Z_\beta \equiv \lim_{N \to \infty} N^{-\beta}(Z_N - \mathbb{E}[Z_N])$.

The equilibrium number of informed traders in the diversifiable noise model satisfies

$$\lim_{N \to \infty} N^{-\beta} m_N(\beta) \equiv \lambda_\beta = \tau \sigma_z \sigma_\epsilon \sqrt{C^{-1} - \sigma_\epsilon^2}; \tag{8}$$

whereas in the systematic noise model (8) holds with $\lambda_1 = \min(\lambda_\beta, 1)$.

The central result of the Proposition is to show that the standard limiting prices, given in (7), occur for a much larger set of primitives than previously established. In particular, the model with diversifiable noise generates the same type of equilibrium, where prices aggregate the disperse pieces of information that agents possess in such a way that individual agents' signals do not affect prices. Prices stay partially revealing while noise per capita tends to zero. As the Proposition shows, a necessary condition for partial revelation of information is for m_N to grow at the rate β, i.e. at the same rate as the standard deviation of the aggregate noise Z_N. In essence, if the number of informed agents grows at a faster (slower) rate than the amount of noise in the economy, then prices become fully revealing (non–informative). Under such circumstances agents would not have incentive to acquire costly information. The condition on C and σ_ϵ^2 simply rules out equilibria in which prices are uninformative: if this condition is not met all agents will optimally stay uninformed.

The parameter λ_β in (8) measures the amount of informed trading per unit of noise. In the standard model, λ_1 corresponds to the fraction of agents that becomes informed. In the diversifiable noise model, the equilibrium fraction of informed agents goes to zero at the rate $N^{-(1-\beta)}$. However, the number of informed agents *per unit of noise* is not zero: the "right" mass of agents becomes informed preventing asset prices from fully revealing traders' information. The intensity of information acquisition, measured by λ_β, coincides for both the systematic and diversifiable noise models, as long as $\lambda_\beta \leq 1$. This is not surprising, since agents have CARA preferences and the value of information takes on the usual certainty equivalence form – the nature of noise does not affect the value of information. It is nonetheless interesting to find that in the systematic noise case there are economic primitives under which all agents in the economy become informed (i.e. for σ_z large enough $\lambda_1 = 1$), whereas this never occurs in economies with diversifiable noise. The population of informed traders in the model with diversifiable noise is negligible in size with respect to the total number of agents in the economy N, so λ_β can take on any value on $\mathbb{R}_+$ when $\beta < 1$.

The proof of the Proposition characterizes the limiting prices P_β in closed–form for models with both types of noise, providing the expressions for a_β, b_β and d_β. As discussed, the models do share many common features. The differences between the equilibrium prices across the two models are less apparent. In intuitive terms, the discrepancies arise due to the size of

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the informed population: while with finite per capita noise a positive fraction of all agents becomes informed, the risk–bearing capacity of the informed with diversifiable noise is negligible. We highlight the implications of these differences by giving the equilibrium properties of the diversifiable noise model in the next Proposition.

Proposition 2. *In the diversifiable risk model, the informational content of prices satisfies*

$$var(X|P_\beta) = C\sigma_\epsilon^2. \tag{9}$$

The ex–ante risk–premia on the risky asset and its price volatility are given by

$$\eta_\beta \equiv \mu_x - \mathbb{E}[P_\beta] = \tau\mu_z C\sigma_\epsilon^2; \qquad var(P_\beta) = 1 - C\sigma_\epsilon^2. \tag{10}$$

Changes in asset prices are serially uncorrelated, $\rho_\beta \equiv \text{cov}[X - P_\beta, P_\beta - \mathbb{E}[P_\beta]] = 0$.

Trading volume, for $\mu_x = \mu_z = 0$, is given by

$$\mathcal{V}_\beta \equiv N^{-\beta} \lim_{N\uparrow\infty} \sum_{i=1}^{N} \mathbb{E}[|\theta_i|] = \sqrt{\frac{2}{\pi}} \left[\lambda_\beta \sqrt{\frac{1+C}{\tau^2\sigma_\epsilon^2}} + \sqrt{\sigma_\epsilon^2\sigma_z^2 C} \right]. \tag{11}$$

Due to the different role in terms of risk–bearing capacity of the informed, the equilibrium risk–premium in the diversifiable noise model is affected by the informed agents only to the extent that their trades affect the information revealed by prices. This drives a wedge between the equilibrium risk–premium in the two models, even when $\lambda_\beta \leq 1$.[12] For example, in the systematic noise model the risk–premium is decreasing in σ_z, whereas with diversifiable noise it is independent of the variance of noise trading.[13] The intuition for this is driven by the fact that price revelation is independent of σ_z in both models: an increase in the amount of noise σ_z increases the intensity of information acquisition λ_β in such a way so that price informativeness is independent of σ_z. Since the actual measure of informed agents in the economy with diversifiable noise only matters for price revelation, and not for risk–sharing purposes, the risk–premium is, as a consequence, independent of σ_z. Although the same feedback on price informativeness occurs with systematic noise, now the size of the informed population does matter for risk–sharing purposes: higher σ_z increases the number of informed agents. In turn, they demand a lower expected return from the risky stock, which reduces the equilibrium risk–premium.

The fact that risk–sharing and informational trading motives are decoupled in the diversifiable noise model makes it a simpler model to work with. This is most apparent in the simple

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[12]The differences between the two models when $\lambda_\beta \geq 1$ are apparent, as in this case the fraction of informed agents in the systematic noise model is always $\lambda_1 = 1$, whereas the informed population under diversifiable noise still satisfies (8).

[13]The equilibrium risk–premium in the systematic noise model is $\eta_1 = \tau\mu_z/(\xi + \tau\sigma_z\sqrt{\xi - 1})$, with $\xi = 1/(C\sigma_\epsilon^2)$.

expression for price volatility in (10). With diversifiable noise, the unconditional price volatility has simple comparative statics with respect to the model's primitives: it is decreasing in σ_ϵ^2, c and τ, and it is independent of σ_z^2. It is worth recalling that these comparative statics do *not* hold in the model with systematic noise, where it is possible for the price volatility to be either increasing or decreasing in these variables, depending on the values of the primitives of the model.

Perhaps more surprising is the fact that prices are martingales in the diversifiable noise two–period model. Price changes are uncorrelated since the aggregate demand of the uninformed sector eliminates price reversals induced by noise traders. In equilibrium, the conditional expectation of the risky asset satisfies $\mathbb{E}\left[X|P_\beta\right] = \eta_\beta + P_\beta$, so pricing is "efficient" in the sense that a dollar movement in the stock price causes uninformed agents to update their expectation by one dollar (Kyle, 1989). In contrast, the standard model generates negative serial correlation in returns: shocks stemming from noise trading move prices that partially reverse next period.[14] This is driven by the fact that a one dollar change in prices yields a smaller change in the expectations of the uninformed,[15] since agents may trade for both risk–sharing and speculative reasons.

Expected trading volume expression in the diversifiable noise case (11) has a simple decomposition. The first term in (11) stems from the trades of the informed agents, who only trade due to informational reasons. The second term is generated by the aggregate trades of the large uninformed population. One can verify from the expressions in the appendix that the trades of the informed, in equilibrium, are given by $\theta_i = (Y_i - P_\beta)/(\tau\sigma_\epsilon^2)$, whereas each uninformed agent, in the limit, trades a negligible amount.[16] In the systematic noise model, the informed's trading strategies can be written as $\theta_i = (Y_i - P_\beta)/(\tau\sigma_\epsilon^2) + \theta_u(P_\beta)$, where $\theta_u(P_\beta)$ denotes the trading strategy of an uninformed agent. Informed agents trade both for risk–sharing and speculative reasons, generating different patterns in trading volume vis à vis the diversifiable noise model.

Throughout the previous discussion we have focused on the comparison between the models with systematic and diversifiable noise, a natural choice given that they both can be generated as limiting economies from a well–defined finite–agent model. An alternative class of competitive rational expectations models is the risk–neutral market maker model discussed in the introduction (Hirshleifer, Subrahmanyam, and Titman, 1994; Vives, 1995). In these models there is a continuum of informed agents, and prices are set by a risk–neutral market maker (à la Kyle, 1985). One can readily verify that if one sets $\lambda_\beta = 1$ and assumes $\mu_z = 0$, the price function

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[14]This is not a general property of rational expectations models. Depending on the information structure and/or persistence of noise traders' demands, one can generally speaking have positive or negative correlation, see Wang (1993), Brennan and Cao (1996).

[15]In the systematic noise model we have that $d\mathbb{E}\left[X|P_\beta\right]/dP_\beta < 1$.

[16]We note that while trading volume is negligible in a per–capita basis, it is increasing in N, consistent with the large trading volume we observe in financial markets.

given in Proposition 1 coincides with that in Vives (1995). The uninformed population in the model with diversifiable noise plays a similar role as the risk–neutral market maker when there is no aggregate risk ($\mu_z = 0$), and prices are efficient in the sense that $P_\beta = \mathbb{E}[X|P_\beta]$.

Despite this similarity, the two models are indeed very different. Obviously, the endogeneity of the information acquisition decision sets the two models apart in terms of comparative statics. But postulating the existence of a risk–neutral market maker has more important consequences. In Proposition 2 we showed that the diversifiable noise model is perfectly compatible with a positive asset risk–premia, whereas a model with a risk–neutral market maker by construction always exhibits a zero risk–premium. Furthermore, trading volume implications are different in the two models, since the model with diversifiable noise explicitly accounts for the trades from the uninformed fringe. Although our model with diversifiable noise provides a foundation for the models with a risk–neutral market maker, it also highlights that starting with a continuum model, instead of from the finite–agent model, generates different implications.

This far we have focused on the purely competitive model where agents act as price takers. We next consider the rational expectations equilibria with imperfect competition discussed in Kyle (1989). In contrast to our previous analysis, agents will now behave strategically, internalizing the impact of their trades in prices.[17] We conjecture that prices in the finite agent economy take the linear form (3). Each agent i conjectures that she faces a residual supply curve of the form $P_N(\theta_i) = P_{Ni} + d_{Ni}\theta_i$, for some intercept P_{Ni} and slope $d_{Ni} > 0$.[18]

The rest of the model's elements are as before. Fixing the number of informed agents m and the total number of traders N, a rational expectations equilibrium is defined by a set of trading strategies θ_i that solve

$$\theta_i \in \arg\max_\theta \mathbb{E}\left[u(\theta_i(X - P_N(\theta_i)))|\mathcal{F}_i\right]; \qquad i = 1, \ldots, N;$$

and a price function of the form (3) such that the market clearing condition (2) holds.

We endogenize the information acquisition activities of traders as in Kyle (1989). At the information acquisition stage we equate the ex–ante expected utilities of informed and uninformed agents. From this condition we determine the number of informed traders $m_N(\beta)$. After that, we take the limit of large N.

The next proposition shows that the limiting equilibria coincide with those in Proposition 1.

Proposition 3. *The economy with imperfect competition with endogenous information acquisi-*

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[17]The Kyle (1989) model is a type of share auction. In contrast to indivisible unit auctions, bidders are allowed to bid fractional amounts of good. See, for example, Wilson (1979), Back and Zender (1993) and Viswanathan, Wang, and Witelski (2001).

[18]This conjecture is verified in equilibrium. See Kyle (1989) for details on the rational expectations equilibrium definition under imperfect competition.

tion exhibits the same limiting prices, limiting optimal trading strategies, and limiting measures of informed trading as the competitive model in Proposition 1, both in the systematic and the diversifiable noise models. Moreover, the existence of large noise is a necessary and sufficient condition for the existence of competitive limiting equilibria.

The Proposition establishes that in large economies with growing noise, either systematic or diversifiable, agents' strategic decisions become irrelevant: they are never pivotal in price determination. Since trading behavior is non–strategic, the limiting equilibria converges to the perfectly competitive one. We note that, although the flavor is similar, the result is not a special case of the limits considered in Kyle (1989), section 9, for neither the systematic nor the diversifiable noise models, since we endogenize the information acquisition decisions as we increase the number of agents and noise.[19]

Moreover, the existence of large noise is a necessary condition for the limiting equilibria to be competitive: if noise is a constant independent of N, $\beta = 0$ in our definition 1, the limiting equilibrium of the Kyle (1989) model will *not* coincide with that obtained when perfectly competitive behavior is assumed. Intuitively, if the economies do not have large noise, then only a finite number of agents will become informed in equilibrium, and strategic effects will be present even as N grows large. Therefore large noise, as defined in section 2 is necessary and sufficient for having limiting competitive equilibria.[20]

The auction literature, when studying information aggregation, has typically focused on perfect revelation of information in markets with large numbers of risk–neutral bidders.[21] As discussed in Jackson (2003), this yields a Grossman and Stiglitz (1980) type of impossibility result if information is costly to acquire. The introduction of noise, typical in rational expectations models, is a simple mechanism that can prevent auction prices from perfectly revealing information, thereby supporting endogenous information acquisition. Proposition 3 provides one particular auction setting in which prices aggregate information and the allocation of information is endogenous. There are some interesting analogies between the size of aggregate supply we discuss in this paper, and the literature in multi–unit auctions with large number of agents.[22] In particular, the diversifiable noise model has a similar flavor to auction models where the fraction of agents who receive a good goes to zero as the number of agents increases. On the other hand, the model with systematic noise can be compared to an auction model where a

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[19]This Proposition is also related to the convergence results in Kovalenkov and Vives (2004). In contrast to their work, we take the limit as N goes to infinity without assuming that there is "a free entry of uninformed speculators," as defined in Kyle (1989).

[20]We should also note that the conclusions of the applications studied in section 4 are also unaffected by modeling strategic interactions.

[21]See Wilson (1977), Milgrom (1979), Milgrom (1981), Pesendorfer and Swinkels (1997), Pesendorfer and Swinkels (2000), and Kremer (2002).

[22]See, for example, Swinkels (2001), Jackson (2003), Jackson and Kremer (2004), Hong and Shum (2004).

positive fraction of the agents receive the good in the limiting economy. Whether the types of noise introduced in this paper may have similar effects in other auction settings seems to be an interesting research route.

4 Applications

After studying the differences between the diversifiable noise and systematic noise models, we further consider the role of noise specification in two important extensions of the basic rational expectations paradigm. In section 4.1 we extend the multi–asset market analysis in Admati (1985) to allow for both endogenous information acquisition and different types of noise. We find that there are significant qualitative differences between the two classes of models, in particular in terms of the complementarities of information acquisition across different classes of assets. In section 4.2 we extend the analysis of information sales of Admati and Pfleiderer (1986) and again find that the implications of the diversifiable noise model differ dramatically from those with systematic noise or a risk–neutral market maker.

4.1 Information acquisition in multi–asset markets

Up to now, we studied the economy with only one risky asset. In this section we analyze what happens to the original asset price and the incentives to gather information when a second risky asset is introduced. For definiteness, we assume that initially there was a systematic noise asset in the economy and consider how the limiting equilibrium changes if we introduce another systematic noise asset, as in Admati (1985). We compare such limiting economy with the case when a diversifiable noise asset is added into the economy, instead. This allows us to extend our main result, Proposition 1, to the case when there are several risky assets in the economy. In doing so, we focus on the role that different types of noise play in the acquisition and aggregation of information in multi–asset markets.

   We denote the payoffs of the two risky assets by the vector $\mathbf{X} = (X_a, X_b)$, a two–dimensional zero–mean normally distributed random variable with variances set equal to one and correlation ρ. There are N agents in the economy with CARA preferences and risk aversion parameter τ. At the information acquisition stage, an agent i can choose to stay uninformed, purchase a signal $Y_{ij} = X_j + \epsilon_{ij}$ on one of the assets $j = a, b$ only (at a cost c_j), or purchase signals on both assets (at a cost $c_d = (1 - \delta)(c_a + c_b)$, for some $\delta \geq 0$; note that we allow for possible economies of scale). The random variables ϵ_{ij} are i.i.d. with $\mathrm{var}(\epsilon_{ij}) = \sigma_\epsilon^2$. We introduce an augmented index $t = a, b, d$ to enumerate three informed agent "types" that can emerge at the information acquisition stage: index a corresponds to agents who purchase only a signal on asset a, index b

for those that become informed only on asset b, and index d for agents who buy signals on both assets. We will refer to agents of types a or b as "specialist," since they trade on the basis of private information on a single asset. In contrast, agents of type d will be called "generalists," since they trade on the basis of signals on both assets. Finally, we define $C_t = e^{2\tau c_t} - 1$, for $t = a, b, d$. All other assumptions and conventions of section 2 apply.

Each of the assets $j = a, b$ is subject to an aggregate zero–mean supply shock Z_{Nj}. As in the economy with one risky asset, an asset j can have diversifiable noise, $\beta_j \in (0, 1)$, or systematic noise, when $\beta_j = 1$. Generalizing the one–asset case definitions, let $Z_{\beta_j} \equiv \lim_{N \to \infty} N^{-\beta_j} Z_{Nj}$, for some $\beta_j \in (0, 1]$ such that Z_{β_j} has finite positive variance, for $j = a, b$. Also define $\mathbf{Z}_\beta = (Z_{\beta_a}, Z_{\beta_b})$. We further assume that $\text{var}(\mathbf{Z}_\beta) = \sigma_z^2 \mathbf{I}$, where $\mathbf{I}$ denotes the two by two identity matrix. While we explicitly describe a symmetric economy, where assets only differ with respect to the type of noise (diversifiable versus systematic), the assumptions that we make are purely for notational convenience and brevity, and do not affect the fundamental conclusions we draw from the model. The proof in the Appendix can be easily adapted to accommodate more general assumptions on $\text{var}(\mathbf{Z}_\beta)$ and $\text{var}(\mathbf{X})$.

We denote the finite–agent economy by $\mathcal{E}_N(\boldsymbol{\beta})$, where vector $\boldsymbol{\beta} = (\beta_a, \beta_b)$ specifies the type of noise in the economy. In what follows we focus on two particular models: $\boldsymbol{\beta} = (1, 0.5)$, which we refer to as a mixed noise model;[23] and $\boldsymbol{\beta} = (1, 1)$, which we refer to as a systematic noise model, and which serves as our benchmark. In both of these models asset a has systematic noise. The difference between the models stems from the asset b: in the mixed noise model it has diversifiable noise, whereas in the benchmark case that asset has systematic noise.[24] The objective of the analysis is to study how the type of noise of one asset, asset b in our case, may change the incentives to acquire information on other securities, asset a, as well as their equilibrium prices.

We now turn to characterize the equilibrium properties of an economy $\mathcal{E}_N(\boldsymbol{\beta})$ when the number of agents N grows without bound. As in previous sections, we let $m_{Nt}(\boldsymbol{\beta})$ and $\mathbf{P}_N(\boldsymbol{\beta})$ denote the endogenously determined number of traders of type t, and the equilibrium price vector of the risky assets respectively. Let us define $\lambda_t(\boldsymbol{\beta}) = \lim_{N \to \infty} N^{-\beta_t} m_{Nt}(\boldsymbol{\beta})$, for $t = a, b$; and $\lambda_d(\boldsymbol{\beta}) = \lim_{N \to \infty} N^{-\beta_b} m_{Nd}(\boldsymbol{\beta})$.

The next proposition generalizes our previous results to this multi–asset setting.

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[23] As before, this mixed model is equivalent to any of the form $\boldsymbol{\beta} = (1, \beta)$ for any $\beta \in (0, 1)$.

[24] Our benchmark model $\boldsymbol{\beta} = (1, 1)$ model is a special case of Admati (1985). In contrast to her work, agents here are allowed to endogenously make information acquisition decisions. Admati and Pfleiderer (1987) study the viability of different allocations of information, whereas we focus on one particular type of information gathering technology. Veldkamp and VanNieuwerburgh (2007) study information acquisition in a multi–asset model under different assumptions on the cost of information.

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Proposition 4. *Consider a sequence of economies $\mathcal{E}_N(\beta)$. The equilibrium price vector satisfies:*

$$\mathbf{P_\beta} \equiv \lim_{N \uparrow \infty} \mathbf{P}_N(\beta) = \mathbf{A}_\beta \mathbf{X} - \mathbf{D}_\beta \mathbf{Z}_\beta; \tag{12}$$

for some $\mathbf{A}_\beta, \mathbf{D}_\beta \in \mathbb{R}^{2 \times 2}$. If the parameter values of the model are such that $\mathbf{D}_\beta^{-1} \mathbf{A}_\beta$ has full rank and $\lambda_d(\beta) > 0$, then:

(i) *In the systematic noise model, $\boldsymbol{\beta} = (1,1)$, the equilibrium number of informed traders satisfies $\lambda_a(\boldsymbol{\beta}) = \lambda_b(\boldsymbol{\beta}) = 0$.*

(ii) *In the mixed noise model, $\boldsymbol{\beta} = (1, 0.5)$, the equilibrium number of informed traders satisfies $\lambda_a(\boldsymbol{\beta}) > 0$, and $\lambda_b(\boldsymbol{\beta}) = 0$.*

The proposition first establishes a convergence result analogous to the single–asset Proposition 1: prices converge to a limiting random variable that is independent of individual agents' signals, and solely depends on the asset payoff vector $\mathbf{X}$ and the noise term $\mathbf{Z}_\beta$. Moreover, when prices satisfy a natural rank condition, namely that agents learn from both prices, the proposition characterizes the equilibrium when the signals are complements, i.e. when there is a strict subset of the population that becomes informed about both assets.[25] The proof contains closed–form solutions for the information acquisition parameters $\lambda_t(\beta)$, which are quite different for the mixed and systematic noise models.

In the systematic noise case, the proposition establishes that if a positive fraction of agents becomes generalist, there would be no agents who would become specialists in either of the assets. This is rather intuitive, since under our symmetry assumptions either all agents become specialists, or all agents become generalists. Intuitively, we would expect to have $\lambda_d(\boldsymbol{\beta}) > 0$ when there are some economies of scale in the information acquisition technology, i.e. for $\delta > 0$. We note that one can obtain the conditions on the primitives such that $\lambda_d(\beta) > 0$ is the unique equilibrium from the expressions in the proof of the proposition.

In the model with mixed noise, asset a has a full measure of informed agents trading on the basis of private information, whereas only a small group of agents (in per–capita terms) gather information on asset b. Moreover, it is immediate that asset b, even though it is small (in terms of the amount of noise trading), affects the equilibrium price, and the amount of information gathered, in asset a. Rather intuitively, since asset b's price reveals information, asset a's market clearing condition is affected. Therefore, both the properties of asset a's price (trading volume, volatility), as well as the incentives to gather information on this asset, change.

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[25]This is precisely the case where the mixed and systematic noise models differ most clearly. See the proof of the proposition for details on how to compute the equilibria in other cases.

The proposition also establishes that, under the stated conditions, there will always be a positive fraction of agents in the mixed noise model who are specialists in asset a, which never occurs under systematic noise. In general, under the rank condition of the proposition, the mixed noise model has two separate markets: one for agents trading on the basis of signals on asset a, and one in which agents possess information on asset b (and maybe also on asset a). The key distinction is that in the mixed noise model the measure of generalists $\lambda_d(\boldsymbol{\beta})$ has no effect on the information revealed by the price of asset a. The intuition for this is that the noise of asset a screens out the informed trades of the generalists, since the equilibrium fraction of generalists tends to zero. On the other hand, in the systematic noise model the information acquisition decisions of agents who purchase both signals affects directly the information revealed by prices on both assets. Clearly, the incentives to gather information where there is mixed noise are different.

To illustrate the perverse effect that the introduction of the diversifiable noise asset b has on asset a, consider the following parameter values: $c_a = c_b = 0.3$, $\delta = 0.2$, and $\sigma_\epsilon^2 = \tau = 1$. For these parameter values, the systematic noise model always yields an interior fraction of agents becoming informed. In particular, the equilibrium fraction of informed agents in the standard model λ_d belongs to the interval $(0.34, 0.79)$, with the boundary points of this interval correspond to the cases of perfectly correlated and uncorrelated assets, respectively. In the model with mixed noise, the equilibrium information acquisition yields both generalists, with $\lambda_d \in (0.68, 1.18)$, and specialists, with $\lambda_a \in (0, 0.47)$. As the correlation between the two assets increases, information gathering activities in asset a decrease, and for $\rho \geq 0.6$ they disappear altogether. The only equilibria that survive as the correlation goes up is one in which a small subset of the population is a generalist, and no agent becomes a specialist on asset a. This is in sharp contrast to the systematic noise equilibria, which highlights the different results, in terms of complementarity/substitutability, that can arise depending on the type of noise in asset markets.

4.2   Markets for information

Our discussion in this subsection follows Admati and Pfleiderer (1986). They use the systematic noise model to study the problem facing a monopolistic seller of information. She has perfect information about the value of the risky asset and sells her information to traders. We assume that there are N agents who consider purchasing the signal from the monopolist. The set of economic primitives is as described in section 2. We focus on the case where the white noise that the monopolist adds to her signal is personalized.[26] In particular, the monopolist sells

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[26]We use the term "white noise" to refer to the error term that the seller of information would like to add to her signal to differentiate it from the concept of noise used elsewhere in the paper. As Admati and Pfleiderer

signals $Y_i = X + \epsilon_i$ to $m_N \leq N$ agents, where ϵ_i are i.i.d. random variables with precision s_N. The monopolistic seller can choose both the number of agents she would like to serve, m_N, and the precision of the signal she offers, s_N. We restrict the precision of the signal sold to be bounded from below by an arbitrary small positive constant ℓ. The economic motivation for the introduction of the lower bound on precision is that the seller of information cannot reasonably expect to sell information without *any* content.

From the model setup it is clear that, after the information sales stage of the game has been completed, the rational expectations equilibrium that arises coincides with the one described in section 2. The interesting question is what happens at the information sales stage. As in Admati and Pfleiderer (1986), the problem that the information seller faces at that stage, for a finite N, reduces to

$$\max_{m_N \leq N, s_N \geq \ell} \quad \frac{1}{2\tau} m_N \log \left(\frac{\text{var}(X|Y_i, P_N)^{-1}}{\text{var}(X|P_N)^{-1}} \right); \tag{13}$$

where the conditional variances in (13) depend both on s_N and m_N, and are stated explicitly in Lemma 1 in the Appendix. Given the model specification, i.e. the choice of β, the equilibrium size of the market for information and the precision of the signals may vary. We denote them as $m_N(\beta)$ and $s_N(\beta)$, respectively, and study their limiting behavior as $N \uparrow \infty$. Let $s_\beta = \lim_{N \to \infty} s_N(\beta)$.

The following proposition extends the results of Admati and Pfleiderer (1986) to the diversifiable noise case.

Proposition 5. *The optimal number of informed agents m_N that the monopolist sells to satisfies (8). In the diversifiable noise model the monopolist sets $s_\beta = \ell$, and the information sales satisfy*

$$\lambda_\beta = \arg\max_\lambda \quad \lambda \log \left(\frac{1 + \ell + \lambda^2 \ell^2/(\tau \sigma_z)^2}{1 + \lambda^2 \ell^2/(\tau \sigma_z)^2} \right). \tag{14}$$

Intuitively, the monopolist is facing a trade–off between, on one hand, selling to as many agents as possible while, on the other, controlling the information that is revealed by the price. In the systematic noise model, as discussed in Admati and Pfleiderer (1986), the information seller extracts surplus from all agents in the economy by serving everyone, i.e. setting $\lambda_1 = 1$, and controls the damaging effects of price informativeness by adding non–trivial amount of white noise to the signal that she sells, namely setting $s_1 = 1/(\tau \sigma_z)$. The nature of the solution in the diversifiable noise model is qualitatively different. If noise is diversifiable the monopolist cannot sell to the whole trader population: given any signal with bounded precision prices will become fully revealing in the large N limit. Therefore, an interior solution for the size of the investor

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(1986) demonstrate in the systematic noise case, adding personalized white noise dominates adding photocopied white noise.

population being served arises (measured by parameter λ_β). In addition, the information seller controls the information revelation through prices by electing the precision of the signal to be as low as possible. These types of optimal sales with diversifiable noise seem to be closer to the empirical evidence in Graham and Harvey (1996), Jaffe and Mahoney (1999) and Metrick (1999) than the predictions from the standard model.

Limiting equilibrium asset prices in both classes of models satisfy the linear functional form (7), but the price coefficients are different. This, in turn, leads to different predictions with respect to price informativeness, expected trading volume and price volatility. We also note that if the lower bound on the signal precision did not exist, i.e. $\ell = 0$, the monopolist would like to design a pricing scheme that would satisfy $\lim_N s_N(\beta) = 0$ and sell to $m_N = N$ agents when $\beta \in (0, 1)$. In this way, she would control the price revelation through the added signal error, and still sell to the whole investor population. This equilibrium is also qualitatively different from the one in Admati and Pfleiderer (1986).

We continue the discussion initiated in section 3 with respect to the relationship between the model with a risk–neutral market maker and the diversifiable noise model. In particular, we discuss next whether it is innocuous to start by assuming the existence of a continuum of agents (as in many models with a risk–neutral market maker), instead of deriving the limiting economy by taking an explicit limit from a finite–agent economy (as we do in this paper). Cespa (2007) studies the information sales problem dealt with in this section within the risk–neutral market maker context. He commences by assuming that the information seller can contact a continuum of agents. They trade in a competitive market with a risk–neutral market maker. Importantly, his main conclusion in the one–period model mirrors that of Admati and Pfleiderer (1986): the monopolist seller will sell to the whole population of traders signals of finite precision given by the expression $s_1 = 1/(\tau \sigma_z^2)$, which coincides with the expression obtained in the systematic noise model. One could have been tempted to conjecture that a similar equilibrium would arise in the diversifiable noise case, since the uninformed population plays a role similar to the risk–neutral market maker. Proposition 5 makes it clear that such conjecture would be wrong. The equilibria are, indeed, strikingly different depending on whether one takes the limit of the trading game prior to studying the information allocation decisions, or one takes the limit of the trading and information sales problems simultaneously. This highlights the fact that starting with a continuum economy, rather than deriving equilibria as the limit of a finite–agent model, is not an innocuous assumption.

5 Conclusion

This paper contributes to the literature on endogenous information acquisition in financial markets. In particular, we have introduced a new notion of large noise, which has been shown to be both necessary and sufficient to yield large economies where (i) prices partially reveal information about traded assets, which allows for endogenous information acquisition; (ii) trading behavior is competitive, i.e. agents act as price–takers. The new equilibria that we uncover, which we refer to as diversifiable noise model, shares many of the properties of the classical model of Hellwig (1980) and Verrecchia (1982). On the other hand, the diversifiable noise model has a life of its own, as our analysis of markets for information, and multi–asset equilibria show. The crucial difference between the two types of equilibria lies in the role of the informed agents, and in particular in their risk–sharing capacity vis–à–vis the group of uninformed investors. In the diversifiable noise model the size of the informed trader population is negligible, and the uninformed are the marginal investors in terms of risk–sharing. On the other hand, the informed agents are still marginal in terms of the information provided by prices, and thereby asymmetric information plays a non–trivial role in determining equilibrium prices.

Our work suggests that the role of noise in this class of rational expectations models deserves more attention, both from a theoretical and empirical perspective. On a theoretical level, it would be interesting to know how many of the results in the literature cited in the introduction are affected by the assumption of diversifiable or systematic noise. Our analysis of information sales and complementarities in information acquisition in multi–asset markets suggests that new predictions may arise from the diversifiable noise model. Finally, the paper opens the door for empirically differentiating the two classes of models from both their explicit equilibrium asset price properties and their implications in applied areas. Extending our work into dynamic settings seems a natural step in terms of fitting the data to the model. Finally, while we have considered the standard notion of noise in rational expectations models, such noise could arise as well from differences of opinion (say from heterogenous priors). Perhaps other measures of the size of noise, along the lines of those proposed in our paper, could be developed for such extensions.

Appendix

We commence by stating a lemma which we will use throughout the proofs. The next lemma solves for the equilibrium price in the finite economy $\mathcal{E}_N(\beta)$ in closed–form, and characterizes the endogenously determined number of informed traders. We let n_N denote the number of uninformed agents, where $n_N = N - m_N$. Moreover, let $V_z^2 = \text{var}(Z_N)$ denote the variance of aggregate supply. Finally, in a slight abuse of notation, we use $\mathcal{F}_I$ and $\mathcal{F}_U$ to denote the information sets of typical informed and uninformed agents, respectively. We drop the subscript N in the statement of the proposition for notational clarity (all price coefficients, as well as m and V_z, depend on N). The proof of the lemma is omitted, and follows as in Hellwig (1980) and Admati and Pfleiderer (1987).

Lemma 1 (Finite–agent economy equilibrium). *There exists a symmetric equilibrium of the form (3), in which[27]*

$$a = \frac{\mu_x N + \mathbb{E}[Z_N]d_u}{\tau d_l}; \qquad d = \frac{1 + d_u}{d_l}; \qquad b = dr; \tag{15}$$

where r is the solution to

$$r^3 + r\frac{V_z^2}{(m-1)\sigma_\epsilon^2} - \frac{V_z^2}{\tau(m-1)\sigma_\epsilon^4} = 0. \tag{16}$$

and d_l and d_u are constants given by

$$
\begin{aligned}
d_u &= \frac{nmr}{\tau(mr^2\sigma_\epsilon^2 + V_z^2)} + \frac{m(m-1)r}{\tau(V_z^2 + r^2\sigma_\epsilon^2(m-1))}; \\
d_l &= \frac{n(r^2m(m+\sigma_\epsilon^2) + V_z^2)}{\tau(mr^2\sigma_\epsilon^2 + V_z^2)} + \frac{m(V_z^2(\sigma_\epsilon^2 + \sigma_x^2) + \sigma_\epsilon^2 r^2(m-1)(m+\sigma_\epsilon^2))}{\tau\sigma_\epsilon^2(V_z^2 + r^2\sigma_\epsilon^2(m-1))}.
\end{aligned}
$$

The equilibrium number of informed traders, ignoring integer constraints,[28] is given by the condition

$$\frac{\text{var}(X|\mathcal{F}_U)}{\text{var}(X|\mathcal{F}_I)} = e^{2\tau c}; \tag{17}$$

where

$$\text{var}(X|\mathcal{F}_U) = \left(1 + \frac{r^2m^2}{mr^2\sigma_\epsilon^2 + V_z^2}\right)^{-1}; \tag{18}$$

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[27]Formally the expressions are true in the case $m \geq 2$. This condition is innocuous, since we are interested in interior solutions, i.e. markets where as $N \uparrow \infty$ the number of informed traders grows without bound, and ignore for the most part corner solutions.

[28]We should note that integer constraints are never an issue for our limiting results. One could define the equilibrium at the ex–ante information acquisition stage by inequalities, for a given finite number of agents, and one would arrive at the same conclusions.

$$var(X|\mathcal{F}_I) = \left(1 + \frac{1}{\sigma_\epsilon^2} + \frac{r^2(m-1)^2}{r^2(m-1)\sigma_\epsilon^2 + V_z^2}\right)^{-1}. \tag{19}$$

Proof of Proposition 1.

Suppose that the endogenously determined number of informed agents m_N satisfies

$$\lim_{N \to \infty} N^{-\alpha} m_N(\beta) = \lambda_\alpha; \tag{20}$$

for some positive real numbers α and λ_α. We argue that a necessary and sufficient condition for prices in the limiting economy to be partially revealing is that $\alpha = \beta$. The price function in (3) can be expressed as

$$P_N = a_N + d_N N^\beta \left[m_N N^{-\beta} r_N \left(X + \bar{e}_N \right) - Z_{\beta N} \right]; \tag{21}$$

where $Z_{\beta N} \equiv N^{-\beta} Z_N$, $\bar{e}_N = (m_N)^{-1} \sum_{i=1}^{m_N} \epsilon_i$, and $r_N = b_N/d_N$. Note that $Z_{\beta N}$ has a non–degenerate limiting distribution by assumption, Definition 1, namely $\lim_{N \uparrow \infty} var(Z_{\beta N}) = \sigma_z^2$. From this observations, it is apparent that price revelation depends solely on the relative size of the coefficients multiplying $Z_{\beta N}$ and X within the brackets in (21), i.e. on the limiting value of $m_N N^{-\beta} r_N$.

First we show sufficiency. If $\alpha = \beta$, then from (16), and using our conjecture (20), we see that $\lim_{N \to \infty} r_N = \frac{1}{\tau \sigma_\epsilon^2} > 0$. Using this limiting result, it is immediate from (18) that the limiting prices are partially revealing as long as $\beta > 0$. Finally note that (7) follows from the previous limits and the law of large numbers.

We show necessity by contradiction. First suppose that $\alpha < \beta$. Then (16) yields $\lim_{N \to \infty} r_N = \frac{1}{\tau \sigma_\epsilon^2} > 0$. From (21) it is immediate that the limiting prices are completely uninformative about X. Now suppose that $\alpha > \beta$. If $\beta \geq \alpha/2$ we again have that $\lim_{N \to \infty} r_N > 0$, and by inspection of (21) we see that prices become fully revealing as $N \to \infty$. If $\beta < \alpha/2$, first note that, letting $\gamma = 2\beta - \alpha < 0$, from (16) we have $r_N = \left(\frac{N^\gamma \sigma_z^2}{\tau \lambda_\alpha \sigma_\epsilon^4}\right)^{1/3} \xi(N)$, where $\lim_{N \to \infty} \xi(N) = 1$. This in turn implies that $\lim_{N \to \infty} m_N N^{-\beta} r_N = \infty$, so that prices also become fully revealing when $\beta < \alpha/2$. This shows that $\alpha = \beta$ is a necessary condition for prices to be asymptotically partially revealing.

We next argue that having asymptotically partially revealing prices is necessary and sufficient for endogenous information acquisition. We start by conjecturing that the number of informed agents satisfies (20). If $\alpha < \beta$, taking limits in (17) we have

$$\lim_{N \to \infty} \frac{var(X|\mathcal{F}_U)}{var(X|\mathcal{F}_I)} = 1 + \frac{1}{\sigma_\epsilon^2};$$

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and this limiting value is larger than $e^{2\tau c}$ by assumption. Therefore $\alpha < \beta$ cannot characterize a limiting equilibrium with endogenous information gathering. On the other hand, if $\alpha > \beta$ then

$$\lim_{N \to \infty} \frac{\mathrm{var}(X|\mathcal{F}_U)}{\mathrm{var}(X|\mathcal{F}_I)} = 1;$$

which obviously cannot be compatible with a limiting equilibrium with information acquisition. This argument implies that in any limiting equilibrium with endogenous information acquisition decisions (8) must hold.

In order to compute the equilibrium measures of informed agents we note that for $\alpha = \beta$ we have

$$\lim_{N \to \infty} \frac{\mathrm{var}(X|\mathcal{F}_U)}{\mathrm{var}(X|\mathcal{F}_I)} = \frac{1 + \frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_z^2}\left(\frac{\lambda_\beta b_\beta}{d_\beta}\right)^2}{1 + \frac{1}{\sigma_z^2}\left(\frac{\lambda_\beta b_\beta}{d_\beta}\right)^2}.$$

Using (17) we immediately arrive at the expressions for λ_β given in the proposition. Finally, taking formal limits in the expressions from Lemma 1 yields the expressions for the price coefficients:

(i) In the systematic noise model, where $\beta = 1$, the price coefficients satisfy:

$$a_1 = \frac{\mu_x - \tau\mu_z}{1 + \tau\lambda_1 r_1 + \frac{(\lambda_1 r_1)^2}{\sigma_z^2}}; \qquad b_1 = \lambda_1 r_1 d_1; \qquad d_1 = \frac{1 + \frac{\lambda_1 r_1}{\tau\sigma_z^2}}{\lambda_1 r_1 + \frac{1}{\tau}\left(1 + \frac{(\lambda_1 r_1)^2}{\sigma_z^2}\right)}; \qquad (22)$$

where $r_1 = \frac{1}{\tau\sigma_\epsilon^2}$.

(ii) In the diversifiable noise model, where $0 < \beta < 1$, the price coefficients are given by

$$a_\beta = \frac{\mu_x - \tau\mu_z}{1 + \lambda_\beta^2 r_\beta^2/\sigma_z^2}; \qquad b_\beta = \lambda_\beta r_\beta d_\beta; \qquad d_\beta = \frac{\lambda_\beta r_\beta/\sigma_z^2}{1 + \frac{(\lambda_\beta r_\beta)^2}{\sigma_z^2}}; \qquad (23)$$

with $r_\beta = \frac{1}{\tau\sigma_\epsilon^2}$.

This concludes the proof. $\square$

Proof of Proposition 2.

Using (8) in the expressions for the price coefficients given in the proof of Proposition 1 one can derive (9) and (10). In order to compute trading volume we note that

$$\mathcal{V}_\beta = N^{-\beta}\sqrt{\frac{2}{\pi}}\left(m_N\sqrt{\mathrm{var}(\theta_I)} + (N - m_N)\sqrt{\mathrm{var}(\theta_U)}\right); \qquad (24)$$

where θ_I and θ_U denote the trading strategy of an informed and uninformed agent respectively. In order to compute the above variances we use

$$\text{var}(\theta_i) = \frac{\text{var}(X - P_x) - \text{var}(X|\mathcal{F}_i)}{\tau^2 \text{var}(X|\mathcal{F}_i)^2},$$

which, after some tedious calculations, yields

$$\lim_{N \uparrow \infty} N^{1-\beta} \sqrt{\text{var}(\theta_U)} = \sqrt{\frac{\sigma_z^2}{1 + r_\beta^2 \lambda_\beta^2 / \sigma_z^2}}.$$

Using the analogous expression for informed agents one arrives at (11). $\square$

Proof of Proposition 3.

Following Kyle (1989), let us conjecture that the optimal trading strategies of the agents are linear, namely let $\theta_i \equiv rY_i - qP_N$; for $i = 1, \ldots, m$, and $\theta_i \equiv -wP_N$, for $i = m+1, \ldots, N$. It is straightforward to verify that, using our notation, the characterization in Theorem 5.2 of Kyle (1989) reduces to the following system of equations for (d, r, q, w):[29]

$$(mq + nw)d = 1; \tag{25}$$
$$(1 - \phi(r))(1 - qd) = 1 - d\eta(r); \tag{26}$$
$$r = \left(\frac{1}{\tau\sigma_\epsilon^2}\right)\frac{(1 - \phi(r))(1 - 2\eta(r)d)}{(1 - \eta(r)d)}; \tag{27}$$
$$\eta(r)d - \gamma(r) = wd\eta(r)\left(\frac{d}{1 - wd} + \tau\text{var}(X|\mathcal{F}_U)\right); \tag{28}$$

where

$$\eta(r) = r\sigma_\epsilon^2\text{var}(X|\mathcal{F}_I)^{-1};$$
$$\phi(r) = \frac{r^2(m-1)\sigma_\epsilon^2}{(m-1)r^2\sigma_\epsilon^2 + V_z^2};$$
$$\gamma(r) = \left(\frac{\text{var}(X|\mathcal{F}_U)}{\text{var}(X|\mathcal{F}_I)}\right)\frac{\sigma_\epsilon^2 r^2 m}{(mr^2\sigma_\epsilon^2 + V_z^2)};$$

with $\text{var}(X|\mathcal{F}_I)$ and $\text{var}(X|\mathcal{F}_U)$ given by the expressions (18) and (19). As before, for notational convenience we have dropped the subscript N from the pricing variables.

Now let's assume that (5) holds, and conjecture (20) for some positive real numbers α and λ_α. As in the proof of Proposition 1, we shall show that the only equilibrium compatible with endogenous information acquisition requires $\alpha = \beta$, and show that the limiting prices converge

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[29]The coefficient b is given as before by $b/d = r$.

to those in Proposition 1. First note that if $\alpha = \beta > 0$, we immediately have from (25) that $\lim_{N\uparrow\infty} w_N d_N = \lim_{N\uparrow\infty} q_N d_N = 0$, and $\lim_{N\uparrow\infty} \phi(r_N) = \lim_{N\uparrow\infty} \gamma(r_N) = 0$. From (28) we then get that $\lim_{N\uparrow\infty} d_N \eta(r_N) = 0$. Finally, from (27) we have that $\lim_{N\uparrow\infty} r_N = 1/(\tau\sigma_\epsilon^2)$. Some straightforward calculations show that $\lim_{N\uparrow\infty} d_N N^\beta = d_\beta$, as given in Proposition 1. Furthermore, one can verify that the limiting trading strategies for both informed and uninformed coincide in the perfect competition and the imperfect competition models, and thereby the expected utilities are given by our previous limiting expressions, and the endogenously determined λ_β is again the same as the one provided in Proposition 1.

This verifies that the equilibrium in the imperfectly competitive market with large noise and where the informed population satisfies (8) does indeed converge to its perfectly competitive counterpart. We are left to check whether there are other limiting equilibria. One can verify, as in the proof of Proposition 1, that prices would become perfectly revealing or completely uninformative if $\alpha \neq \beta$, from which we can conclude, by comparing the expected utilities of informed and uninformed agents, that $\alpha = \beta$ is also a necessary condition for existence of a limiting economy with endogenous information acquisition in the imperfectly competitive model.

Finally, we note that if $\beta = 0$ in Definition 1, i.e. when noise does not grow with N, one can easily verify from Lemma 1 that only a finite number of agents will become informed in equilibrium. Some straightforward calculations show that the equilibrium prices, as characterized by (25)–(28), will differ from those stated in Lemma 1. This completes the proof. $\square$

Proof of Proposition 4.

Prices, in the finite agent economy, are conjectured to be linear in the signals received by the agents:

$$\mathbf{P_N} = \sum_{i=1}^{m_N} \mathbf{B_i Y_i} - \mathbf{D Z_N}; \tag{29}$$

where $m_N = m_{Na} + m_{Nb} + m_{Nd}$ denotes the total number of informed, $\mathbf{Z_N} = (Z_{Na}, Z_{Nb})$ is the vector of aggregate supplies, and $\mathbf{B_i} \in \mathbb{R}^{2\times2}$ and $\mathbf{D} \in \mathbb{R}^{2\times2}$ are the equilibrium price coefficients. The ex–ante utility (gross of information costs) of an agent whose information at the time of trading is given by the filtration $\mathcal{F}_i$ is proportional to $-|\mathrm{var}(\mathbf{X}|\mathcal{F}_i)|^{-1/2}$. The (endogenous) number of informed traders of each of the types is given by the natural indifference conditions (and corresponding inequalities), as in (17).

By similar arguments to those in the proof of Proposition 1 we have that λ_a, λ_b and λ_d must have finite limits (possibly zero), otherwise prices will perfectly reveal information. Formally

taking limits in (29), we arrive at an expression of the form (12), where in the mixed noise model

$$\mathbf{D}_\beta^{-1}\mathbf{A}_\beta = \begin{bmatrix} \left(\frac{\lambda_a(\beta)}{\tau\sigma_{\epsilon a}^2}\right) & 0 \\ 0 & \left(\frac{\lambda_b(\beta)+\lambda_d(\beta)}{\tau\sigma_{\epsilon b}^2}\right) \end{bmatrix}; \tag{30}$$

and in the systematic noise case

$$\mathbf{D}_\beta^{-1}\mathbf{A}_\beta = \begin{bmatrix} \left(\frac{\lambda_a(\beta)+\lambda_d(\beta)}{\tau\sigma_{\epsilon a}^2}\right) & 0 \\ 0 & \left(\frac{\lambda_b(\beta)+\lambda_d(\beta)}{\tau\sigma_{\epsilon b}^2}\right) \end{bmatrix}. \tag{31}$$

The following three conditions are the candidates for characterizing the equilibrium measures of informed trading λ_a, λ_b and λ_d in both models:

$$\kappa_a|\text{var}(\mathbf{X}|\mathcal{F}_a)^{-1}| = |\text{var}(\mathbf{X}|\mathcal{F}_U)^{-1}|; \tag{32}$$
$$\kappa_b|\text{var}(\mathbf{X}|\mathcal{F}_b)^{-1}| = |\text{var}(\mathbf{X}|\mathcal{F}_U)^{-1}|; \tag{33}$$
$$\kappa_d|\text{var}(\mathbf{X}|\mathcal{F}_d)^{-1}| = |\text{var}(\mathbf{X}|\mathcal{F}_U)^{-1}|; \tag{34}$$

where $\mathcal{F}_t$ denotes the information possessed by an agent of type t, and $\kappa_t \equiv 1/(1 + C_t)$, for $t = a, b, d$. Equations (32)–(34) represent the set of indifference conditions for agents who purchase one signal on asset a, one signal on asset b, or signals on both assets.

Let $s_\epsilon \equiv 1/\sigma_\epsilon^2$ denote the precision of the signal error. Define $\mathbf{H} = \mathbf{V}_\mathbf{x}^{-1}$, and let H_{ij} denote the ijth component of the matrix $\mathbf{H}$, which we subscript using the asset indexes a and b. Some simple manipulations yield that in the mixed noise model

$$\text{var}(\mathbf{X}|\mathbf{P}_\beta)^{-1} = \begin{bmatrix} x_a & -H_{ab} \\ -H_{ab} & x_b \end{bmatrix}; \tag{35}$$

where

$$x_a \equiv H_{aa} + \left(\frac{\lambda_a s_\epsilon}{\tau}\right)^2 \frac{1}{\sigma_z^2}; \tag{36}$$

$$x_b \equiv H_{bb} + \left(\frac{(\lambda_b + \lambda_d)s_\epsilon}{\tau}\right)^2 \frac{1}{\sigma_z^2}. \tag{37}$$

With this notation, the system (32)–(34) reduces to

$$\kappa_a((x_a + s_\epsilon)x_b - \kappa_0) = x_a x_b - \kappa_0; \tag{38}$$
$$\kappa_b(x_a(x_b + s_\epsilon) - \kappa_0) = x_a x_b - \kappa_0; \tag{39}$$
$$\kappa_d((x_a + s_\epsilon)(x_b + s_\epsilon) - \kappa_0) = x_a x_b - \kappa_0; \tag{40}$$

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where $\kappa_0 = H_{ab}^2$.

By inspection, it is immediate that (38)–(40) cannot all be satisfied (with the exception of subsets of the parameter space of measure zero), and therefore at most two of those equations will bind at the optimal solution. Moreover, when $\mathbf{D}_\beta^{-1}\mathbf{A}_\beta$ has full rank, it is immediate from (30) that in the mixed noise model (38) must bind. If $\lambda_d > 0$, then (40) must also bind.

Letting ϕ denote the solution to

$$\phi^2(C_d - C_a) - \frac{(1 + C_a)}{\sigma_\epsilon^2}\phi - \frac{\rho^2}{C_a(1 - \rho^2)^2} = 0;$$

we have that the solutions to (38) and (40) expressed in terms of the λ_t's using (36) and (37) are given by

$$\lambda_a(\boldsymbol{\beta}) = \tau \sigma_z \sigma_\epsilon^2 \sqrt{\frac{\rho^2}{\phi(1 - \rho^2)^2} + \frac{1}{C_a \sigma_\epsilon^2} - \frac{1}{1 - \rho^2}};$$

$$\lambda_d(\boldsymbol{\beta}) = \tau \sigma_z \sigma_\epsilon^2 \sqrt{\phi - \frac{1}{1 - \rho^2}}.$$

In the systematic noise model, information acquisition decisions are characterized by the same system (38)–(40), where x_b is also given by (37), but instead of (36) we have

$$x_a \equiv H_{aa} + \left(\frac{(\lambda_a + \lambda_d)s_\epsilon}{\tau}\right)^2 \frac{1}{\sigma_z^2}. \tag{41}$$

The systematic noise model differs from the one with mixed noise in this mapping from the λ_t's to the variables x_a and x_b, and also on the set of constraints that bind. In particular, on top of the system (38)–(40), we need to restrict, for the obvious economic reasons, $\lambda_t \geq 0$, for $t = 0, a, b, d$, and $\lambda_a + \lambda_b + \lambda_d \leq 1$. It is straightforward to check that if $\lambda_d > 0$, so that (40) binds, then (38) or (39) cannot bind in the symmetric model. Note that in this case only one of the indifference conditions (38)–(40) binds. Some straightforward calculations yield

$$\lambda_d(\boldsymbol{\beta}) = \tau \sigma_z \sigma_\epsilon^2 \sqrt{\frac{(1 + \sqrt{1 + \omega})}{\sigma_\epsilon^2 C_d} - \frac{1}{1 - \rho^2}}; \tag{42}$$

where $\omega = C_d\left(1 + (\rho\sigma_\epsilon^2)^2 C_d/(1 - \rho^2)^2\right)$. This completes the proof. $\square$

Proof of Proposition 5.

By arguments similar to those in Proposition 1 we see that the monopolist seller of information optimal number of sales must satisfy (8), for some constant λ_β. One can easily verify

that

$$\lim_{N \uparrow \infty} \text{var}(X|\mathcal{F}_I)^{-1} = 1 + s_\beta + \left(\frac{\lambda_\beta s_\beta}{\tau} \right)^2 \frac{1}{\sigma_z^2};$$

$$\lim_{N \uparrow \infty} \text{var}(X|\mathcal{F}_U)^{-1} = 1 + \left(\frac{\lambda_\beta s_\beta}{\tau} \right)^2 \frac{1}{\sigma_z^2}.$$

From these and (13) it is immediate that the monopolist problem reduces to

$$\max_{\lambda, s} \quad \lambda \log \left(\frac{1 + s + \lambda^2 s^2 / (\tau \sigma_z)^2}{1 + \lambda^2 s^2 / (\tau \sigma_z^2)} \right);$$

with the added constraints $s \geq \ell > 0$ (for both the systematic and diversifiable noise models), and $\lambda \leq 1$ (for the systematic noise model).

For a fixed λ, the optimal noise added is characterized by $s = \tau \sigma_Z / \lambda$. Some simple calculations show that $s \geq \ell$ must bind at the optimal solution in the diversifiable noise model, whereas in the systematic noise model $\lambda_1 \leq 1$ will be the binding constraint. The statements in the proposition follow from these observations. $\square$

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