

Information sales and strategic trading*

Diego García[†]

Francesco Sangiorgi[‡]

April 1, 2011

Abstract

We study information sales in financial markets with strategic risk-averse traders. The optimal selling mechanism is one of the following two: (i) sell to as many agents as possible very imprecise information; (ii) sell to a small number of agents information as precise as possible. As risk sharing considerations prevail over the negative effects of competition, the *newsletters* or *rumors* associated with (i) dominate the *exclusivity* contract in (ii). These allocations of information have distinct implications for price informativeness and trading volume, and thus our paper provides a direct link between properties of asset prices and financial intermediation. Moreover, as more information is sold when the externality in its valuation is relatively less intense, we find a ranking reversal of the informational content of prices between (a) market structures (market-orders vs. limit-orders), and (b) models of traders' behavior (imperfect vs. perfect competition).

JEL classification: D82, G14.

Keywords: markets for information, imperfect competition.

*We thank Anat Admati, Greg Brown, Paolo Fulghieri, Pab Jotikasthira, Ohad Kadan, Christian Lundblad, Günter Strobl, Laura Veldkamp and Pierre-Olivier Weill for comments on an early draft. We would also like to thank seminar participants at the London School of Economics, Toulouse University, Stockholm School of Economics, Carnegie Mellon University, Duke, UNC at Chapel-Hill, Collegio Carlo Alberto, and participants at the WFA 2010 and FIRS 2008 meetings.

[†]Diego García, UNC at Chapel Hill, Chapel Hill, NC, 27599-3490, USA, tel: 1-919-962-8404, fax: 1-919-962-2068, email: diego_garcia@unc.edu, webpage: <http://www.unc.edu/~garciadi>.

[‡]Francesco Sangiorgi, Stockholm School of Economics, Sveavägen 65, Box 6501, SE-113 83, Stockholm, Sweden, tel: +46-8-736-9161, fax: +46-8-312-327, e-mail: francesco.sangiorgi@hhs.se.

1 Introduction

In modern security markets information is sold and distributed to investors in a variety of ways. Brokerage (sell-side) analysts distribute reports and newsletters to a large number of clients, while buy-side employees and independent investment research firms offer investment advice to a small number of customers, often providing it only to the proprietary desk that commissions the research. Widely distributed investment advice seems to have little informational content, while the opposite is expected from more expensive personalized research.¹ This heterogeneity raises a number of interesting questions: Why do such different allocations of information arise? What are the consequences for asset pricing properties such as informational efficiency and trading volume?

To answer these questions we study the problem of a financial intermediary selling information to strategic risk-averse traders across different types of markets. In essence, our paper takes the information sales problem of Admati and Pfleiderer (1986), and extends it to the non-competitive markets of Kyle (1985) and Kyle (1989). Our main contribution is to show that the contracts that arise endogenously as the solution to the information sales problem resemble the dichotomy observed in actual markets for information: either (i) sell to as many agents as possible very imprecise information, or (ii) sell to a single agent (or a small group of agents) information as precise as possible.

As in Admati and Pfleiderer (1988), the main tradeoff in the information sales problem is between maximizing aggregate expected profits and ex-ante risk-sharing. By extending their analysis to more general information structures, our research shows that the problem is highly convex, which yields these two corner solutions as the optimal selling mechanisms. The *newsletters* or *rumors* associated with solution (i) maximize ex-ante risk-sharing by splitting the information in such a way that agents hold very small risky portfolios. The *exclusivity* contracts in (ii) maximize expected trading profits by allowing a few well informed traders to profit from liquidity traders. The optimal allocation of information depends on the level of noise trading per unit of risk-tolerance of the agents. We show that there is a threshold level above which risk sharing gains dominate the costs induced by competition and noisy signals, and the newsletters allocation is optimal; below the threshold the opposite is true.

In the newsletter equilibrium, a large number of traders are heterogeneously informed, and disagreement on the value of the asset generates high trading volume. The disagreement among many traders is an equilibrium outcome in our model. It provides a rational perspective on the empirical evidence documenting high trading volume. Since trading profits are lower when information is dispersed in the market than when is concentrated, our model predicts

¹See, in the context of newsletters, Graham and Harvey (1996), Jaffe and Mahoney (1999) and Metrick (1999).

a negative relationship between trading volume and the aggregate trading profits earned by strategic agents.

The endogeneity of the information allocations in our model is a critical departure point from much of the literature on information and asset pricing. With exogenous information, many different empirical implications can be obtained by varying the set of signals available to traders. Even models with endogenous information limit the equilibrium information allocations that can arise.² Our paper considers very general signal structures, from the case of photocopied information (Admati and Pfleiderer, 1988) to signals with conditionally independent errors (personalized noise, as in Admati and Pfleiderer, 1986). While choosing from a large set of different informational arrangements, our model provides sharp and strikingly simple predictions regarding equilibrium information allocations and asset prices.

The model also brings new cross-sectional implications. Growth stocks, more visible firms, and companies with volatile cash flows, should have markets for information with noisy newsletters. On the other hand, less visible firms should be associated with exclusive contracts for information, which is consistent with evidence regarding independent research firms covering smaller companies.³ Our results are also consistent with the fact that newsletters are often purchased by small investors, who are more risk averse, and with the fact that customers of independent research are typically hedge funds and mutual funds, who can afford expensive research (i.e. to have their research produced in-house) and they are able to recover those expenses by trading large quantities.

Market-order and limit-order exchanges share the same discontinuity of the optimal information allocation with respect to the primitives of the model (noise trading per unit of risk-tolerance).⁴ Comparing equilibrium properties at the optimal information allocations, we find a ranking reversal of the informational efficiency of prices across markets and models. The ranking in the exogenous information case is driven by the different usage of private information: limit-orders yield more efficient prices than market-orders because execution-price risk dampens trading aggressiveness (Brown and Zhang, 1997); models in which competitive behavior is assumed are more informative than their imperfect competition counterpart because traders do not internalize their price impact (Kyle, 1989). As the seller of information wants to

²Most of the literature analyzes “natural” benchmarks. For example, Grossman and Stiglitz (1980) study the case where all agents get the same signal, whereas in Verrecchia (1982) and Kyle (1989) signals are conditionally i.i.d.

³See, for example, the article “Independents move to fill the information gap” in the Financial Times on May 1, 2006, page 9.

⁴We use the references “market-” and “limit-orders” to refer to the Kyle (1985) and Kyle (1989) models respectively, following the original paper by Kyle (1989). We remark that the interpretation of the Kyle (1989) model as one with limit-orders hinges in the equivalence of demand schedules and a collection of limit-orders. See also Brown and Zhang (1997) and Bernhardt and Taub (2006). We note that our limit-orders model corresponds to the version of Kyle (1989) with free entry of uninformed speculators, and that our market-orders model differs from Kyle (1985) in that we allow traders to be risk-averse.

maximize its value, she finds it optimal to sell *less* information when the negative externality associated with information leakage is relatively more intense. In equilibrium, the financial intermediary sells sufficiently less information so as to reverse the rankings: prices in markets with limit-orders reveal less information than in market-orders based exchanges, and markets populated by competitive traders are less informative than markets in which traders behave strategically.

Our analysis yields two other ancillary results. First, the paper provides an example of a large auction market where assuming imperfect competition yields different equilibria than using a purely competitive equilibrium concept. In particular, we complement the examples in Kyle (1989) and Kremer (2002),⁵ by providing a simple economic problem where the type of limiting economy studied in these papers arises endogenously (with information precision vanishing as the number of informed agents increases). Second, we find that, assuming perfect competition at the trading stage, the seller of information will always choose to sell noisy newsletters to traders, i.e., the exclusivity contracts, that arise when traders are strategic, are never optimal if one assumes competitive behavior. Intuitively, the fact that traders internalize their impact on price drives the dominance of the exclusive contracts versus the newsletters. Thereby, models with and without the price-taking assumption yield *qualitatively* different implications.⁶

Our paper is closely related to previous work on information sales, as well as the literature on mutual funds and analysts.⁷ We model information sales as *direct*, in the sense defined in Admati and Pfleiderer (1986). The paper by Admati and Pfleiderer (1988), that studies information sales in a Kyle (1985) framework with risk-averse traders, is the closest to our model. Admati and Pfleiderer (1988) show that in the context of *photocopied* noise (see Admati and Pfleiderer, 1986) a monopolistic seller of information would like to sell to a small number of traders, depending on their risk-aversion. We extend their analysis by letting the information seller choose among a larger class of informational structures, that nest the photocopied and personalized noise as special cases. We show that *personalized* noise strictly dominates photocopied noise, since it dampens the effects of competition between traders without compromising the quality of the signals offered. Our paper brings out a very simple bang-bang solution. This

⁵See pages 339-340 of Kyle (1989), and in particular Theorem 9.1. Kremer (2002)'s example concludes his paper.

⁶Kovalenkov and Vives (2008) find that the competitive and strategic models we study (in particular the competitive and strategic versions of Kyle, 1989) have similar equilibrium properties as long as the size of the market and risk aversion are not too small. In our application the models are truly different, both quantitatively and qualitatively, even when there are large numbers of agents.

⁷Some early theoretical work includes Admati and Pfleiderer (1986, 1987, 1988, 1990), Benabou and Laroque (1992), and Fishman and Hagerty (1995). See Dridi and Germain (2009), Ross (2000), Biais and Germain (2002), Veldkamp (2006), Morgan and Stocken (2003), van Bommel (2003), Germain (2005), Dierker (2006), Cespa (2008), García and Vanden (2009) and Lee (2009) for some recent theoretical contributions. We discuss related empirical studies in section 5.

dichotomy in the solution of the problem within the same model is in sharp contrast with the rest of the literature on direct information sales, in which the quantity of information sold is only of one type: the papers on direct information sales cited before (Admati and Pfleiderer, 1986, 1988; Cespa, 2008), all have interior solutions for the precision of the information sold and/or the number of agents the information is sold to.

Our paper also contributes to the literature on information aggregation in auctions.⁸ Share auctions, first studied in Wilson (1979), allow bidders to receive fractional amounts of the good for sale. The Kyle (1989) model we study is essentially a share auction with non-discriminatory pricing. Examples in financial markets abound, from auctions of Treasury securities, the actual opening mechanism in the NYSE, to auctions of equity stakes at IPOs. Our contribution is to endogenize the allocation of information in two special types of share auctions with risk-averse buyers.⁹ Prices in the share auctions we study indeed aggregate the diverse pieces of information that agents receive from the monopolist seller. On the other hand, the type of information received by agents is a non-trivial function of the number of informed agents in the equilibrium that yields optimal rumors, in sharp contrast to most of the limiting equilibria studied in the literature, where signals' precision are typically held constant as new traders are added to the auction. Our results highlight the importance of endogenizing the allocation of information when studying issues of information aggregation.

The paper is structured as follows. In section 2 we discuss the main ingredients of our model and setup the financial intermediary's problem. Section 3 contains our main analysis, where we characterize the problem and solve for the optimal information sales under personalized noise. Section 4 discusses the robustness of the main results under more general information allocations. In section 5 we relate our results to the empirical literature. Section 6 concludes. All proofs are relegated to the Appendix.

2 The model

In this section we present the main ingredients of our economy with endogenous asymmetric information. We discuss the elements of the financial market under study, the information allocations we consider in our base case, and the equilibrium concepts we use throughout the paper.

⁸Some classic papers on information aggregation include Wilson (1977), Milgrom (1979), and Milgrom (2000). See Pesendorfer and Swinkels (1997), Pesendorfer and Swinkels (2000), Hong and Shum (2004), Reny and Perry (2006) and Cripps and Swinkels (2006) for some recent work on the area.

⁹For work on information acquisition in auctions in general see Matthews (1984), Hausch and Li (1993), Guzman and Kolstad (1997), Persico (2000), Moresi (2000), Gaier and Katzman (2002), Jackson (2003).

2.1 Preferences and assets

All agents have CARA preferences with a risk aversion parameter r . Thus, given a final payoff π_i , each agent i derives the expected utility $\mathbb{E}[u(\pi_i)] = \mathbb{E}[-\exp(-r\pi_i)]$. There are two assets in the economy: a risk-less asset in perfectly elastic supply, and a risky asset with a random final payoff $X \in \mathbb{R}$ and variance normalized to 1.¹⁰ All random variables, unless stated otherwise, are normally distributed, uncorrelated, and have zero mean. There is random noise trader demand Z for the risky asset. This variable has the usual role of preventing private information from being revealed perfectly to other market participants. We let σ_z^2 denote the variance of Z . We use θ_i to denote the trading strategy of agent i , i.e. the number of shares of the risky asset that agent i acquires. With this notation, the final wealth for agent i is given by $\pi_i = \theta_i(X - P_x)$, where P_x denotes the price of the risky asset.¹¹ As usual in the literature, price informativeness (or informational efficiency) is measured by the inverse of the variance of the payoff conditional on the equilibrium price, $\text{var}(X|P_x)^{-1}$.

2.2 The monopolist information seller

There is one information seller, the monopolist, who has perfect knowledge about the payoff from the risky asset X . On the main body of the paper we will focus on sales of *personalized* information (see Admati and Pfleiderer, 1986), i.e. the case in which the seller of information gives agent i a signal of the form $Y_i = X + \epsilon_i$, with ϵ_i i.i.d. Due to the assumption of homogeneous risk-aversion, we first consider the case in which the signals sold have the same precision, which we denote by $s_\epsilon \equiv \text{var}(\epsilon_i)^{-1}$. In section 4 we shall generalize both the i.i.d. assumption and the symmetry assumptions in order to encompass other models in the literature (i.e. Admati and Pfleiderer, 1988). The information seller can write contracts for the delivery of signals Y_i to N agents, where N is large. The monopolist freely chooses the signals' quality s_ϵ , and also to how many agents $m \leq N$ to sell the information to. We will refer to the total amount of information sold, $y = ms_\epsilon$, as the “stock of private information.”

In our stylized setting, the seller of information can add independent noise terms to the signal she possesses. The i.i.d. assumption on $\{\epsilon_i\}$ can be justified theoretically as in the literature on rational inattention (Sims, 2003, 2005). Also, as Admati and Pfleiderer (1986) point out, signals can be personalized in an indirect way, by selling reports which are intentionally vague, so that customers themselves make personal independent errors in the interpretation of the information. In practice, this can be implemented whenever financial analysts transmit

¹⁰The normalization of the variance of X is without loss of generality. All the results in the paper would go through assuming an arbitrary variance σ_x^2 , see footnote 18.

¹¹We normalize the agents' initial wealth and the risk-free rate to zero. This is without loss of generality due to the CARA preference assumption.

information to investment customers directly by telephone, or notify the information to salespeople in the brokerage firm where they work, who in turn call the customers (for a description of the process of delivery of recommendations from sell-side analysts see, for example Michaely and Womack, 2005).

Figure 1 sketches the stages of the model. The monopolist seller of information contacts m agents and offers them signals as specified above for a price c . If an agent accepts he pays the fee c , and next period he receives the signal Y_i , which he will use to make his portfolio decision. If an agent declines he trades as an uninformed investor when financial markets open. Traders are not allowed to resell the information they receive to other traders and the precision of the signals is assumed to be contractible. The type of information sales we are considering can be thought as subscriptions to some future advice, for which trades pay some ex-ante price c , and later get to observe information about the risky asset.

We should emphasize that all the assumptions on the information seller of Admati and Pfleiderer (1986) are in place. In particular, there is no reliability problem between the information seller and the buyers, in the sense that she can commit to truthfully revealing the signal Y_i with the precision s_ϵ that she promised. Furthermore, the information seller is not allowed to trade on her information. The model just described is the simplest setting in which to discuss information sales with strategic traders. Section 4 considers a number of generalizations of this framework, among them allowing the seller to trade. The main economic forces behind the model, and the monopolist's optimal sales, are shown there to be robust to such generalizations.

2.3 The equilibrium at the trading stage

We perform our analysis of information sales across market structures and models. In the main body of the paper we consider two different setups building on: (1) the limit-orders model of Kyle (1989), (2) the market-orders model of Kyle (1985). In section 4 we also consider the competitive version of Kyle (1989). In (1), we assume that a large number of uninformed traders participate in the stock market alongside the traders who can become informed. With this specification the setup is identical, post information sales, to the one discussed in Kyle (1989) under the assumption of free entry of uninformed speculators. This assumption is equivalent to the existence of a competitive market-making sector that clears the market, as in Kyle (1985). This equivalence allows us to compare the limit and market-orders models on equal footing (Bernhardt and Taub, 2006).

We should emphasize that agents do *not* act as price takers in (1) and (2) – they anticipate the dependence of prices on their trading strategies. The basic difference across these market

structures is that in (1) traders observe the market clearing price and strategies are demand schedules (a collection of limit orders), while in (2) they do not observe the market clearing price and strategies are quantities. A Technical Appendix that accompanies the paper contains the details of the solution and the characterization of the equilibrium for the three models considered. The Appendix in the paper contains the proofs of the Propositions, as well a succinct equilibrium characterization of the three types of markets just described.

The purpose of this comprehensive analysis is to compare the asset pricing properties of different markets at the *endogenous* information allocation. In this way we extend previous findings in the literature, that assumed exogenous information allocations, i.e., taking the number of informed traders m and the quality of their information s_ϵ as given. Kyle (1989) shows that prices in the equilibrium with imperfect competition are less informative than in the equilibrium with perfect competition, because agents who internalize their price impact trade less aggressively on their private information. Brown and Zhang (1997) and Bernhardt and Taub (2006) show that limit-orders yield more informative prices than market-orders.¹² The intuition for their results is that agents who can submit demand schedules, instead of market-orders, trade more aggressively on their information: on the one hand they face less execution-price risk due to noise-trading; on the other each of them internalizes the order-reducing effect of his order on the trades of other speculators, increasing competition. The ranking of informational efficiency across markets and models in the exogenous information case is therefore driven by the different usage of information that traders do.

3 Optimal information sales

We start our analysis by simplifying the monopolist's problem, we then describe the main forces that drive the model, and we characterize the optimal information sales.

3.1 The monopolist's problem

The monopolist seller of information would charge a price c that makes each of the agents just indifferent between accepting the monopolist's offer or trading as an uninformed agent. Due to the assumption of free entry in Kyle (1989) and Bertrand competition by risk-neutral market makers in Kyle (1985), uninformed agents have no gains from trade. Thus, the profits earned by the seller of information from a particular allocation equals the sum of each informed

¹²The models in these two papers are not isomorphic to the ones we study here. Brown and Zhang (1997) look at a version of the Vives (1995) model, with a continuum of competitive agents. The analysis in Bernhardt and Taub (2006) differs from ours along two dimensions: they only consider the risk neutral case, and the information structure is of the form $\sum_{i=1}^m Y_i = X$.

agent's certainty equivalent. Next Proposition provides a novel expression for the monopolist's profits, which is valid across all models introduced in the previous section.

Proposition 1. *The monopolist's problem can be expressed as*

$$\max_{m \in \{1, \dots, N\}, s_\epsilon \in \mathbb{R}_+} \mathcal{C}(m, s_\epsilon) = \frac{m}{2r} \log(1 + 2r\mathbb{E}[\chi_i]); \quad (1)$$

where χ_i denotes the interim certainty equivalent of informed agent i , namely

$$\chi_i = \mathbb{E}[\pi_i | \mathcal{F}_i] - \frac{r}{2} \text{var}(\pi_i | \mathcal{F}_i), \quad (2)$$

with $\pi_i = \theta_i(X - P_x)$ and \mathcal{F}_i denoting the trading profits and the information set at the trading stage of agent i respectively. The monopolist's problem in (1) is subject to one of the three constraints (3)-(5) in the Appendix, depending on the type of market and trader behavior under consideration.

We remark that the interim certainty equivalent χ_i is precisely the objective function that trader i maximizes at the interim stage (this is $t = 2$ in Figure 1). The expression in Proposition 1 formalizes the tradeoff between risk-sharing gains, via the concavity of the log function, and competition, captured by the expected interim certainty equivalent term. The relative strength of competition and risk sharing considerations depends on the primitives of the model, namely traders' risk-aversion r and noise trading volatility σ_z . Ceteris paribus, the value of information to each trader decreases with risk-aversion and increases with noise trading, so r and σ_z have opposite effects on the consumer surplus. Nevertheless, the optimal information allocation, the solution to (1), only depends on the product of the two, which we denote as $\kappa = r\sigma_z$. We refer to κ as the *noise per unit of risk-tolerance* in the economy. Notice that the risk neutral case implies $\kappa = 0$ for any σ_z , so assuming risk neutrality is with loss of generality.

For a given value of s_ϵ , selling information to more traders has two opposite effects on the value of information. On the one hand, increasing m increases ex-ante risk-sharing gains, as noise trader risk is being shared among more risk-averse traders. On the other hand more informed agents compete more aggressively, thereby reducing expected interim profits – the first term in (2). Admati and Pfleiderer (1988) consider a market order model in which the allocation of information is constrained to photocopied noise. In this setup, it is suboptimal for the monopolist to add any noise to the signals. As a consequence, it is never optimal to sell to too many traders – the optimal number of informed traders is finite. Outside the Admati and Pfleiderer (1988) setup, as we further show below, adding noise to the signals could be beneficial. In this case the interaction of strategic trading, externalities in the valuation of information and risk sharing considerations makes the solution to the monopolist's problem far from trivial.

Before solving the full problem, we explore the optimal noise added by the monopolist in the limit-order case for a fixed number of informed traders m . We remark that in this case maximizing consumer surplus reduces to maximizing the interim certainty equivalent $\mathbb{E}[\chi_i]$ in (1).

Proposition 2. *In the limit-order model, when $m = 1$, the monopolist sells her information with no noise added, i.e. $\sigma_\epsilon^2 \equiv 1/s_\epsilon = 0$. For $m \geq 2$, the optimal precision of the signals increases in the noise per unit of risk-tolerance parameter κ , $ds_\epsilon/d\kappa > 0$.*

In absence of competition, a single informed trader fully internalizes the price impact of his trades, and the seller maximizes the value of the signal giving him full information. On the other hand, if she were to sell perfect information to multiple traders, speculators would compete aggressively. In the limit-order model the competition is so fierce that prices would reveal all their private information and driving profits to zero (see Kyle, 1989, Theorem 7.5). This is driven by the possibility of agents to submit demand schedules, rather than just market-orders, which makes their trading strategies riskless arbitrage opportunities with perfect information.

In order to mitigate the effects of competition, the seller optimally adds noise to the signals she sells, dampening the competition problem, but at the expenses of more payoff uncertainty.¹³ The amount of noise that optimizes this trade-off is characterized explicitly in the proof of Proposition 2. The higher noise trading per unit of risk tolerance, κ , the smaller the negative externality of price revelation. Rather intuitively, higher risk-aversion makes agents trade less aggressively, and higher noise-trading makes the price less informative. Proposition 2 establishes that the monopolist responds to a decrease in the information externality by selling more precise information to the m traders. This intuition appears throughout our analysis, playing a key role in determining the equilibrium level of price informativeness across models at the endogenous information allocation.

The market-order case shares similar results. The main qualitative difference arises from the fact that agents cannot condition their trades on price. The resulting execution-price risk makes agents trade less aggressively on their information and gives more weight to risk-sharing considerations. One can verify that for low values of m the monopolist would optimally give agents perfect signals, as in the $m = 1$ case of Proposition 2. In general, fixing m , the seller of information would add noise to the signals if and only if $\kappa \leq \bar{\kappa}_m$, where $\bar{\kappa}_m$ is increasing in m .

¹³This resembles the optimality of adding noise from Admati and Pfleiderer (1986). The motivations for adding noise in our paper are related, but not identical to theirs. For instance, the seller would never sell perfect information to a single trader in the competitive model of Admati and Pfleiderer (1986). In section 4.3 we further compare our results to a case closely related to Admati and Pfleiderer (1986), where agents act as price takers.

3.2 Optimal exclusivity contracts and noisy newsletters

We start this section studying the problem in (1) for open sets around the zero risk-aversion per unit of noise trading, and the large risk-aversion per unit of noise trading cases.

Proposition 3. *There exists $\underline{\kappa}$ such that if $\kappa < \underline{\kappa}$, the monopolist optimally sells to a single agent, $m = 1$, and sets $s_\epsilon = \infty$, i.e. tells the agent what she knows.*

In the risk neutral case the monopolist's problem reduces to that of maximizing expected profits, and can be solved in closed form (details are provided in the proof). When agents are almost risk neutral, selling to more traders does not bring significant risk-sharing gains. On the other hand, competition decreases aggregate profits, which makes the concentrated information allocation with $m = 1$ optimal on open sets around $\kappa = 0$. This result coincides with the results in Admati and Pfleiderer (1988) for the risk neutral case, although here the allocations with $m \geq 2$ are allowed to have personalized noise.¹⁴

In the limit-order case, half of the information of the seller gets impounded into prices, i.e. the conditional volatility of the risky asset is exactly one half the unconditional volatility, irrespective of the level of noise trading. Speculator's effective risk aversion is zero as he receives a signal with no noise and faces no execution-price risk. As the risk neutral monopolist trader in Kyle (1985), he optimally adjusts his trading strategy so as to offset any variation in noise trading.

The next Proposition describes the allocation of information that arises with a large number of traders, and establishes its optimality when the monopolist faces an economy with highly risk-averse traders and/or an asset with large amounts of noise.

Proposition 4. *There exists some $\bar{\kappa}_N$ such that for all $\kappa > \bar{\kappa}_N$ the monopolist's problem (1) is solved by selling signals to all agents, $m = N$. As $N \uparrow \infty$, for $\kappa > \bar{\kappa}_N$, the precision of each informed trader's signal vanishes and trading volume grows without bound. Equilibrium price informativeness coincides, under such optimal sales, across the market and limit-order models.*

More risk averse speculators trade less aggressively on information, and more noise trading makes prices less informative, so for κ large the negative effects of competition are relatively small. The monopolist could still sell perfect information to a single trader, maximizing the interim certainty equivalent. Nevertheless, relative to selling to many agents, the ex-ante value of information would be very low, due to either the large risk aversion or noise-trading risk. As a consequence, risk sharing gains dominate competition effects, driving the optimality of selling to as many agents as possible. The optimal allocation of information with large number

¹⁴We discuss the relationship between our model and that of Admati and Pfleiderer (1988) at more length in section 4.

of traders does indeed resemble very noisy newsletters, as individual precision in each trader’s signal vanishes in the large N limit.

The proof of the Proposition gives an explicit characterization of the optimal stock of information sold to agents, y , as a function of the model primitives. As in Proposition 2, the optimal stock of private information sold is shown to be increasing in κ . We further show that informational efficiency is always greater than in the exclusivity contract case. Prices aggregate the information dispersed in the economy and reveal more than under the equilibrium with a perfectly informed monopolist trader.

The disagreement among traders, all of whom get heterogeneous information about the value of the asset, drives trading volume to be higher than in the equilibrium with a concentrated information allocation. As the proof shows, trading volume grows without bound in the large N limit. Indeed, for N large, the model exhibits “trading frenzies,” in the sense of a significant spike of trading volume when the optimal sales of information are as in Proposition 4.

In the optimal contracts in Proposition 4, the monopolist sells signals in such a way as to have a large number of informed agents monopolistically competing against each other as in the leading example of section 9 of Kyle (1989), and the concluding example in Kremer (2002). These two examples are built abstractly by taking the large N limit in an auction setting letting the precision of the signal vanish as N increases. As highlighted by Kyle (1989), even in the large N limit, when agents are “small” in terms of their informational advantage, they internalize their price impact. Proposition 4 presents a simple economic setting where such a limiting economy arises endogenously. We should also remark that, as shown in the proof of the Proposition, the limit-order and market-order equilibria coincide in the large N limit.

After establishing that the optimal solution is non-interior for two open sets of $\kappa \in \mathbb{R}_+$, we further analyze the problem in this section to assess how tight the bounds $[0, \underline{\kappa})$ and $(\bar{\kappa}_N, \infty)$ actually are. Proposition 3 only establishes the existence of an open set $[0, \underline{\kappa})$, whereas Proposition 4 does not address whether the bound $\bar{\kappa}_N$ has a finite limit if we let $N \uparrow \infty$. As equilibria with risk-averse traders in the Kyle (1985) or Kyle (1989) models can only be characterized via a non-linear equation (Subrahmanyam, 1991), the general problem in (1), for an arbitrary κ , is particularly challenging analytically.¹⁵

We solve the model numerically.¹⁶ We first consider the limit-order case. For each m , we use the characterization of the optimal sales from Proposition 2 to solve for the optimal s_ϵ and the equilibrium price, obtaining the maximum consumer surplus for each given number

¹⁵For instance, there exist open sets of κ such that optimal profits as a function of m exhibit both local maximum and minimum which are not the global maximum (or minimum).

¹⁶We note that the problem is rather straightforward in terms of finding numerical solutions. For a fixed m , it reduces to the maximization of a function over two variables, subject to a single non-linear constraint.

of informed agents m . Figure 2 plots the profits obtained by the monopolist from selling to $m = 2, \dots, 40$ (dotted lines), as well as the profit functions corresponding to $m = 1$ and $m = N$ (solid lines). As Figure 2 makes clear, the profit functions with $m = 1$ and $m = N$ (for N large) form an upper envelope that dominates any allocation of information to m informed agents. The corresponding functions $\mathcal{C}_1(r, \sigma_z)$ and $\mathcal{C}_\infty(r, \sigma_z)$ are defined in (13) and (32) in the Appendix. In other terms, Figure 2 shows that for N large, $\bar{\kappa}_N$ has a finite limit, $\bar{\kappa}$, and $\underline{\kappa}$ and $\bar{\kappa}$ coincide.

Figure 3 summarizes our numerical analysis in the market-order model. We solve the model optimizing over s_ϵ , obtaining the maximum consumer surplus for each m . We do this for a fine grid of values for κ , and report the resulting consumer surplus for different m . Comparing Figure 3 to Figure 2, we see that the upper envelope now consists of the fragments of six different profit lines, those that encompass $m \leq 5$ and the m large case.¹⁷ On the lower range for κ the monopolist sells to a small number of traders, and she does not add any noise to the signals. For κ sufficiently high the optimal allocation is again virtually the opposite: give low-precision information to a large number of traders.

The following Theorem summarizes our main findings.

Theorem 1. *The optimal information sales involves either: (i) selling signals with no noise added to a finite number of traders, or (ii) selling signals with vanishing precision, in the large N limit, to all traders.*

We qualify the term “finite” in the Theorem as meaning a small number of agents $m < N$, as Figures 2 and 3 suggest, although formally the interpretation of a finite number of traders hinges on the normalization $\sigma_x = 1$.¹⁸ We should emphasize that interior allocations, in which finitely many traders acquire noisy signals, are never optimal in our setup. The optimal information allocation is either extremely concentrated, with one or few traders acquiring precise information, or extremely diffuse, with the whole market receiving the noisy newsletters from Proposition 4.

Figure 4 summarizes the implications of Theorem 1 for price informativeness. The solid line represents the conditional precision of the asset payoff given prices in the limit-order model. The dotted line, with five discrete jumps, corresponds to the market-order model, whereas the dashed line plots price informativeness treating m as a continuous variable in the same

¹⁷We remind the reader that we have normalized $\sigma_x^2 = 1$, which affects the maximum number of agents to which the monopolist may want to sell signals with no noise in the market-order case. All the qualitative results are unaffected by this normalization.

¹⁸The statement in Theorem 1 is independent of the normalization $\sigma_x = 1$; the results extend to the general case by letting $\kappa = r\sigma_z\sigma_x$. For large values of σ_x the risk-sharing gains in the market-order model yield a larger number of possible optimal “finite” m allocations. We also remark that in the limit-order model only the $m = 1$ allocation is ever optimal.

model. The main implication of Figure 4 is that prices are more informative in the model with market-orders than in the model with limit-orders. Precisely because agents trade more aggressively on their information when they can submit price-contingent orders, the monopolist seller constraints the amount of information they get, and in equilibrium asset prices are less informative. As a result the ranking of informational efficiency across markets is reversed with respect to the exogenous information case.

4 Extensions and robustness

In this section we ascertain the robustness of the main predictions of our model by extending the analysis to a larger class of information allocations. We first examine the case of correlated noise in the signals, which nests the “photocopied information” of Admati and Pfleiderer (1988). We then look into the case where the monopolist seller of information can trade, and discuss other asymmetric allocations. Next, we study our model under the assumption of price-taking behavior.

4.1 Correlated noise

Throughout the paper, we have assumed that noise added to the signals is i.i.d. across buyers. This class of signals is what Admati and Pfleiderer (1986) refer to as allocations with “personalized noise,” in contrast to the case where the noise terms are perfectly correlated (as in Admati and Pfleiderer, 1988). A natural question to ask is whether the i.i.d. assumption is without loss of generality. Is “photocopied information,” where all agents get the same signal, potentially better? Furthermore, the seller could sell signals with correlated error terms. Consider the case of two agents: the monopolist could report $Y_1 = X + \epsilon$ to one agent and $Y_2 = X - \epsilon$ to the other, giving them “mixed signals.” These signals are ex-ante identical, but agents may receive opposite reports at the trading stage. We note this is not quite “lying,” the seller of information may just break up the information she has on X into different pieces, each of which has some value.

We should emphasize that we are the first ones to study within the same theoretical framework, and across different market structures, such a comprehensive set of information allocations.¹⁹ The three allocations under study can be given slightly different interpretations. Under photocopied noise, the monopolist seller is sending the same message to all traders, who process it without error. The case with conditionally independent errors, the focus of section 3, can be interpreted as agents committing errors (as in the rational inattention literature,

¹⁹Clearly, the CARA/Gaussian assumption, and its tractability, are critical to examine general allocations of information.

Sims, 2003, 2005), or simply allowing the seller to give agents different pieces of her signal. The general case is somewhat more abstract, in the sense that we allow the seller to choose from a rather large set of signals.

We consider the case of perfectly correlated signals first, and then the case of optimally correlated signals. Following Admati and Pfleiderer (1988), we allow the information seller to give $m \leq N$ traders the same signal $Y = X + \epsilon$. The information seller can control the distributional properties of $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. The next Theorem summarizes the results of Admati and Pfleiderer (1988) on the optimal information sales and extends them to the limit order model of Kyle (1989).

Theorem 2. *In the market-order model, the monopolist seller never adds noise to her signal, i.e. she optimally sets $\sigma_\epsilon^2 = 0$ when constrained to sell the same signal to traders. In both market-order and limit-order driven models, for all κ , the monopolist sells to a finite number of traders.*

The main qualitative difference with respect to our analysis in Section 3 is the fact that the newsletter equilibrium of Theorem 1 does not arise. Rather intuitively, selling the same signal to a large number of traders dissipates the value of the information very quickly. It is the ability of the seller of information to add conditionally independent noise that drives the optimality of the dispersed information allocation in Proposition 4 and Theorem 1. Another qualitative difference is on the noise added to the signals. In the limit order model of Kyle (1989) the seller adds photocopied noise to the signal as soon as $m \geq 2$, as with i.i.d. noise, while in the market-order model she never does.

Since the nature of the solution changes, it is interesting to check whether the results on informational efficiency across models hold in this setup. Figure 5 mirrors Figure 4 for the case of “photocopied information.” The solid line presents the conditional precision of the asset’s payoff given prices in the limit-order model, whereas the dotted line does the same for the market-order model (as before, the dashed line is the equilibrium values when m is treated as a continuous variable in the market-order model). We again observe that the market-order model yields significantly more informative prices than the limit-order model. The mechanism via which this happens is the same as before: the monopolist optimally sells more information whenever the externality associated with the usage of information is less intense, which results in more informative prices in equilibrium.

While Theorem 2 analyzes the “photocopied information” case, it does not compare it to the “personalized information” allocations of Section 3, or to more general information allocations. In order to establish the optimality of the different types of allocations, we next consider the case where the monopolist can choose arbitrary signals. Assume that the monopolist seller of information markets signals of the form $Y = X + \epsilon_i$. In the most general case, the

monopolist chooses the full variance-covariance matrix $\Sigma_\epsilon = \text{var}(\epsilon)$, where $\epsilon = (\epsilon_1, \dots, \epsilon_m)$. The characterization of the equilibrium, fixing m and Σ_ϵ is standard, and included in the Technical Appendix, where we also provide further details on the different cases discussed below. For now, we restrict attention to symmetric information allocations, where all agents get the same signal precision, but the signal errors are allowed to be correlated. For a fixed m , the monopolist's choice variables are the precision s_ϵ of the signal errors ϵ_i , as well as the correlation between signals sold to different traders, which we shall denote by ρ . We remark that this variation of the problem nests both the Admati and Pfleiderer (1988) model and the i.i.d. case studied in Section 3. Also note the correlation must satisfy the lower bound $\rho \geq -1/(m-1)$ for $m \geq 2$, in order for Σ_ϵ to be positive-semidefinite.

The next Proposition solves for the optimal correlation.

Proposition 5. *Fixing $m \geq 2$, consumer surplus is strictly decreasing in ρ , and the monopolist optimally sets ρ to be as low as possible, namely $\rho = -1/(m-1)$.*

This result implies that the profits for the monopolist in our base case (Theorem 1) are higher than those in the photocopied noise case discussed in Theorem 2. The solution to the information sales problem, with an unrestricted error correlation, is again particularly simple: make agents' signals as different as possible (conditional on the asset's final payoff). For example, in the $m = 2$ case, the information seller should give traders perfectly negatively correlated report errors, or what we previously referred to as "mixed signals." Such reports make the information contained in prices more valuable, and as a consequence they raise the ex-ante value of the monopolist's reports (Admati and Pfleiderer, 1987). One can check numerically that the optimal m , when allowed to sell signals with negatively correlated error terms, as in Proposition 5, is always at the upper boundary, $m = N$.²⁰ Making signals conditionally negatively correlated allows the seller to bank on risk-sharing gains, while mitigating the effects of competition.

4.2 Asymmetric allocations of information

This section further extends the previous analysis by considering non-symmetric allocations of information, i.e. relaxing the assumption in section 2.2 that the precision of the signal given to each trader is the same. We also study the case where agents have different risk-aversion coefficients, as well as the case where the seller of information is allowed to trade. The common element of each of these variations of the main model is whether allocations of information

²⁰As $\rho \uparrow 0$ in the large N limit, the equilibrium consumer surplus does not converge to the one in the conditionally i.i.d. case. When $\kappa \approx 0$ the monopolist earns the same profits selling to one agent (giving him her information) as she would selling to a very large number of agents very noisy (slightly tilted correlation-wise) information.

where different traders may possess information of different quality can arise as the solution to the information seller’s problem.

Asymmetric allocations. In the main body of the paper the seller of information chooses the number of informed agents m , as well as the quality of the information given to each of these agents, under the constraint that all agents get the same signal precision. If we let $\Sigma_\epsilon = \text{var}(\epsilon)$, where $\epsilon = (\epsilon_1, \dots, \epsilon_m)$, the main body of the paper considers the case where $\Sigma_\epsilon = \sigma_\epsilon^2 \mathbf{I}_m$ for some $\sigma_\epsilon^2 \in \mathbb{R}_+$. In this section, we relax this assumption, allowing the seller to offer signals with different (non-zero) precision to multiple traders.²¹

In particular, we will consider the case where the seller of information offers signals of precision s_A to m_A traders, and signals of precision s_B to m_B traders.²² We maintain throughout this section the assumption of homogeneous risk aversion and conditionally independent signals, i.e. Σ_ϵ is diagonal, and we focus our analysis in the limit-order market.

We solve the model numerically (see Table 5 in the Technical Appendix for details). Our analysis shows that there are three regions of κ that yield qualitatively different information allocations: (a) for κ sufficiently low, the concentrated allocations of Proposition 3 are optimal, (b) for κ sufficiently high, the seller finds it optimal to disperse information as in Proposition 4, (c) for intermediate values of κ , the seller offers signals with heterogeneous precision to multiple traders. In the latter case, there is an open interval of values of κ such that the monopolist maximizes profits by selling information with no noise added to a single trader while at the same time selling noisy signals to a large number of agents. Thus, the different allocations of information that we describe in the main body of the paper can *co-exist* even in the case where all agents are ex-ante identical.

Heterogeneous risk-aversion. The previous analysis shows how a convex combination of the optimal solutions from Theorem 1 can arise in our model. We discuss next how this asymmetric allocation of information can emerge as a result of heterogeneous risk-aversion on the buyers side of the model, which generates sharper implications as to whom receives what type of signals. We shall assume there is one “large trader” with risk-aversion r_T , and a large set of “small retail investors” with risk-aversion r_R , such that $r_T < r_R$. The monopolist seller can potentially sell conditionally independent signals to both the large trader and the retail investors. Throughout the following discussion, we study a limit-order market.

Consider the particular case where $r_T = 2$ and $r_R = 10$. Concentrating the information in the hands of the risk-tolerant trader would give a consumer surplus $\mathcal{C} = 0.27$, while diffusing

²¹We should remark that the term “asymmetric” is a slight abuse – in the setting of section 3 the allocations among the potential N clients of the information seller are asymmetric in the concentrated allocation of Proposition 3.

²²We have also looked at the case where the seller offers three different types of signals to traders. The case with two groups that we consider dominates such allocations.

noisy information to all retail investors gives $\mathcal{C} = 0.26$. By selling information with asymmetric precision, the monopolist maximizes profits by giving the large trader a very precise signal, and at the same time selling very noisy information to all the risk-averse agents, which results in a consumer surplus equal to $\mathcal{C} = 0.28$. Rather intuitively, the monopolist allocates the information so that agents with high risk-tolerance receive expensive high-precision signals, whereas those that are more risk-averse end up with cheaper and noisier information.

Allowing the information seller to trade. We next consider the case where the information seller can trade, as well as choose the number of agents m to whom sell signals of the form $Y_i = X + \epsilon_i$, with $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$ i.i.d. In order to highlight the differences with Admati and Pfleiderer (1988), we focus on the market-order model, and ignore reliability problems, i.e. we assume the seller tells her clients the signal that she promised. This assumption reduces the problem to the one studied in the main body of the paper, with the added constraint that there will be at least one perfectly informed trader, the information seller. The optimal information sales depend on the absolute and relative values of the risk-aversion of the monopolist seller, which we denote by r_S , and the risk-aversion of the buyers of information, denoted by r_T .

As in Admati and Pfleiderer (1988), if the information seller is risk neutral, then she will not sell the information to any other trader. Similarly, if the seller is sufficiently more risk averse than the other traders, then she would optimally sell her information and commit not to trade. Selling information might still be optimal in this case if such a commitment is not possible. Our key departing point, vis-à-vis Admati and Pfleiderer (1988), comes from the ability to sell conditionally i.i.d. signals. For illustrative purposes, let the risk-aversion of the monopolist be $r_S = 10$. The consumer surplus of the allocation where she trades on her own account and does not sell her information is $\mathcal{C} = 0.12$. If the other traders have a similar risk-aversion, $r_T = 10$, the information seller would optimally sell to many of them giving them very noisy signals, as in the diffuse allocations from section 3. Consumer surplus in this allocation jumps up to $\mathcal{C} = 0.17$. On the other hand, if the other traders would be more risk-tolerant, $r_T = 2$, then the monopolist would sell her information to one-single trader, generating a consumer surplus of $\mathcal{C} = 0.18$.

The previous examples are robust to different parameter configurations and show how, when she chooses to sell information to other traders, the monopolist would either: (i) contact a few traders and give them high-precision signals, or (ii) contact all traders and give them very noisy information. We thus conclude that the main qualitative results of the paper are robust to allowing the seller to trade.

4.3 Competitive behavior

All our previous results were derived assuming agents acted strategically when trading, i.e. they anticipated the effect of their trading on equilibrium asset prices. Of course, one could solve the model under the alternative competitive assumption, that is, assuming that agents act like price takers. Although the equilibria have different characterizations, it is not clear to what extent the competitive assumption affects the qualitative aspects of the optimal contracts. The next Proposition shows that indeed it does: assuming perfect competition the exclusive contracts are never optimal.

Proposition 6. *If agents act as price takers, the information seller sells to as many agents as she can, with signals' precision vanishing as the number of traders diverges to infinity. For any value of the primitives κ , equilibrium prices at the optimal sales are less informative if agents act as price takers than if they act strategically.*

Under perfect competition the information seller always chooses to sell to as many agents as possible, controlling the damaging effects of information leakage by giving agents very imprecise signals. The optimality of selling to a single trader disappears. It is therefore critical to model the price impact of traders in this type of information sales problem. Rather intuitively, it is precisely the fact that the monopolist trader internalizes his trades' impact on prices that drives the optimality of the exclusivity contract in section 3. The result has a similar flavor to that in Admati and Pfleiderer (1986), where it is shown, in a large market with perfectly competitive traders, that the seller of information would always optimally sell to all agents.

Proposition 6 shows that prices are less informative under the competitive assumption than under strategic trading. This result may be surprising, since, as Kyle (1989) convincingly shows, strategic trading makes agents more cautious with their trades, implying less informative prices (see Theorem 7.1 in Kyle, 1989). The intuition for the result reversal is that the information seller will give agents more informative signals under imperfect competition precisely because these traders, in contrast to competitive traders, will marginally internalize the effect of their trades on prices. In terms of the information revealed by prices the former effect dominates the usual effect of less aggressive trading.

In the proof of Proposition 6 we further show that in the competitive case, contrary to the exogenous information model, informational efficiency is *increasing* in risk aversion. For low risk aversion the seller is forced to sell very noisy signals to control for the dilution in the value of information via information leakage. Risk tolerant agents therefore end up with noisier signals, which makes the equilibrium prices more informative as risk-aversion is increased. This is not only in contrast with the exogenous information model, but also with the model with strategic traders, where for $\kappa > \kappa^*$ we have that price informativeness decreases in κ .

We conclude that the price-taking assumption eliminates the $m = 1$ equilibria and generates contrary comparative static results with respect to the solution where agents anticipate their impact on prices.

5 Discussion and empirical implications

Our paper provides a stylized model that ties together financial intermediation with asset pricing properties. We discuss in this section how it relates to previous literature on price informativeness, trading volume, and observed institutional arrangements. We further highlight several new testable implications that arise from our analysis.

High trading volume in the stock market is considered to be a puzzle for the rational expectations literature. It is typically attributed to forms of investor irrationality such as overconfidence (Odean, 1999), or (exogenous) differences of opinion (Harris and Raviv, 1993). The former interpretation seems to be particularly compelling when associated with the poor trading profits documented for individual investors (Barber and Odean, 2000; Barber, Lee, Liu, and Odean, 2009). Alternatively, the newsletter equilibrium of our model provides a rational perspective that is also consistent with such evidence. When noisy information is diffuse, disagreement among a large number of investors generates high trading volume, while individual trading profits vanish. Rather than irrational behavior, it is the endogeneity of the information allocation, which scatters the information in the market, that drives high trading volume.

The different information allocations imply a distinct relationship between trading volume and aggregate trading profits of informed traders.²³ In the concentrated information equilibrium, a small group of well informed traders controls the leakage of information through prices, which results in low trading volume and high trading profits. In contrast, in the diffuse information equilibrium, high trading volume is associated with low trading profits. Thus, our model offers the testable prediction of a negative relation between aggregate trading volume and aggregate trading profits of informed traders. To the best of our knowledge this relationship has not been empirically investigated, but data on informed investor holdings (such as institutional investors) on high and low trading volume assets could be used to validate it.

Our model mirrors several institutional arrangements we observe on Wall Street. The concentrated allocation can be mapped into buy-side research: analysts hired by financial institutions to provide detailed research for their proprietary trading in exchange for their wages. Similarly, it is consistent with independent research firms that provide customized

²³It is important here to note that we are not considering the trading profits of the noise traders, since by construction aggregate trading profits are zero.

research for a small number of clients. The diffuse allocation, can be mapped into sell-side analysts selling wide-spread newsletters (Graham and Harvey, 1996; Metrick, 1999), media and analyst reports (Womack, 1996), and even institutions such as Yahoo newsgroups (Antweiler and Frank, 2004) or TV shows (Engelberg, Sasseville, and Williams, 2009). Indeed, most of these channels of information seem to have very low informational content but generate large trading volume, as predicted by our theory.

A common practice in the business of sell-side research is that of analysts “tipping” institutional traders before disseminating their reports to the general public (see, among others, Irvine, Lipson, and Puckett, 2007). This process, being inherently dynamic, cannot be fully accommodated by our model. Nevertheless, the analysis in section 4.2 captures its economic essence. In particular, we show how different information allocations can coexist in the same market: agents with low risk-aversion receive precise information, while at the same time a large number of high risk-aversion agents receive noisy newsletters. In the context of our model, an agent’s risk-tolerance captures his trading capacity, yielding a natural interpretation of low risk-aversion agents as larger institutional investors and high risk-aversion as retail investors. While studying a truly dynamic model, where some agents receive information and trade before others, is an interesting route to pursue, our model can accommodate equilibria that mimic the “tipping” of sell-side research.

Models with exogenous information yield the unambiguous prediction that transparency of the execution price and price informativeness move in tandem (Glosten, 1994; Baruch, 2005; Biais, Glosten, and Spatt, 2005). The prediction of our model is rather different. On the one hand, when noise trading per unit of risk-tolerance is low, the information seller constraints the number of agents that acquire information when agents can condition their trades on prices. This results in lower informational efficiency vis-à-vis the case where the transaction price is not observable. On the other hand, when noise trading per unit of risk-tolerance is high, transparency is not related to price efficiency (Proposition 4). In this respect, our model contributes to the debate in the empirical literature on the effects of transparency on informational efficiency (see, among others, Boehmer, Saar, and Yu, 2005; Madhavan, Porter, and Weaver, 2005). In particular, the mixed evidence on such effects is consistent with the predictions of our model. Finally, our results also suggest new experimental studies that compare situations with exogenous information (Bloomfield and O’Hara, 1999) to those where information is endogenously determined.

The comparison between optimal information sales across the limit-orders model of Kyle (1989) and market-orders model of Kyle (1985) has implications for price informativeness across markets, as illustrated in Figure 4 and Figure 5. These implications are in stark contrast with models in which information is exogenous, suggesting a way of testing the alternative theories.

A challenge for this is that the theoretical structures we analyze have no direct counterpart in real markets – exchanges allow for different types of orders, which only loosely correspond to our theoretical constructs. However, there are several observable market characteristics that capture the key economic differences between them – execution price risk and competition among informed agents. One such market characteristic is fragmentation. In more fragmented markets, speculators have multiple venues to trade on their information, potentially reducing competition among informed traders. Moreover, by submitting demand functions to all execution venues, a trader incurs the risk of executing too many shares (relative to the desired position). Hence, a higher degree of fragmentation resembles the market-order exchange structure in the paper. In our setup, such fragmentation would result in more information being sold to traders, and, as a consequence, in more informative prices. This prediction is consistent with recent findings in O’Hara and Ye (2010), that find more fragmentation to be associated with more efficient prices.

Another market characteristic that relates to our results on limit versus market-order exchanges is transparency. Some markets allow the submission of hidden or reserve (i.e., partially hidden) orders, whereas others are more transparent. When a trader sends a market order, the existence of hidden liquidity introduces uncertainty over the execution price. Markets where only reserve orders are allowed would have less execution-price risk than markets in which completely hidden orders are allowed. A higher degree of such “dark liquidity” resembles the market-order exchange of our theoretical model. In general, market characteristics that enhance competition among traders or reduce execution price risk, should be associated with more aggressive trading and more efficient prices with exogenous information, but less (or equal) price efficiency when information is endogenously determined.

The model generates cross sectional predictions for the type of information allocation that is likely to arise. The relative importance of sell-side versus buy-side research as a vehicle of information transmission and acquisition for a given market or asset should be driven by the relevance of the information externality. In the context of our model, glamour stocks, more visible firms, and companies that show up prominently in the news are more likely to attract liquidity trading. This makes newsletters the optimal vehicle used by information providers to sell their information. The same prediction holds for firms with more volatile cash flows: the risk sharing motive should induce information providers to diffuse information among the general public. On the other hand, for less visible firms, illiquid stocks, and companies with relatively stable cash flows, competition among informed traders should induce information providers to concentrate the information they sell. In such markets information should be sold mainly in an exclusive way.

More broadly, our model predicts heterogeneity in the information allocations, and distribu-

tion channels, across different asset classes. While we see very active markets for information at the retail level in stock markets, other asset classes receive less retail attention and attract more institutional players. Corporate bonds, for example, have more stable cash flows than stocks. Based on the volatility of its cash flows, and/or less noise trading per unit of risk-tolerance, our model predicts that corporate bonds would have concentrated information allocations, whereas stocks are more likely to exhibit diffuse allocations. The lower coverage of the corporate bond market for retail investors, relative to stock markets, is thus consistent with our model.

The discontinuity in the equilibrium as a function of κ establishes a rationale for regime shifts between the exclusivity contracts and the newsletters equilibria. Namely, consider a repeated sequence of economies, identical and independent from each other. Assume that any of the two exogenous parameters, the noise-trading intensity σ_z , and/or the risk-aversion r , is a stochastic process taking values on the positive real line. As the variable $\kappa = r\sigma_z$ crosses the κ^* barrier the allocation of information in the economy will shift from one type of equilibrium to the other. This will in turn generate regimes with different asset pricing properties, depending on whether κ is above or below κ^* . For example, in periods of high noise-trading prices will be more informative, aggregate trading profits lower, and trading volume will be high. When noise-trading drops below the cutoff, trading volume will dry up, as only a few agents will become informed. Trading profits will be high and price revelation low. These time-series implications of the model offer novel empirical predictions tying liquidity and asset price behavior through financial intermediaries.

6 Conclusion

This paper explores the allocation of information that arises when information is sold by a monopolist to a set of strategic risk-averse traders. We find that the optimal sales of information takes on one of the following two forms: (i) sell to many agents very imprecise information; (ii) sell to a small group of agents signals as precise as possible. We show that the optimality of one type of contract versus the other is driven by the tradeoff between maximizing aggregate expected profits and ex-ante risk-sharing. The newsletters equilibrium (i) maximizes ex-ante risk-sharing generating disagreement among traders and high trading volume, while the exclusivity contracts (ii) maximizes expected trading profits concentrating information in the hands of a few agents. Comparing equilibrium properties at the optimal information allocations, we find a ranking reversal of the informational efficiency of prices across markets and models: as the seller of information wants to maximize its value, she finds it optimal to sell *less* information when the negative externality associated with information leakage is relatively more intense. As a consequence, the market-orders model yields more efficient prices than with limit-orders,

and the strategic model is more informational efficient than its competitive counterpart. Our results complement the previous literature on information sales and highlight the importance of endogenizing information allocations when studying information aggregation and efficiency.

Finally, we acknowledge the fact that the sell-side industry is far from a monopoly. While our qualitative results on the type of information sales that arise are likely to be robust to allowing for competition in the market for information, further research may uncover other predictions that may arise with competition among multiple information sellers. We also recognize that much of the information that is distributed to the market is produced by sell-side analysts that work for brokerage houses, and who might be interested in maximizing not only consumers' surplus, as in our analysis, but also trading volume. The newsletter equilibrium would fit the bill for this modified objective function: generating disagreements among traders as to the value of the asset maximizes trading volume.

Appendix

Before proceeding with the proofs, we introduce some basic notation, following Kyle (1989). The Technical Appendix that accompanies the paper contains a full characterization of the equilibria of the models studied in the paper. The interested reader is urged to consult this Technical Appendix for further details.

We characterize informed agents' trading strategies in the limit-orders model by two positive constants (β, γ) , defined by $\theta_i = \beta Y_i - \gamma P_x$, for informed agent $i = 1, \dots, m$; whereas we use $\theta_i = \beta Y_i$ in the market-orders model. We reserve the index i for informed agents, and we will use u to denote quantities related to the uninformed agents. From the market-clearing condition prices will be of the form

$$P_x = \lambda \left(\beta \sum_{i=1}^m Y_i - Z \right);$$

for some $\lambda > 0$. We define the informational content parameter ψ by $\text{var}(X|P_x)^{-1} \equiv \tau_u = 1 + \psi y$, where $y \equiv m s_\epsilon$ is the “stock of private information” in the economy. The variable ψ measures the fraction of the informed agents' precision that is revealed by prices. Furthermore, we define the conditional precision of payoffs and trading profits for an informed agent as $\tau_i \equiv \text{var}(X|\mathcal{F}_i)^{-1}$ and $\tau_\pi \equiv \text{var}(X - P_x|\mathcal{F}_i)^{-1}$ respectively. In the market-orders model $\mathcal{F}_i = \sigma(Y_i)$, whereas in the limit-orders model $\mathcal{F}_i = \sigma(Y_i, P_x)$, so that in the later case $\tau_i = \tau_\pi$. Finally, we define the informational incidence parameter $\zeta \equiv dP/d\mathbb{E}[X|\mathcal{F}_i]$ and the price impact parameter $\lambda_i \equiv dP_x/d\theta_i$.

The equilibrium in the limit-orders market of Kyle (1989) is characterized, for a given information allocation m and s_ϵ , via the system of equations

$$\kappa \sqrt{\frac{\psi}{(1-\psi)y}} = m \frac{(1-\psi)(1-2\zeta)}{(m-\psi)(1-\zeta)}; \quad \frac{\zeta}{\tau_i} = \frac{\psi}{\tau_u}; \quad (3)$$

where $\kappa = r\sigma_z$. The corresponding system in the market-orders model is given by

$$\kappa \sqrt{\frac{\psi}{(1-\psi)y}} = \frac{\tau_\pi(1-\psi)(1-2\zeta)}{\tau_i\tau_u(1-\zeta)}; \quad \frac{\zeta}{\tau_i} = \frac{\psi}{1+\psi s_\epsilon}; \quad (4)$$

and in the competitive version of Kyle (1989) by

$$\kappa \sqrt{\frac{\psi}{(1-\psi)y}} = m \frac{(1-\psi)}{(m-\psi)}. \quad (5)$$

Proof of Proposition 1.

For each trader, the certainty equivalent of wealth is the constant c that solves $\mathbb{E}[u(\pi_i)] = u(c)$, and the CARA assumption implies

$$c = -\frac{1}{r} \log(-\mathbb{E}[u(\pi_i)]). \quad (6)$$

The first-order condition of an informed trader θ_i satisfies (Kyle, 1985, 1989)

$$\theta_i = \frac{\mathbb{E}[X - P_x | \mathcal{F}_i]}{r \text{var}[X - P | \mathcal{F}_i] + \lambda_i}; \quad (7)$$

for some $\lambda_i > 0$ (in the competitive model $\lambda_i = 0$).

In order to compute (6) we start by noticing that, at the interim stage, profits are conditionally normal, so we can write

$$c = -\frac{1}{r} \log(\mathbb{E}[\mathbb{E}[\exp(-r\pi_i) | \mathcal{F}_i]]) = -\frac{1}{r} \log(\mathbb{E}[\exp(-r\chi_i)]), \quad (8)$$

where

$$\chi_i = \mathbb{E}[\pi_i | \mathcal{F}_i] - \frac{r}{2} \text{var}(\pi_i | \mathcal{F}_i). \quad (9)$$

Define $\eta_i \equiv \mathbb{E}[X - P_x | \mathcal{F}_i]$ and rewrite (9), using the definition of profits $\pi_i = \theta_i(X - P_x)$ and the first-order condition (7), as

$$\chi_i = \eta_i^2 \left(\frac{\lambda_i + \frac{r}{2} \text{var}[X - P_x | \mathcal{F}_i]}{(\lambda_i + r \text{var}[X - P_x | \mathcal{F}_i])^2} \right). \quad (10)$$

As $P_x = \mathbb{E}[X | \mathcal{F}_u]$ and $\mathbb{E}[X] = 0$, we have that $\mathbb{E}[X - P_x] = 0$. As a consequence η_i has the form $\eta_i = l_1 Y_i + l_2 P_x$ if $\mathcal{F}_i = \sigma(Y_i, P_x)$ (limit-orders model) and $\eta_i = l_3 Y_i$ if $\mathcal{F}_i = \sigma(Y_i)$ (market-orders model), for some constants l_1, l_2, l_3 . Therefore η_i is normally distributed and $\mathbb{E}[\eta_i] = 0$. Standard results on the expectation of quadratic forms of Gaussian random variables yield²⁴

$$\begin{aligned} \mathbb{E}[\exp(-r\chi_i)] &= \left(1 + 2r\sigma_\eta^2 \left(\frac{\lambda_i + \frac{r}{2} \text{var}[X - P_x | \mathcal{F}_i]}{(\lambda_i + r \text{var}[X - P_x | \mathcal{F}_i])^2} \right) \right)^{-1/2} \\ &= (1 + 2r\mathbb{E}[\chi_i])^{-1/2}, \end{aligned}$$

²⁴The expression follows from well known properties of multivariate normal distributions. In particular, if $X \sim \mathcal{N}(\mu, \Sigma)$ is an n -dimensional Gaussian random vector, $b \in \mathbb{R}^n$ is a vector, $A \in \mathbb{R}^{n \times n}$ is a symmetric matrix, it is well known that

$$\mathbb{E} \left[\exp \left(b^\top X + X^\top A X \right) \right] = |I - 2\Sigma A|^{-1/2} \exp \left[b^\top \mu + \mu^\top A \mu + \frac{1}{2} (b + 2A\mu)^\top (I - 2\Sigma A)^{-1} \Sigma (b + 2A\mu) \right];$$

as long as the matrix $I - 2\Sigma A$ is positive definite.

where the second equality follows by taking expectations in (10), and $\sigma_\eta^2 = \text{var}(\eta_i)$. Using the above expression in (8) and rearranging completes the proof. \square

Proof of Proposition 2.

We first show that the monopolist sets $s_\epsilon = \infty$ when $m = 1$. The expected interim certainty equivalent can be expressed as

$$\mathbb{E}[\chi_i] = \frac{1}{2r} \left(\frac{\tau_i}{\tau_u} - 1 \right) \frac{(1 - 2\zeta)}{(1 - \zeta)^2} = \frac{1}{2r} \left(\frac{\zeta - \psi}{\psi} \right) \frac{(1 - 2\zeta)}{(1 - \zeta)^2}, \quad (11)$$

where the first equality is shown in the Technical Appendix (section 4) and the second follows by (3). Using the equilibrium definitions for ψ and ζ we have that, for any $\kappa \geq 0$, the monopolist's problem when she sells to one agent reduces to

$$\max_{s_\epsilon} \mathcal{C} = \max_{s_\epsilon} \frac{1}{2r} \log \left(1 + r\sigma_z \frac{\sqrt{r^2\sigma_z^2 + 4s_\epsilon(1 + s_\epsilon)} - r\sigma_z}{2(1 + s_\epsilon)} \right). \quad (12)$$

It is easy to verify that the above function is strictly increasing in s_ϵ , so that the optimal solution is to set $s_\epsilon = \infty$. Furthermore, taking limits in (12) we have that when the monopolist sells to a single agent her profits are given by

$$\mathcal{C}_1(r, \sigma_z) = \frac{1}{2r} \log(1 + r\sigma_z). \quad (13)$$

Let us introduce the variable ϕ , defined as $\tau_i = \text{var}(X|Y_i, P_x)^{-1} = 1 + s_\epsilon + (m - 1)\phi s_\epsilon$. In order to derive the optimality condition that defines y , we note that using (3), the problem in (1) for a fixed m can be equivalently stated as

$$\max_{y, \phi, \mu} \frac{m}{2r} \log(1 + \Omega(y, \phi)) - \mu G(y, \phi) \quad (14)$$

where

$$\Omega(y, \phi) = \frac{2\kappa}{1 + \phi y} \sqrt{\frac{\phi y}{m(m - 1)(1 - \phi)}}; \quad (15)$$

such that

$$G(y, \phi) = \kappa \left[\frac{\phi}{y(1 - 1/m)(1 - \phi)} \right]^{1/2} - 1 + 2\phi + \frac{\phi(1 + y)}{(m - 1)(1 + \phi y)} = 0. \quad (16)$$

Equations (15) and (16) follow from (11) and the equilibrium condition (3) expressed in terms of ϕ , using the definitions of ζ and ψ (see the Technical Appendix for details). We note that $\Omega = 2r\mathbb{E}[\chi_i]$, i.e. it is a simple rescaling of the expected interim certainty equivalent.

The first-order conditions for y and ϕ from (14) are

$$\Omega \left(\frac{1}{2y} - \frac{\phi}{1 + \phi y} \right) = \mu \left[\frac{\phi(1 - \phi)}{(m - 1)(1 + \phi y)^2} - \frac{\Omega m(1 + \phi y)}{2y^2} \right]; \quad (17)$$

$$\Omega \left(\frac{1}{2\phi(1 - \phi)} - \frac{y}{1 + \phi y} \right) = \mu \left[2 + \frac{1 + y}{(m - 1)(1 + \phi y)^2} + \frac{\Omega m(1 + \phi y)}{2\phi(1 - \phi)y} \right]. \quad (18)$$

Define $x = \phi y$. Using the constraint $G(y, \phi) = 0$ and the first-order conditions (17)-(18) one can verify after some simple algebraic manipulations that, at an interior optima, the following must be satisfied

$$Ax^2 + Bx + C = 0; \quad (19)$$

with $A = (m - 2)(1 - \phi)$, $B = (m - 1)\phi(2\phi - 1) + 1 + \phi(1 - \phi)$, and $C = (m - 1)(2\phi^2 - 1) + \phi^2$. The above quadratic equation implicitly defines the optimal y for given m and ϕ . Together with (16), it characterizes the equilibrium at the optimal y for a given m . One can readily verify that, for $m \geq 2$, (19) yields a strictly positive and finite value for x , and thus for the optimal noise s_ϵ .

We next show that the optimal noise s_ϵ is increasing in κ . The system of equations (16) and (19) defines the equilibrium optimal values for y and ϕ . Applying the implicit function theorem at this system, one can verify that $dy/d\kappa > 0$ if and only if

$$y\phi \left(2 + \phi \left(y + 4y\phi + y^2\phi(\phi - 1) - 2(1 + \phi) \right) \right) > 1. \quad (20)$$

When $\kappa = 0$, one can immediately use (16) to verify that (20) holds. In order to see that it holds for all κ , we make an argument by contradiction. In particular, if there exists some κ' such that (20) does not hold, by continuity there must exist some κ^* for which (20) holds as an equality. Equivalently, there exists some $y(\kappa^*)$ that solves (20) as an equality. But this cubic equation, for all $\phi \in [0, 1/2]$, has a unique real solution that is negative. This yields a contradiction. Thus $ds_\epsilon/d\kappa > 0$. \square

Proof of Proposition 3.

Limit-order model. The consumer surplus in the case $m = 1$ is given by $\mathcal{C}_1(r, \sigma_z)$ in (13). The consumer surplus in the case $m \geq 2$ is given

$$\mathcal{C}_m(r, \sigma_z) = \frac{m}{2r} \log \left(1 + \kappa \hat{\Omega}(\phi, x) \right)$$

where x is defined as in the proof of Proposition 2, $\hat{\Omega} = \Omega/\kappa$, where Ω is defined in (15), and ϕ is given implicitly by (16).

In order to characterize the optimal sales of the monopolist, we define the ratio of her profits in the case where she sells to one agent to those where she contracts with $m \leq N$:

$$\mathcal{R}_m(\kappa) \equiv \frac{\mathcal{C}_1(r, \sigma_z)}{\mathcal{C}_m(r, \sigma_z)}.$$

Taking limits we have that

$$\lim_{\kappa \downarrow 0} \mathcal{R}_m(\kappa) = \frac{1}{\hat{\Omega}_0(m)},$$

with

$$\hat{\Omega}_0(m) = \lim_{\kappa \downarrow 0} m\hat{\Omega} = \frac{1}{1+x} \sqrt{\frac{mx}{(1-\phi)(m-1)}}. \quad (21)$$

We will show that $\hat{\Omega}_0(m) < 1$ for all $m \geq 2$, which implies the result in the Proposition. The first-order condition (19) and the equilibrium condition (16) imply that as $\kappa \downarrow 0$

$$x = \frac{w}{2(m-1)}; \quad \phi = \frac{2(m-1)^2 + w(m-1) - w}{2(m-1)(1 + 2(m-1) + w)}.$$

where $w = -1 + \sqrt{1 + 2(m-1) + 4(m-1)^2}$. We finally note that $x < 1$ and that $w \in (2(m-2), 2(m-1))$. Using these expressions in (21) one can verify that $\Omega_0(m)$ is a strictly decreasing function of m , which achieves a maximum at $\Omega_0(2) = 0.79 < 1$. Since this is a strict inequality, and the profit function for any m is a continuous function of κ , the monopolist optimally sells to a single agent. Using the first part of Proposition 2 completes the proof for the limit-order model.

Market-order model. We first characterize the problem as in Proposition 2. The expression for the interim certainty equivalent in the Technical Appendix and (4) imply that the monopolist's problem reduces to

$$\max_{y, \psi} \mathcal{C}_m(r, \sigma_z) = \frac{1}{2r} \log(1 + \kappa\Omega(y, \psi)) \quad (22)$$

such that

$$G(y, \psi) \equiv \kappa J(y, \psi) - \left(1 - \psi \frac{y}{m} - 2\psi\right) = 0; \quad (23)$$

where

$$\Omega(y, \psi) = \frac{(m + \psi y)}{m^2(1 + \psi y)} \sqrt{\frac{\psi y}{1 - \psi}}; \quad J(y, \psi) = \frac{\Omega(y, \psi)m [(m + y)(1 + \psi y) - y(1 - \psi)^2]}{y(m + \psi y)}$$

The first-order conditions to this problem imply that

$$\frac{d\Omega}{dy} \left(\kappa \frac{dJ}{d\psi} + \frac{y}{m} + 2 \right) = \left(\kappa \frac{dJ}{dy} + \frac{\psi}{m} \right) \frac{d\Omega}{d\psi}. \quad (24)$$

The expressions (23) and (24) are the system of equations that characterize the optimal information sales for a fixed m , so long as the optimal y is interior. The non-interior y can be dealt with similarly, by formally taking limits in the above expressions for consumer surplus and the equilibrium constraint. In particular, one can show that the profits at the corner solutions $y = \infty$ are equal to

$$\mathcal{C}_m^*(r, \sigma_z) = \frac{m}{2r} \log \left(1 + \kappa \frac{\sqrt{t(m+t)}}{m^2(1+t)} \right) \quad (25)$$

with t being the solution to the following quartic

$$\kappa t^3 = (1+t^2)(m-t)^2.$$

Define the ratio of the profits in the $m = 1$ case, where the monopolist sets $y = \infty$, and in the $m \geq 2$ cases at interior solutions:

$$\mathcal{R}_m(\kappa) \equiv \frac{\mathcal{C}_1^*(r, \sigma_z)}{\mathcal{C}_m^*(r, \sigma_z)}.$$

Taking limits in the above expression, using (22) and (25) one has that

$$\lim_{\kappa \downarrow 0} \mathcal{R}_m(\kappa) = \frac{m^2(1+y\psi)}{(m+\psi y)} \sqrt{\frac{(1-\psi)}{\psi y}}.$$

where y and ψ are given by (23) and (24) evaluated at $\kappa = 0$. These two equations can be solved explicitly to find $y = 2m/(m-3)$ and $\psi = (m-3)/(2(m-2))$. Note that there are interior solutions for y when $\kappa \downarrow 0$ if and only if $m \geq 4$. These expressions further imply

$$\lim_{\kappa \downarrow 0} \mathcal{R}_m(\kappa) = \sqrt{2(m-1)},$$

which is clearly greater than 1 for $m \geq 2$, so the information allocation where the monopolist sells to a single agent perfect information dominates any of the allocations with interior solutions as $\kappa \downarrow 0$.

To complete the proof, we define the ratio of the profits in the $m = 1$ case and in the $m \geq 2$

cases where the monopolist sets $y = \infty$:

$$\mathcal{R}_m^*(\kappa) \equiv \frac{\mathcal{C}_1^*(r, \sigma_z)}{\mathcal{C}_m^*(r, \sigma_z)}.$$

Simple algebra shows that

$$\lim_{\kappa \downarrow 0} \mathcal{R}_m^*(\kappa) = \frac{(1+m)}{2\sqrt{m}},$$

which is clearly greater than 1 for any $m \geq 2$. The open set statement follows from continuity. This completes the proof of the Proposition. \square

Proof of Proposition 4.

Limit-order model. We first show that profits are increasing in m for κ large enough. Define $\hat{\kappa} \equiv 1/\kappa$ and let y be given. The proof of Lemma 4 in the Technical Appendix shows that the equilibrium condition $\zeta = \psi\tau_i/\tau_u$ in (3) can be written as

$$y = \frac{(m-\psi)(\zeta-\psi)}{\psi(1+\psi(m-2)-\zeta(m-\psi))}. \quad (26)$$

For (3) to hold as $\hat{\kappa} \rightarrow 0$ we must have that $\psi \rightarrow 0$, $\zeta \rightarrow \psi$ and $(\zeta - \psi)/\psi \rightarrow y/m$. As a consequence,

$$\lim_{\hat{\kappa} \downarrow 0} 2r\mathbb{E}[\chi_i] = \lim_{\hat{\kappa} \downarrow 0} \frac{(\zeta - \psi)(1 - 2\zeta)}{\psi(1 - \zeta)^2} = \frac{y}{m}.$$

Taking the derivative of (1) with respect to m , and using the above expression, we have that as $\hat{\kappa}$ goes to zero

$$\text{sign}\left(\frac{d\mathcal{C}}{dm}\right) = \text{sign}\left(\log(1 + y/m) - \frac{y/m}{(1 + y/m)}\right) > 0.$$

From this it is immediate that the monopolist optimally sells to $m = N$ agents. The open set statement in the Proposition then follows from continuity of the problem with respect to $\hat{\kappa}$.

Next we characterize the solution of the monopolist's problem as $m \rightarrow \infty$. For (26) to hold as an equality in the limit as $m \rightarrow \infty$, it is clear that we must have $\lim_{m \uparrow \infty} \zeta = \lim_{m \uparrow \infty} \psi \equiv \psi_\infty$. Then, taking limits as $m \rightarrow \infty$ in (1) and using (3)

$$\lim_{m \uparrow \infty} \mathcal{C} = \frac{y(1 - 2\psi_\infty)}{2r(1 + y\psi_\infty)}, \quad (27)$$

and that the constraint (3) reduces to

$$\kappa \sqrt{\frac{\psi_\infty}{y(1-\psi_\infty)}} = 1 - 2\psi_\infty. \quad (28)$$

Using (28) to eliminate y in (27) the monopolist's problem becomes

$$\max_{\psi_\infty} \frac{\kappa^2}{2r} \frac{\psi_\infty(1-2\psi_\infty)}{((1-\psi_\infty)(1-2\psi_\infty)^2 + \kappa^2\psi_\infty^2)}$$

Equating to zero the derivative of the above expression yields the optimality condition for ψ_∞ as the unique real solution in $[0, 1/2]$ to

$$\psi_\infty^4 - \psi_\infty^3 + \frac{(\kappa^2 - 2)}{8}\psi_\infty^2 + \frac{1}{2}\psi_\infty - \frac{1}{8} = 0. \quad (29)$$

Moreover, from (29) we have $\kappa^2\psi_\infty^2 = (1-2\psi_\infty)^2(1-2\psi_\infty^2)$, and using this into (28) yields the optimal stock of private information sold:

$$y = \frac{1-2\psi_\infty^2}{\psi_\infty(1-\psi_\infty)}, \quad (30)$$

and informational efficiency

$$\text{var}(X|P_x)^{-1} = 1 + \frac{1-2\psi_\infty^2}{(1-\psi_\infty)}. \quad (31)$$

Using (27) and (30), the monopolist's profits

$$\mathcal{C}_\infty(r, \sigma_z) \equiv \frac{(1-2\psi_\infty^2)(1-2\psi_\infty)}{2r\psi_\infty(2(1-\psi_\infty^2) - \psi_\infty)}. \quad (32)$$

Since $\psi_\infty \in [0, 1/2]$, equation (30) shows that $\tau_u \geq 2$. Applying the implicit function theorem to (29) one can verify that $\psi_\infty(\kappa)$ defines a monotonically decreasing function of κ , so using (30), y is increasing in κ . We remark that since the stock of private information sold is finite, when $N \uparrow \infty$ the precision of the signal received by each agent vanishes. This completes the proof in the limit-order model.

Market-order model. We follow a similar argument as the one used in the limit-order case. Solving the equilibrium condition $\zeta = \psi\tau_i/(1+\psi s_\epsilon)$ in (4) for s_ϵ yields

$$s_\epsilon = \frac{(\zeta - \psi)}{\psi(1 - \zeta)}, \quad (33)$$

so that for (4) to hold as $\hat{\kappa} \rightarrow 0$, we must have $\psi \rightarrow 0$, $\zeta \rightarrow \psi$ and $(\zeta - \psi)/\psi \rightarrow y/m$; moreover $\psi \rightarrow 0$ implies $\tau_\pi \rightarrow \tau_i$. As a consequence,

$$\lim_{\hat{\kappa} \downarrow 0} 2r\mathbb{E}[\chi_i] = \lim_{\hat{\kappa} \downarrow 0} \frac{(\tau_\pi - \tau_u)(1 - 2\zeta)}{\tau_u(1 - \zeta)^2} = \frac{y}{m}.$$

In order to prove the second part, we show that as $m \rightarrow \infty$ the monopolist's problem in the model with market orders and with limit orders coincide. Consider the problem for fixed y . Multiplying both sides of (33) by m yields

$$y = \frac{m(\zeta - \psi)}{\psi(1 - \zeta)}.$$

For the above equation to hold as $m \rightarrow \infty$, it is clear that we must have $\lim_{m \uparrow \infty} \zeta = \lim_{m \uparrow \infty} \psi = \psi_\infty$. Taking limits as $m \rightarrow \infty$ in the profit function one obtains

$$\lim_{m \uparrow \infty} \mathcal{C} = \frac{y(1 - 2\psi_\infty)}{2r(1 + y\psi_\infty)},$$

and the constraint (4) reduces to

$$\kappa \sqrt{\frac{\psi_\infty}{y(1 - \psi_\infty)}} = 1 - 2\psi_\infty.$$

As the last two equations are identical to (27) and (28), the problems with limit and market orders coincide.

We define trading volume as usual (Vives, 2008):

$$\begin{aligned} \mathcal{V} &= \frac{1}{2} \mathbb{E} \left[\sum_{i=1}^m |\theta_i| + |Z| + |\omega| \right] \\ &= \sqrt{\frac{1}{2\pi}} (m\sigma_\theta + \sigma_z + \sigma_\omega); \end{aligned}$$

where

$$\begin{aligned} \sigma_\theta &= \frac{1}{r} \sqrt{\frac{(\zeta - \psi)(1 + \psi y)\zeta}{\psi^2} \left(\frac{1 - 2\zeta}{1 - \zeta} \right)}, \\ \sigma_\omega &= \sigma_z \sqrt{\frac{1 - \psi}{1 + \psi y}}. \end{aligned}$$

It is straightforward to verify that under the exclusivity contract of Proposition 3 we have

$$\mathcal{V} = \sqrt{\frac{1}{2\pi}} \sigma_z (1 + \sqrt{2}). \quad (34)$$

Furthermore, under the optimal sales in Proposition 4 we have

$$\lim_{N \uparrow \infty} \sqrt{m} \sigma_\theta = \frac{\sqrt{y}(1 - 2\psi_\infty)}{r}, \quad (35)$$

so the trading volume of the informed agents grows without bound. This concludes the proof. \square

Proof of Proposition 5.

Some simple manipulations of the objective function and the equilibrium conditions from the Technical Appendix show that the monopolist's problem reduces to

$$\max_{m, h, s_\epsilon, \beta} \frac{m}{2r} \log \left(1 + \frac{2\beta\sigma_z^2}{(\sigma_z^2 + m\beta^2h)} \frac{f(\beta, h; r/2)}{f(\beta, h; r)} \right) \quad (36)$$

such that

$$m\beta^2(1 + \sigma_\epsilon^2) - rm^2\beta^3(1 + \sigma_\epsilon^2) + r\beta(1 + \sigma_\epsilon^2)(m\beta^2h + \sigma_z^2) - \frac{\beta r \sigma_z^4}{(m\beta^2h + \sigma_z^2)} - \sigma_z^2 = 0; \quad (37)$$

where $h = m + \sigma_\epsilon^2(1 + \rho(m - 1))$ represents the correlation choice (so $h \geq m$ in order to satisfy positive semi-definiteness), $\sigma_\epsilon^2 = 1/s_\epsilon$ is the variance of the signal errors, and

$$f(\beta, h; r) \equiv m\beta(\sigma_z^2 + m\beta^2h) - rm^2\beta^2(\sigma_z^2 + m\beta^2h) + r(\sigma_z^2 + m\beta^2h)^2 - r\sigma_z^4/(1 + \sigma_\epsilon^2).$$

We consider the choice of the correlation coefficient and signal precision, for a fixed m . The monopolist's problem reduces to

$$\max_{h, \beta, s_\epsilon} F(h, \beta) \equiv \frac{\beta}{(\sigma_z^2 + m\beta^2h)} \frac{f(\beta, h; r/2)}{f(\beta, h; r)}$$

such that $h \geq m$ and (37) holds. We shall show that $dF/dh < 0$. In order to do this we first note that

$$\frac{dF}{dh} = \frac{\partial F}{\partial h} + \frac{\partial F}{\partial \beta} \frac{\partial \beta}{\partial h}; \quad (38)$$

where we are treating β as an implicit function of h defined by the equilibrium constraint (37).

It is straightforward to verify that $\partial F/\partial h < 0$ and that $\partial \beta/\partial h < 0$. In order to sign (38) we first use the first-order condition for the noise choice: $\frac{\partial F}{\partial \sigma_\epsilon^2} + \frac{\partial F}{\partial \beta} \frac{\partial \beta}{\partial \sigma_\epsilon^2} = 0$, from which one

obtains $\frac{\partial F}{\partial \beta} = -\frac{\partial F}{\partial \sigma_\epsilon^2} / \frac{\partial \beta}{\partial \sigma_\epsilon^2}$. Simplifying the resulting expressions into (38), one can verify that $\frac{dF}{dh} < 0$, which shows that the monopolist should set h as low as possible, or $\rho = -1/(m-1)$. This completes the proof. \square

Proof of Proposition 6.

Using the constraint (5) to eliminate s_ϵ one can verify that, for a fixed m , the monopolist seller of information is solving

$$\max_{\phi} \quad \mathcal{C} = \frac{m}{2r} \log(1 + \Omega); \quad (39)$$

where

$$\Omega = \frac{\kappa^2 \phi}{(1 - \phi) \left[m - 1 + \phi + \frac{(\kappa m \phi)^2}{(m-1)(1-\phi)^3} \right]}.$$

The first-order condition for ϕ yields :

$$\frac{(\kappa m \phi)^2}{(m-1)(1-\phi)^3} = \frac{m-1+\phi^2}{1+\phi}. \quad (40)$$

By the envelope theorem

$$\frac{d\mathcal{C}}{dm} = \frac{1}{2r} \left[\log(1 + \Omega) - \alpha \frac{\Omega}{1 + \Omega} \right], \quad (41)$$

where

$$\alpha = \frac{m + \frac{(m-2)(\kappa m \phi)^2}{(m-1)^2(1-\phi)^3}}{m-1+\phi + \frac{(\kappa m \phi)^2}{(m-1)(1-\phi)^3}}. \quad (42)$$

From (41), a sufficient condition for profits to be increasing in m is that $\alpha \leq 1$. Substituting from (40) into (42) and rearranging one can verify that indeed $\alpha \leq 1$, so the sufficient condition is satisfied and, as a consequence, the monopolist finds optimal to sell to $m = N$. Taking limits as $m \rightarrow \infty$ in (39) and (5), using (40), it is immediate to get that the monopolist sets

$$y = \frac{\kappa^2 \psi_\infty}{(1 - \psi_\infty)^3} \quad (43)$$

where ψ_∞ solves

$$(1 + \kappa^2)\psi_\infty^3 + (\kappa^2 - 3)\psi_\infty^2 + 3\psi_\infty - 1 = 0; \quad (44)$$

and that monopolist's profits are given by

$$\lim_{N \uparrow \infty} \mathcal{C}(m, s_\epsilon) = \frac{1}{2r} \frac{(1 - \psi_\infty)^2}{\psi_\infty(2 + \psi_\infty)}. \quad (45)$$

Informational efficiency is measured by the precision conditional on the market price, given by $\tau_u = 1 + y\psi$. Notice that in the limit as $m \rightarrow \infty$, we have $\phi \rightarrow \psi$. In the competitive case we can express (44) as $(\kappa\phi_\infty)^2 = \frac{(1-\phi_\infty)^3}{1+\phi_\infty}$ and substitute into (43) to get $y\phi_\infty = \frac{1}{1+\phi_\infty}$; as $\phi_\infty \in [0, 1]$, we have $\tau_u \in [1.5, 2]$. Since $\tau_u \geq 2$ in the strategic model (see Propositions 3 and 4), prices are less informative in the competitive equilibrium. Finally, applying the implicit function theorem to (44) one can verify that, as in the imperfectly competitive case, $\phi_\infty(\kappa)$ is a monotonically decreasing function of κ . In turn, this implies that τ_u is monotonically *increasing* in κ in the competitive equilibrium. This concludes the proof. \square

References

- Admati, A. R., and P. Pfleiderer, 1986, “A monopolistic market for information,” *Journal of Economic Theory*, pp. 400 – 438.
- , 1987, “Viable allocations of information in financial markets,” *Journal of Economic Theory*, 43, 76–115.
- , 1988, “Selling and trading on information in financial markets,” *American Economic Review*, 78(2).
- , 1990, “Direct and indirect sale of information,” *Econometrica*, 58(4), 901–928.
- Antweiler, W., and M. Z. Frank, 2004, “Is all that talk just noise? The information content of Internet stock message boards,” *Journal of Finance*, 59(3), 1259–1294.
- Barber, B. M., Y.-T. Lee, Y.-J. Liu, and T. Odean, 2009, “Just how much do individual investors lose by trading?,” *Review of Financial Studies*, 22(2), 609–632.
- Barber, B. M., and T. Odean, 2000, “Trading is hazardous to your wealth: The common stock investment performance of individual investors,” *Journal of Finance*, 55, 773–806.
- Baruch, S., 2005, “Who benefits from an open limit-order book?,” *Journal of Business*, 78(4), 1267–1306.
- Benabou, R., and G. Laroque, 1992, “Using privileged information to manipulate markets: insiders, gurus, and credibility,” *The Quarterly Journal of Economics*, pp. 921–958.
- Bernhardt, D., and B. Taub, 2006, “Kyle vs. Kyle (85 v. 89),” *Annals of Finance*, 2(1), 23–38.
- Biais, B., and L. Germain, 2002, “Incentive compatible contracts for the sale of information,” *Review of Financial Studies*, 15, 987–1003.
- Biais, B., L. Glosten, and C. Spatt, 2005, “Market microstructure: a survey of microfoundations, empirical results and policy implications,” *Journal of Financial Markets*, 8(2), 217–264.
- Bloomfield, R. J., and M. O’Hara, 1999, “Market transparency: who wins and who loses,” *Review of Financial Studies*, 12, 5–35.
- Boehmer, E., G. Saar, and L. Yu, 2005, “Lifting the veil: an analysis of pre-trade transparency at the NYSE,” *Journal of Finance*, 60(2), 783–815.
- Brown, D. P., and Z. M. Zhang, 1997, “Market orders and market efficiency,” *Journal of Finance*, 52, 277–308.
- Cespa, G., 2008, “Information sales and insider trading,” *Journal of Finance*, 63(2), 639 – 672.
- Cripps, M. W., and J. M. Swinkels, 2006, “Efficiency of large double auctions,” *Econometrica*, 74(1), 47–92.

- Dierker, M., 2006, "Endogenous information acquisition with Cournot competition," *Annals of Finance*, 2, 369–395.
- Dridi, R., and L. Germain, 2009, "Noise and competition in strategic oligopoly," *Journal of Financial Intermediation*, 18(2), 311–327.
- Engelberg, J., C. Sasseville, and J. Williams, 2009, "Market madness? The case of Mad Money," working paper, UNC at Chapel Hill.
- Fishman, M. J., and K. M. Hagerty, 1995, "The incentive to sell financial market information," *Journal of Financial Intermediation*, 4, 95–115.
- Gaier, E. M., and B. Katzman, 2002, "Endogeneous information quality in common value auctions," working paper, University of Miami.
- García, D., and J. M. Vanden, 2009, "Information acquisition and mutual funds," *Journal of Economic Theory*, 144(5), 1965–1995.
- Germain, L., 2005, "Strategic noise in competitive markets for the sale of information," *Journal of Financial Intermediation*, 14, 179–209.
- Glosten, L. R., 1994, "Is the electronic open limit order book inevitable?," *Journal of Finance*, 49(4), 1127–1161.
- Graham, J. R., and C. R. Harvey, 1996, "Market timing ability and volatility implied in investment newsletters' asset allocation recommendations," *Journal of Financial Economics*, 42, 397–421.
- Grossman, S. J., and J. E. Stiglitz, 1980, "On the impossibility of informationally efficient markets," *American Economic Review*, 70(3), 393–408.
- Guzman, R. M., and C. D. Kolstad, 1997, "Auction equilibrium with costly information acquisition," working paper, University of Illinois at Urbana-Champaign.
- Harris, M., and A. Raviv, 1993, "Differences of opinion make a horse race," *Review of Financial Studies*, 6(3), 473–506.
- Hausch, D. B., and L. Li, 1993, "A common value auction model with endogenous entry and information acquisition," *Economic Theory*, 3(2), 315–334.
- Hong, H., and M. Shum, 2004, "Rates of information aggregation in common value auctions," *Journal of Economic Theory*, 116, 1–40.
- Irvine, P., M. Lipson, and A. Puckett, 2007, "Tipping," *Review of Financial Studies*, 20(3), 741–768.
- Jackson, M. O., 2003, "Efficiency and information aggregation in auctions with costly information," *Review of Economic Design*, 8(2), 121–141.
- Jaffe, J. F., and J. M. Mahoney, 1999, "The performance of investment newsletters," *Journal of Financial Economics*, 53, 289–307.

- Kovalenkov, A., and X. Vives, 2008, “Competitive rational expectations equilibria without apology,” working paper, IESE.
- Kremer, I., 2002, “Information aggregation in common value auctions,” *Econometrica*, 70(4), 1675–1682.
- Kyle, A. S., 1985, “Continuous auctions and insider trading,” *Econometrica*, 53(6), 1315–1336.
- , 1989, “Informed speculation with imperfect competition,” *Review of Economic Studies*, 56(3), 317–355.
- Lee, S., 2009, “Market liquidity, active investment, and markets for information,” working paper, NYU.
- Madhavan, A., D. Porter, and D. Weaver, 2005, “Should securities markets be transparent?,” *Journal of Financial Markets*, 8, 266–288.
- Matthews, S., 1984, “Information acquisition in discriminatory auctions,” in *Bayesian models in economic theory*, ed. by M. Boyer, and R. Kihlstrom. chap. 11, pp. 181–207.
- Metrick, A., 1999, “Performance evaluation with transactions data: the stock selection of investment newsletters,” *Journal of Finance*, 54(5), 1743–1775.
- Michaely, R., and K. L. Womack, 2005, “Brokerage recommendations: stylized characteristics, market responses and biases,” in *Advances in Behavioral Finance, Volume 2*, ed. by R. Thaler. Princeton University Press, pp. 354–389.
- Milgrom, P. R., 1979, “A convergence theorem for competitive bidding with differential information,” *Econometrica*, 47(3), 679–688.
- , 2000, “Rational expectations, information acquisition and competitive bidding,” *The Quarterly Journal of Economics*, 140(2), 341–388.
- Moresi, S., 2000, “Information acquisition and research differentiation prior to an open-bid auction,” *International Journal of Industrial Organization*, 18, 723–746.
- Morgan, J., and P. Stocken, 2003, “An analysis of stock recommendations,” *RAND Journal of Economics*, 34(1), 183–203.
- Odean, T., 1999, “Do investors trade too much?,” *American Economic Review*, 89, 1279–1298.
- O’Hara, M., and M. Ye, 2010, “Is market fragmentation harming market quality?,” *Journal of Financial Economics*, forthcoming.
- Persico, N., 2000, “Information acquisition in auctions,” *Econometrica*, 68(1), 135–148.
- Pesendorfer, W., and J. M. Swinkels, 1997, “The loser’s curse and information aggregation in common value auctions,” *Econometrica*, 65(6), 1247–1281.
- , 2000, “Efficiency and information aggregation in auctions,” *American Economic Review*, 90(3), 499–525.

- Reny, P. J., and M. Perry, 2006, "Toward a strategic foundation for rational expectations equilibrium," *Econometrica*, 74(5), 1231–1269.
- Ross, S. A., 2000, "Markets for agents: fund management," working paper, MIT.
- Sims, C. A., 2003, "Implications of rational inattention," *Journal of Monetary Economics*, 50(3), 665–690.
- , 2005, "Rational inattention: a research agenda," working paper, Princeton University.
- Subrahmanyam, A., 1991, "Risk aversion, market liquidity, and price efficiency," *Review of Financial Studies*, 4(3), 417–441.
- van Bommel, J., 2003, "Rumors," *The Journal of Finance*, 58(4), 1499–1520.
- Veldkamp, L., 2006, "Media frenzies in markets for financial information," *American Economic Review*, 96(3), 577–601.
- Verrecchia, R. E., 1982, "Information acquisition in a noisy rational expectations economy," *Econometrica*, 50(6), 1415–1430.
- Vives, X., 1995, "Short-term investment and the informational efficiency of the market," *Review of Financial Studies*, 8(1), 125–160.
- , 2008, *Information and learning in markets*. Princeton University Press.
- Wilson, R., 1977, "A bidding model of perfect competition," *Review of Economic Studies*, 44(3), 511–518.
- , 1979, "Auctions of shares," *The Quarterly Journal of Economics*, 93(4), 675–689.
- Womack, K., 1996, "Do brokerage analysts' recommendations have investment value?," *Journal of Finance*, 54, 137–157.

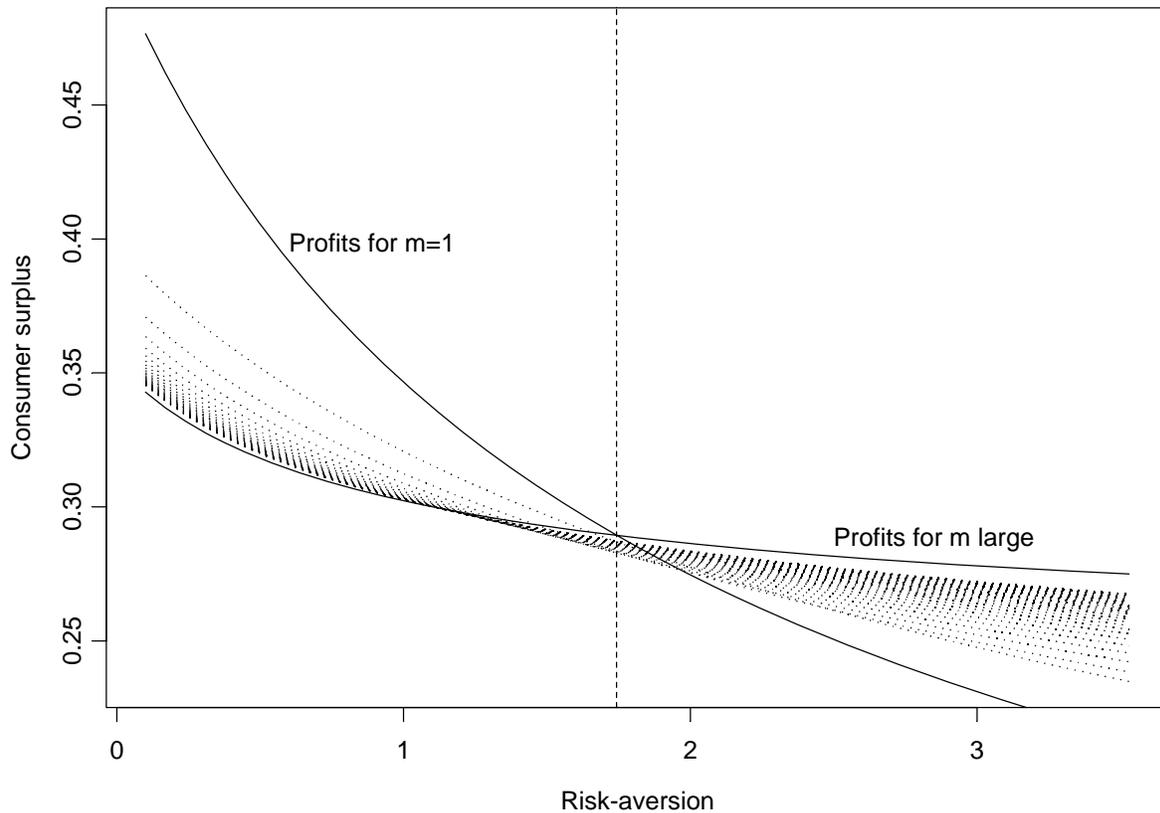


Figure 2: Equilibrium consumer surplus for different values of informed agents m and risk-aversion r , at the optimal noise s_ϵ , in the Kyle (1989) model. The solid lines correspond to consumer surplus C when $m = 1$ and $m = N$ (for large N), whereas the dashed lines correspond to values of $m = 2, \dots, 40$. The vertical line gives the breakpoint between regions where different type of information sales, $m = 1$ versus $m = N$, are optimal. Noise trading intensity is set at $\sigma_z = 1$.

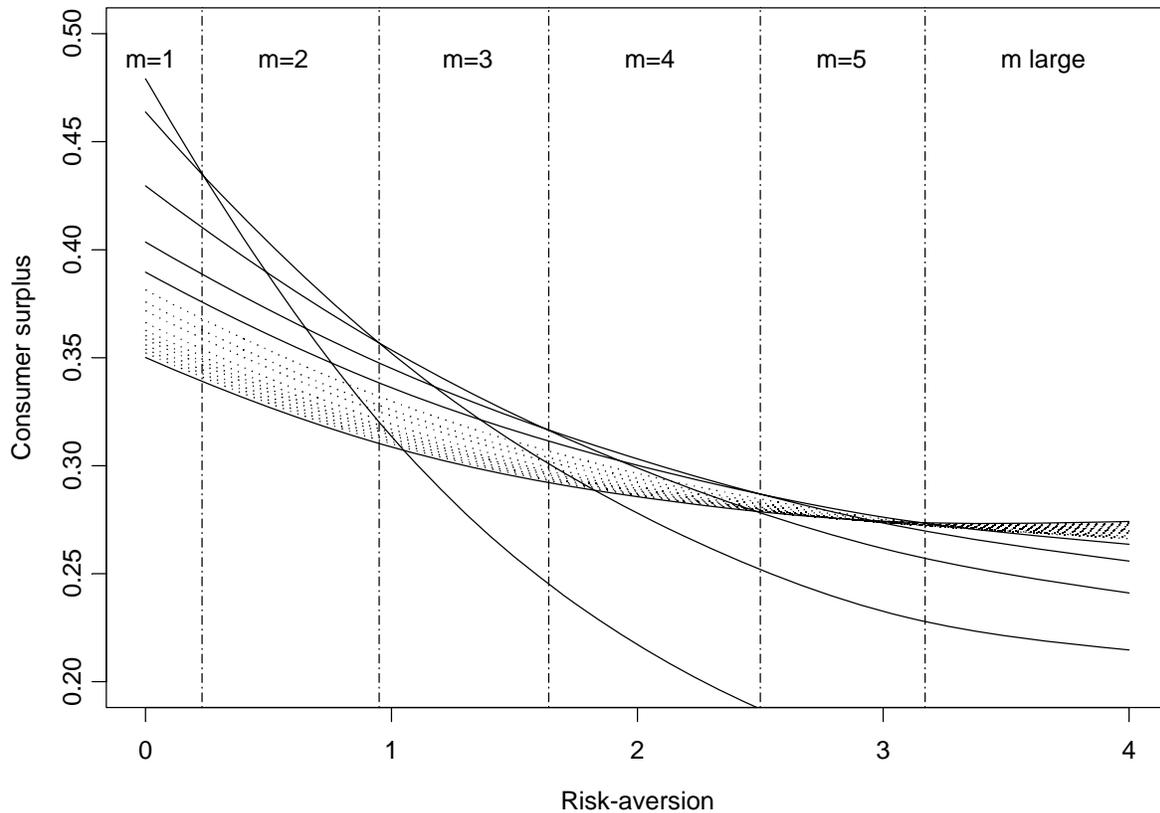


Figure 3: Equilibrium consumer surplus for different values of informed agents m and risk-aversion r , at the optimal noise s_ϵ , in the Kyle (1985) model. The solid lines correspond to consumer surplus \mathcal{C} when $m = 1, 2, 3, 4, 5$ and $m = N$ (for large N), whereas the dotted lines correspond to values of $m = 6, \dots, 40$. The vertical lines give the breakpoints between regions where different m are optimal. Noise trading intensity is set at $\sigma_z = 1$.

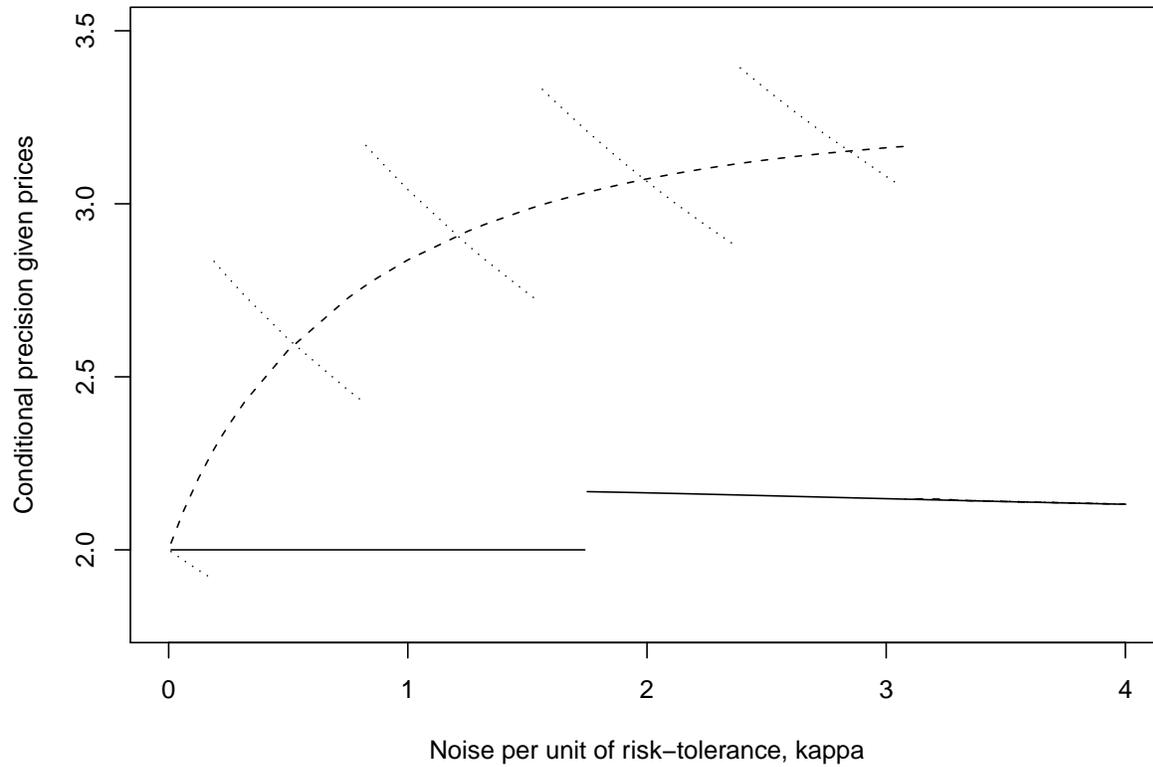


Figure 4: Equilibrium values for $\text{var}(X|P_x)^{-1}$ for different values κ , when the monopolist sells signals with i.i.d. error terms. The solid lines corresponds to the model with limit orders (Kyle, 1989). The dotted and long-dash lines correspond to the model with market-orders (Kyle, 1985). The dotted lines correspond to the case where m is treated as an integer, whereas the dashed lines present the case where m is treated as a continuous variable.

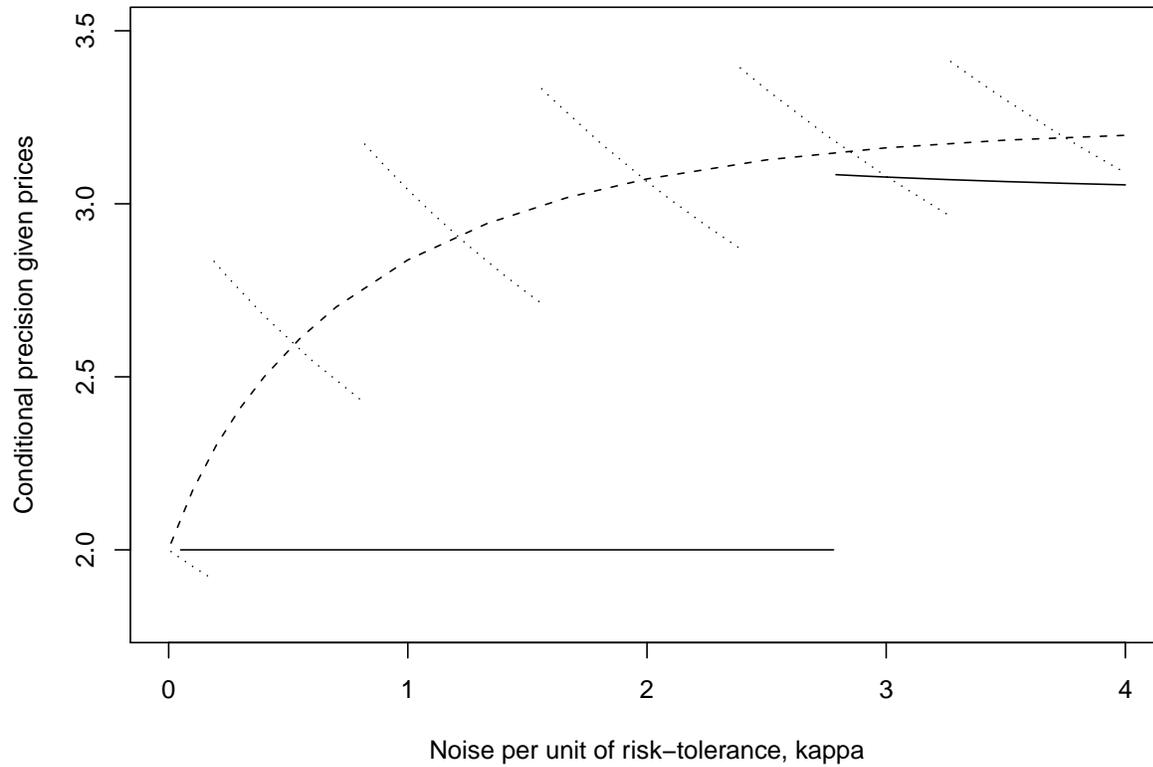


Figure 5: Equilibrium values for $\text{var}(X|P_x)^{-1}$ for different values κ , when the monopolist is constrained to sell photocopied signals to traders. The solid lines corresponds to the model with limit orders (Kyle, 1989). The dotted and long-dash lines correspond to the model with market-orders (Kyle, 1985). The dotted lines correspond to the case where m is treated as an integer, whereas the dashed lines present the case where m is treated as a continuous variable.