# Homework Assignment – Week 2

## Chapter 3

1. Write down the formula that is used to calculate the yield to maturity on a 20-year 10% coupon bond with $1,000 face value that sells for $2,000. Assume yearly coupons.

$2000  $100/(1  *i*)  $100/(1  *i*)2 ⋅ ⋅ ⋅  $100/(1  *i*)20  $1000/(1  *i*)20

1. If there is a decline in interest rates, which would you rather be holding, long-term bonds or short-term bonds? Why? Which type of bond has the greater interest-rate risk?

You would rather be holding long-term bonds because their price would increase more than the price of the short-term bonds, giving them a higher return.

1. A financial advisor has just given you the following advice: “Long-term bonds are a great investment because their interest rate is over 20%.” Is the financial advisor necessarily correct?

No. If interest rates rise sharply in the future, long-term bonds may suffer such a sharp fall in price that their return might be quite low, possibly even negative.

1. If mortgage rates rise from 5% to 10%, but the expected rate of increase in housing prices rises from 2% to 9%, are people more or less likely to buy houses?

People are more likely to buy houses because the real interest rate when purchasing a house has fallen from 3 percent (5 percent –2 percent) to 1 percent (10 percent  9 percent). The real cost of financing the house is thus lower, even though mortgage rates have risen. (If the tax deductibility of interest payments is allowed for, then it becomes even more likely that people will buy houses.)

## Chapter 3 - Quantitative Questions

1. Calculate the present value of a $1,000 zero-coupon bond with five years to maturity if the yield to maturity is 6%.

*PV*  *FV*/(1  *i*)*n* where *FV*  1000, *i*  0.06, *n*  5 🡺 PV = 747.25

1. A lottery claims their grand price is $10 million, payable over 20 years at $500,000 per year. If the first payment is made immediately, what is this grand prize really worth? Use an interest rate of 6%.

In excel you can use the function PV(0.06,20,500000,,1) = 6,079,058. Alternatively you can lay out 20 cashflows in excel and discount each one using an interest rate of 0.06 (remembering that the first cashflow is immediate). The last cashflow is 500000\*(1.06)^19 = 165,257.

1. Consider a bond with a 7% annual coupon and a face value of $1,000. Complete the following table. What relationships do you observe between maturity and discount rate and the current price?

When the coupon rate is equal to the yield to maturity then the current bond price is equal to the bond’s face value for any maturity. When the yield to maturity is above the annual coupon then the bond’s current price is below the face value. When it is below then the bond’s current price is above the face value. If the yield to maturity is not equal to the coupon rate and is kept constant for different maturities then the shorter maturity bond will have a price closer to the current price than the longer maturity bond.

|  |  |  |
| --- | --- | --- |
| Years to Maturity | Yield to Maturity | Current Price |
| 3 | 5% | 1,054.46 |
| 3 | 7% | 1,000.00 |
| 3 | 9% | 949.37 |
| 6 | 7% | 1,000.00 |
| 9 | 5% | 1,142.16 |
| 9 | 9% | 880.10 |

1. Consider a coupon bond that has a $1,000 par value and a coupon rate of 10%. The bond is currently selling for $1,150 and has eight years to maturity? What is the bond’s yield to maturity?

To compute the yield to maturity using excel you can use the function =Rate(8,-100,1150,-1000) = 7.44%

1. You are willing to pay $15,625 now to purchase a perpetuity that will pay you and your heirs $1,250 each year, forever, starting at the end of this year. If your required rate of return does not change, how much would you be willing to pay if this were a 20-year, annual payment, ordinary annuity instead of perpetuity?

To find your “yield to maturity” on the perpetuity you take PV = CF/I 🡪15,625 = 1,250/I 🡪 I = 1,250/15,625 = 8%. From here you need to use the PV function in excel PV(8%,20,-1250) = &12,272.68.

1. What is the price of a perpetuity that has a coupon of $50 per year and a yield of 2.5%? If the yield doubles, what happens to the price?

PV = 50/2.5% = $2,000. If the yield doubles then the PV is cut in half – PV = 50/5% = $1,000.

1. Property taxes in DeKalb County are roughly 2.66% of the purchase price every year. If you just bought a $100,000 home, what is the PV of all future property tax payments? Assume that the house remains worth $100,000 forever, property tax rates never change and that a 9% interest rate is used for discounting.

The annual payment is $2,660. To value a perpetuity you have PV = 2,660/0.09 = $29,556.

1. Assume you just deposited $1,000 into a bank account. The current real interest rate is 2%, and inflation is expected to be 6% over the next year. What nominal rate would you require from the bank over the next year? How much money will you have at the end of 1 year? If you are saving to buy a stereo that currently sells for $1,050, will you have enough to buy it?

The nominal interest rate needs to be 2%+6%=8%. At the end of one year you will have $1,080 in your bank account but the stereo will now cost $1,050\*(1.06) = 1,113. You will be short by 33 dollars.

1. A 10-year, 7% coupon bond with a face value of $1,000 is currently selling for $871.65. Coupon your rate of return if you sell the bond next year for $880.10.

This can be solved in a couple of different ways. First in excel you can use the function =RATE(1,-70,871.65,-880.10) = 9%. Alternatively as you have only a single payment you know that 871.65 = (880.10+70)/(1+I) 🡪 I = (950.10-871.65)/871.65 = 9%

1. You have paid $980.30 for an 8% coupon bond with a face value of $1,000 that matures in 5 years. You plan on holding the bond for 1 year. If you want to earn a 9% rate of return on this investment, what price must you sell the bond for? For this to occur what must happen to the yield to maturity on the bond over that year?

You can use the excel function =-FV(0.09,1,-80,980.3) = $988.53. You have bought this bond with a yield to maturity of 8.5%. If you are to sell the bond in one year at a price of 988.53 then you need the yield to maturity to drop to 8.35%.

1. Calculate the duration of a $1,000, 6% coupon bond with three years to maturity. Assume that all market interest rates are 7%.

There are two ways to do this calculation. In excel = DURATION(TODAY(), TODAY()+3\*365, 0.06, 0.07,1) = 2.831. Alternatively you can compute the sum of the time weighted PV of the Cashflows divided by the PV = 2,756.71/973.76 = 2.831.

|  |  |  |  |
| --- | --- | --- | --- |
| Years | Cashflow | PV of Cashflow | Time Weighted PV of Cashflow |
| 1 | 60 | 56.07 | 56.07 |
| 2 | 60 | 52.41 | 104.82 |
| 3 | 1060 | 865.28 | 2595.83 |

1. Consider the bond in the previous question. Calculate the expected price change in interest rates drop to 6.75% using the duration approximation. Calculate the actual price change using discounted cash flows.

 So the new price is 973.76+6.44 = 980.20. Using excel’s PV function you find =PV(0.0675,3,-60,-1000) =980.23. The approximation is very good in this instance (short maturity and a small change in yield).

1. The duration of a $100 million portfolio is 10 years. $40 million in new securities are added to the portfolio, increasing the duration of the portfolio to 12.5 years. What is the duration of the $40 million in new securities?

Note that the portfolio has $140 million dollars in it. The duration formula can be written as 12.5 = (100\*10/140)+(40\*D/140) 🡪 D = (12.5-100/14)\*140/40 = 18.75.

1. (Note numbers are different than in the text) A bank has two 3-year commercial loans with a present value of $70 million. The first is a $30m loan that requires a single payment of $37.8 million in three years with no other payments until then. The second loan is for $40m. It requires an annual interest payment of $3.6m. The principal of $40m is due in three years.
	1. What is the duration of the bank’s commercial loan portfolio?
	2. What will happen to the value of its portfolio if the general level of interest rates increases from 8% to 8.5%?

This problem can be solved in a couple of different ways. The two bonds have a yield to maturity of 8.55%. Thus you could consider this a single bond with maturity 3 years, a couple of 4.11% and a face value of 77.8 = (40 + 37.8). In excel you can use the DURATION function =DURATION(TODAY(),TODAY()+365\*3,0.411,0.08,1) = 2.876. Alternatively you can lay out the cashflows, discount them using the 8% interest rate and compute the time weighted PV of the cashflows divided by the PV of the cashflows. If the general level of interest rates rises to 8.5% then the value of the portfolio falls to 69.08. Again one can do this using the PV function in excel, by valuing the cashflows using the 8.5% interest rate or by using the duration formula which gives an answer of 69.07.

1. Consider a bond that promises the following cash flows. The yield to maturity is 12%. You plan to buy this bond, hold it for 2.5 years, and then sell the bond
	1. What total cash will you receive from the bond after the 2.5 years? Assume that periodic cash flows are reinvested at 12%.

Given the required discount rate is 12% you need to grow any cash that you receive to 2.5 years from the date you receive it. For example the cashflow you receive at 1 year will become 160\*1.12^1.5 = 189.65. You then need to take the PV of the remaining cashflows at the 2.5 year point. For example the cashflow at year 3 becomes 180/1.12^0.5. The value of the cashflows becomes $733.69.

* 1. If immediately after buying this bond all market interest rates drop to 11% (including your reinvestment rate), what will be the impact on your total cash flow after 2.5 years? How does this compare to part (a)?

This is solved in the same way as for part (a) and the value of the cashflows becomes $733.74.

* 1. Assuming all market interest rates are 12%, what is the duration of this bond?

This is solved by dividing the sum of the the time weighted present value of the cashflows by the present value of the cashflows. The duration is 2.5. Note that as the duration and the holding period in parts (a) and (b) are the same you are insulated from changes in interest rates (essentially the losses in reinvestment rates for the early cashflows are offset by the gains in value for the cashflows paid later).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Year | 0 | 1 | 2 | 3 | 4 |
| Promised Payment | 160 | 160 | 170 | 180 | 230 |

## Chapter 11 Questions

1. What characteristics define the money markets?

The money markets can be characterized as having securities that trade in one year or less, are of large denomination, and are very liquid.

1. Is a Treasury Bond issued 29 years ago with 6 months remaining before it matures a money market instrument?

Money market securities have an original maturity of less than one year, so the bond would not be considered a money market security.

1. Why do banks not eliminate the need for money markets?

Banks have higher costs than the money market owing to the need to maintain reserve requirements. The lower cost structure of the money markets, coupled with the economies of scale resulting from high volume and large-denomination securities, allows for higher interest rates.

1. Distinguish between a term security and a demand security?

Term securities have a specific maturity date. Demand securities can be redeemed at any time. A six-month certificate of deposit is a term security. A checking account is a demand security.

1. What was the purpose motivating regulators to impose interest ceilings on bank savings accounts? What effect did this eventually have on the money markets?

Following the Great Depression, regulators were primarily concerned with stopping banks from failing. By removing interest-rate competition, bank risk was substantially reduced. The problem with these regulations was that when market interest rates rose above the established interest-rate ceiling, investors withdrew their funds from banks.

1. Why does the U.S. Government use the money markets?

The U.S. government sells large numbers of securities in the money markets to support government spending. Over the past several decades, the government has spent more each year than it has received in tax revenues. It makes up the difference by borrowing. Part of what it borrows comes from the money markets.

1. Why do businesses use the money markets?

Businesses both invest and borrow in the money markets. They borrow to meet short-term cash flow needs, often by issuing commercial paper. They invest in all types of money market securities as an alternative to holding idle cash balances.

1. What purpose initially motivated Merrill Lynch to offer money market mutual funds to its customers?

Merrill Lynch initially felt that it could better service it’s regular customers by making it easier to buy and sell securities from an account held at the brokerage house. The brokerage could offer a market interest rate on these funds by investing them in the money markets.

1. Why are more funds from property and casualty insurance companies than funds from life insurance companies invested in the money markets?

Life insurance companies can invest for the long term because the timing for their liabilities is known with reasonable accuracy. Property and casualty insurance companies cannot predict the natural disasters that cause large payouts on policies.

1. Which of the money market securities is the most liquid and considered the most risk-free? Why?

Treasury bills are usually viewed as the most liquid and least risky of securities because they are backed by the strength of the U.S. government and trade in extremely large volumes.

1. Distinguish between competitive bidding and non-competitive bidding for Treasury Securities.

A treasury auction will sell a certain face amount of securities. The non-competitive bids will all settle at the yield dictated by the auction. Thus the face amount of non-competitive bids are removed from the total amount to be auctioned by competitive bid. The competitive bids are ranked according to yield and the auction is settled at the lowest yield for which there are enough bids at a lower yield to clear the remaining auction amount. Winning competitive and non-competitive bids are settled at this yield.

1. Who issues federal funds, and what is the usual purpose of these funds?

Federal funds are sold by banks to other banks. They are used to invest excess reserves and to raise reserves if a bank is short.

1. Does the Federal Reserve directly set the federal funds interest rate? How does the Fed influence this rate?

The Federal Reserve cannot directly set the federal funds rate of interest. It can influence the interest rate by adding funds to or withdrawing reserves from the economy.

1. Who issues commercial paper and for what purpose?

Large businesses with very good credit standings sell commercial paper to raise short-term funds. The most common use of these funds it to extend short-term loans to customers for the purchase of the firm’s products.

1. Why are banker’s acceptances so popular for international transactions?

Banker’s acceptances substitute the creditworthiness of a bank for that of a business. When a company sells a product to a company it is unfamiliar with, it often prefers to have the promise of a bank that payment will be made.

## Chapter 11 Quantitative Problems

T-Bill Math exercises – Fill in the blanks (problems 1-4,6,8 from text)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Price | Principal | Maturity (days) | Annualized Discount Rate | Annualized Investment Rate |
| $4,925 | $5,000 | 182 | 2.967% | 3.054% |
| $9,940 | $10,000 | 91 | 2.374% | 2.421% |
| $4955.76 | $5,000 | 91 | 3.5% | 3.580% |
| $9,900 | $10,000 | 91 | 3.956% | 4.052% |
| $9909.00 | $10,000 | 182 | 1.8% | 1.842% |
| $9696.67 | $10,000 | 364 | 3% | 3.137% |

 The relevant formulae are:

$$PV=Principal×\left(1-Discount Rate×\frac{days}{360}\right)$$

$$PV=\frac{Principal}{1+Investment Rate×\frac{days}{365}}$$

or re-written

$$Discount Rate=\left(1-\frac{PV}{Principal}\right)×\frac{360}{days}$$

$$Investment Rate=\left(\frac{Principal}{PV}-1\right)×\frac{365}{days}$$

1. The price of 182-day commercial paper is $7,840. If the annualized investment rate is 4.093% what will the paper pay at maturity?

$$\begin{matrix}FV&=&PV×\left(1+Investment Rate×\frac{days}{365}\right)\\FV&=&\$7840×\left(1+4.093\%×\frac{182}{365}\right)\\FV&=&\$8000.01\end{matrix}$$

1. The price of $8,000 face value commercial paper is $7,930. If the annualized discount rate is 4%, when will the paper mature? If the annualized investment rate is 4%, when will the paper mature?

The relevant formulas are:

$$Discount Rate=\left(1-\frac{PV}{Principal}\right)×\frac{360}{days}$$

$$Investment Rate=\left(\frac{Principal}{PV}-1\right)×\frac{365}{days}$$

You then find that using a discount rate the number of days is 78.75. Using an investment rate of 4% you find that the number of days to maturity is 80.55.

1. The annualized discount rate on a particular money market instrument is 3.75%. The face value is $200,000 and it matures in 51 days. What is its price? What would be the price if it had 71 days to maturity?

If it matures in 51 days its price would be $198,937.50. If it matures in 71 days its price would be $198,520.83.

1. The annualized yield is 3% for 91-day commercial paper and 3.5% for 182-day commercial paper. What is the expected 91-day commercial paper rate 91 days from now?

Intuitively you should expect this to be approximately 4% - you invest for half the period at 3% and the other half at 4% you will end up with an average yield of around 3.5%. You know if you invest in 91-day commercial paper today and invest the proceeds in 91-day commercial paper this would be equivalent to investing in 182-day commercial paper. Thus

$$\left(1+3\%\frac{91}{365}\right)×\left(1+R\frac{91}{365}\right)=\left(1+3.5\%\frac{182}{365}\right)$$

The actual answer is 3.97%.

1. In a Treasury auction of $2.1 billion par value 91-day T-bills, the following bids were submitted. If only these competitive bids are received, who will receive T-Bill, in what quantity and at what price? If the Treasury also received $800m in non-competitive bids then who will receive T-bills, in what quantity and at what price?

|  |  |  |
| --- | --- | --- |
| Bidder | Bid Amount | Price |
| 1 | $500mn | $0.9940 |
| 2 | $750mn | $0.9901 |
| 3 | $1.5bn | $0.9925 |
| 4 | $1bn | $0.9936 |
| 5 | $600m | $0.9939 |

If there are only competitive bids then the auction will settle at the highest price which clears the auction amount. This includes bidders 1, 4 and 5. The winning price will be $0.9936.

If there is an additional $800m in non-competitive bids then only $1.3 billion will be auctioned competitively. This means that bidders 1, 4& 5 will win the auction. All bills will be settled at a price of $0.9936.

## Additional Questions

1. In the book there is an example of a 25-year fixed payment loan worth $1,000 with a fixed annual payment of $85.81. This gives a yield to maturity of 7%. Set up an excel spreadsheet that
	1. Splits the fixed payment into interest and principal for each cash flow. Note this will be a different amount for each cash flow.
	2. Create a graph with two lines – one for the interest payment and one for the principal payment.



* 1. Explain why the two lines look like they do

At the beginning of the loan the principal outstanding is largest which means that the majority of the payment goes towards interest. As the loan payments continue the principal is slowly reduced and thus the proportion of principal paid increases. The last payment must pay off the remaining loan balance so the interest paid becomes very small.

1. During the 1980’s recession the Federal Reserve chairman, Paul Volker, raised interest rates significantly to try to reduce inflation. In what ways can raising interest rates reduce inflation?
2. In exercise 11 of chapter 11’s quantitative problems there is a mistake in the bidding process. What is it?

The bids are in terms of price rather than yield. In an actual auction the yield is converted to a price and rounded to 6th decimal place (1000 face amount rounded to the nearest penny).

1. How often are T-bills auctioned? Find the announcement and results of the latest 4W T-bill auction on the Treasury Direct website. Find
	1. The amount to be auctioned
	2. The amount that went to non-competitive bidders
	3. The amount that went to competitive bidders

T-bills are auctioned weekly although not all maturities are auctioned every week. Generally 13-week and 26-week bills are auctioned on Monday and 4-week bills are auctioned on Tuesday. 52-week bills are auctioned every 4 weeks. A tentative auction schedule is published on the web through next January. At the 28 day T-bill auction on 4 September 2012, 40bn T-bills were to be auctioned. There were 173.5bn competitive bids and 256m non-competitive bids. This led to 39.7bn of accepted competitive bids and 256m of non-competitive bids.

1. Fill in the blanks in the following table (assume annual coupons and a face value of 100)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Bond | Maturity | Coupon | Yield to Maturity | Present Value | Duration |
| A | 3 | 7% | 3.5% | 109.81 | 2.82 |
| B | 3 | 6% | 8% | 94.85 | 2.83 |
| C | 5 | 2% | 5% | 87.01 | 4.79 |
| D | 5 | 7% | 3.5% | 115.80 | 4.44 |
| E | 7 | 6% | 8% | 89.59 | 5.85 |

1. From the above example, what is the duration of a portfolio consisting of bonds A & B? 2.82 years What is the duration of a portfolio consisting of two units of bond C with 5 units of bond E? 5.55 years