

# **Marketability and Value: Measuring the Illiquidity Discount**

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## **Marketability and Value: Measuring the Illiquidity Discount**

Should investors be willing to pay higher prices for more liquid assets than for otherwise similar assets that are less liquid? If the answer is yes, how much should the premium be for liquid assets? Conversely, how do we estimate the discount for illiquid assets? In this paper, we argue that it is a mistake to think of some assets as illiquid and others as liquid and that liquidity is a continuum, where some assets are more liquid than others. We then examine why liquid assets may be priced more highly than otherwise similar illiquid assets and why some investors value liquidity more than others. We follow up by presenting the empirical evidence that has accumulated over time and across different assets – financial and real – on the cost of illiquidity. Finally, we consider how we can use the theory and evidence on illiquidity to estimate the effect of illiquidity on the value of an asset or business.

When you buy a stock, bond, real asset or a business, you sometimes face buyer's remorse, where you want to reverse your decision and sell what you just bought. The cost of illiquidity is the cost of this remorse. In the case of publicly traded stock in a heavily traded company, this cost should be small. It will be larger for stock in a small, over-the-counter stock and will escalate for a private business, where there are relatively few potential buyers. It can also vary for different types of assets, with higher costs for real assets and lower costs for financial assets. In this paper, we will examine the reasons why investors value liquidity and the empirical evidence on how much they value it. We will follow up by looking at how the perceived liquidity or illiquidity of an asset affects the price you would be willing to pay for it and how best to incorporate illiquidity into valuations.

### **Measuring Illiquidity**

You can sell any asset, no matter how illiquid it is perceived to be, if you are willing to accept a lower price for it. Consequently, we should not categorize assets into liquid and illiquid assets but allow for a continuum on liquidity, where all assets are illiquid but the degree of illiquidity varies across them. One way of capturing the cost of illiquidity is through transactions costs, with less liquid assets bearing higher transactions costs (as a percent of asset value) than more liquid assets. In this section, we consider the components of transactions costs for publicly traded assets first and then extend the analysis to cover non-traded assets.

#### ***Transactions Costs on Publicly Traded Assets***

There are some investors who undoubtedly operate under the misconception that the only cost of trading is the brokerage commission that they pay when they buy or sell assets. While this might be the only cost that they pay explicitly, there are other costs that they incur in the course of trading that generally dwarf the commission cost. When trading any asset, there are three other ingredients that go into the trading costs. The first is the spread between the price at which you can buy an asset (the dealer's ask price) and the price at which you can sell the same asset at the same point in time (the dealer's bid

price). The second is the price impact that an investor can create by trading on an asset, pushing the price up when buying the asset and pushing it down while selling. The third cost, which was first proposed by Jack Treynor in his article<sup>1</sup> on transactions costs, is the opportunity cost associated with waiting to trade. While being a patient trader may reduce the first two components of trading cost, the waiting can cost profits both on trades that are made and in terms of trades that would have been profitable if made instantaneously but which became unprofitable as a result of the waiting. It is the sum of these costs, in conjunction with the commission costs that makes up the trading cost on an asset.

### *The Bid-Ask Spread*

There is a difference between what a buyer will pay and the seller will receive, at the same point in time for the same asset, in almost every traded asset market. The bid-ask spread refers to this difference. In the section that follows, we will examine why this difference exists, how large it is as a cost and the determinants of its magnitude.

#### *Why is there a bid-ask spread?*

In most markets, there is a dealer or market maker who sets the bid-ask spread, and there are three types of costs that the dealer faces that the spread is designed to cover. The first is the cost of holding inventory; the second is the cost of processing orders and the final cost is the cost of trading with more informed investors. The spread has to be large enough to cover these costs and yield a reasonable profit to the market maker on his or her investment in the profession.

#### *1. The Inventory Rationale*

Consider a market maker or a specialist on the floor of the exchange who has to quote bid prices and ask prices, at which she is obligated to execute buy and sell orders from investors. These investors, themselves, could be trading because of information they have received (informed traders), for liquidity (liquidity traders) or based upon their belief that an asset is under or over valued (value traders). In such a market, if the market makers set the bid price too high, they will accumulate an inventory of the stock. If

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<sup>1</sup> This was proposed in his article titled What does it take to win the trading game? published in the *Financial Analysts Journal*, January-February 1981.

market makers set the ask price too low, they will find themselves with a large short position in the stock. In either case, there is a cost to the market makers that they will attempt to recover by increasing the spread between the bid and ask prices.

Market makers also operate with inventory constraints, some of which are externally imposed (by the exchanges or regulatory agencies) and some of which are internally imposed (due to limited capital and risk). As the market makers' inventory positions deviates from their optimal positions, they bear a cost and will try to adjust the bid and ask prices to get back to their desired position. If the inventory is too high, the prices will be lower than the preferred prices; if the inventory is too low, the prices will be higher than the preferred prices.

### *2. The Processing Cost Argument*

Since market makers incur a processing cost with the paperwork and fees associated with orders, the bid-ask spread has to cover, at the minimum, these costs. While these costs are likely to be very small for large orders of stocks traded on the exchanges, they become larger for small orders of stocks that might be traded only through a dealership market. Furthermore, since a large proportion of this cost is fixed, these costs as a percentage of the price will generally be higher for low-priced stocks than for high-priced stocks.

Technology clearly has reduced the processing cost associated with trades as computerized systems take over from traditional record keepers. These cost reductions should be greatest for stocks where the bulk of the trades are small trades - small stocks held by individual rather than institutional investors.

### *3. The Adverse Selection Problem*

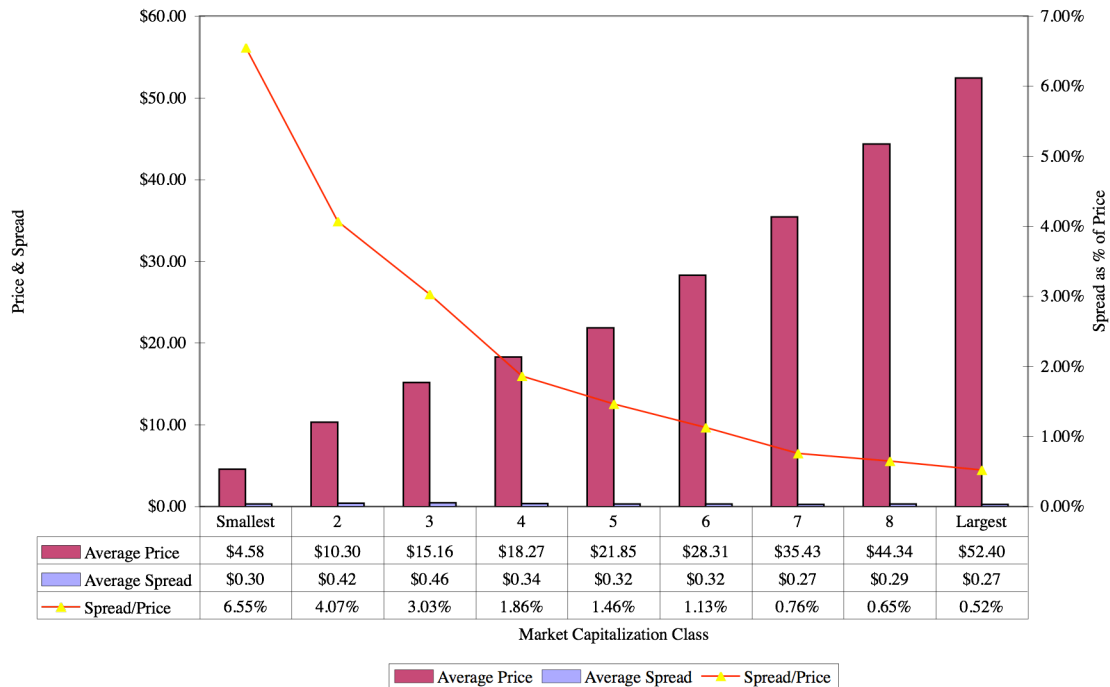
The adverse selection problem arises from the different motives investors have for trading on an asset - liquidity, information and views on valuation. Since investors do not announce their reasons for trading at the time of the trades, the market maker always runs the risk of trading against more informed investors. Since the expected profits from such trading are negative, the market maker has to charge an average spread that is large enough to compensate for such losses. This theory would suggest that spreads will increase with the proportion of informed traders in an asset market, the "differential"

information possessed, on average, by these traders and uncertainty about future information on the asset.

### *The Magnitude of the Bid-Ask Spread*

The New York Stock Exchange reported<sup>2</sup> that the average bid-ask spread across all NYSE stocks in 1996 was \$0.23, which seems trivial especially when one considers the fact that the average price of a NYSE stock is between \$ 40 and \$ 50. This average, however, obscures the large differences in the cost as a percentage of the price across stocks, based upon capitalization, stock price level and trading volume. A study<sup>3</sup> by Thomas Loeb in 1983, for instance, reported the spread for small orders as a percentage of the stock price for companies as a function of their marker capitalization. These results are summarized in Figure 1:

*Figure 1: Prices and Spreads by Market Cap*



While the dollar spread is not that different across market capitalization classes, the smallest companies also tend to have lower priced stocks. Consequently, the spread is as

<sup>2</sup> See NYSE Fact Book for a listing of the average spread across all NYSE stocks, by month.

<sup>3</sup> See "Trading Costs: The Critical Link Between Investment Information and Results" in the *Financial Analysts Journal*, May/June 1983.

high as 6.55% of the price, for small capitalization stocks and as low as 0.52% of the price for large capitalization companies. Another study by Huang and Stoll found that the stocks in the top 20% in terms of trading volume had an average spread of only 0.62% as a percent of the market price while the stocks in the bottom 20% had a spread of 2.06%. There are also large differences in bid-ask spreads across different exchanges in the United States. Looking at only NASDAQ stocks, Kothare and Laux (1995) found<sup>4</sup> that the average was almost 6% of the price in 1992, and much higher for low-price stocks on the exchange. Some of the difference can be attributed to the fact that NASDAQ stocks are generally much smaller ( in terms of market capitalization) and riskier than stocks listed on the NYSE or AMEX.

You could argue that these studies are dated and that there have been significant changes in both the way markets are structured and spreads are set in financial markets. In particular, after studies found that spreads on the NASDAQ might have been manipulated by specialists, there was significant legal pressure brought on the exchange to alter the way in which spreads were set. This was followed by the New York Stock Exchange shifting from a long tradition of quoting spreads in 1/16 and 1/8 to decimals. Have these changes made a dramatic difference? On average, spreads have declined but the drop has been much greater for smaller, less liquid stocks.

While these studies looked at traded U.S. equities, there are bid-ask spreads in other markets as well. While no single comprehensive study of all these spreads exists, the following conclusions seem warranted:

1. The spreads in U.S. government securities are much lower than the spreads on traded stocks in the United States. For instance, the typical bid-ask spread on a Treasury bill is less than 0.1% of the price.
2. The spreads on corporate bonds tend to be larger than the spreads on government bonds, with safer (higher rated) and more liquid corporate bonds having lower spreads than riskier (lower rated) and less liquid corporate bonds.

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<sup>4</sup> See “Trading Costs and the Trading Systems for NASDAQ stocks” by M. Kothare and P.A. Laux in *Financial Analysts Journal* (March/April 1995)

3. The spreads in non-U.S. equity markets are generally much higher than the spreads on U.S. markets, reflecting the lower liquidity in those markets and the smaller market capitalization of the traded firms.
4. While the spreads in the traded commodity markets are similar to those in the financial asset markets, the spreads in other real asset markets tend to be much larger.

#### *The Determinants of the Bid-Ask Spread*

A number of studies have looked at the variables that determine (or, at the very least, correlate with) the bid-ask spread. Studies<sup>5</sup> find that the spread as a percentage of the price is correlated negatively with the price level, volume and the number of market makers, and positively with volatility. Each of these findings is consistent with the theory on the bid-ask spread. The negative correlation with price level can be explained by the higher processing cost as a percentage of the price. Higher volume reduces the need for market makers to maintain inventory and also allows them to turn over their inventory rapidly, resulting in lower inventory costs. The higher volatility leads to higher bid-ask spreads partly because the adverse selection problem is greater for more volatile stocks; there will generally be more informed traders, a greater “information differential” and greater uncertainty about future information on these stocks. It is also worth noting that variables such as price level, volatility and trading volume are not only correlated with each other, but are also correlated with other variables such as firm size.

The study quoted in the previous section, by Kothare and Laux, that looked at average spreads on the NASDAQ also looked at differences in bid-ask spreads across stocks on the NASDAQ. In addition to noting similar correlations between the bid-ask spreads, price level and trading volume, they uncovered an interesting new variable. They found that stocks where institutional activity increased significantly had the biggest increase in bid-ask spreads. While some of this can be attributed to the concurrent increase in volatility in these stocks, it might also reflect the perception on the part of market makers that institutional investors tend to be informed investors with more or

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<sup>5</sup> See “Competition and the Pricing of Dealer Service in the Over-the-Counter Market” by S.Tinic and R. West in *Journal of Financial and Quantitative Analysis* (June 1972), “The Pricing of Security Dealer Services: An Empirical Analysis of NASDAQ stocks” by H. Stoll in *Journal of Finance* (November 1978)



better information. Note, though, that institutional investors also increase liquidity which should reduce the order processing cost component of the bid-ask spread, and in some cases the net effect can lead to a lower spread.<sup>6</sup>

Can firms affect the bid-ask spreads that their stock trades at? There is some evidence that they can by improving the quality of information that they disclose the financial markets, thus reducing the advantages that informed traders may have relative to the rest of the market. Heflin, Shaw and Wild (2001) look at 221 firms and examine the relationship between information disclosure quality – they measure this using disclosure quality scores assigned by the Corporate Information Committee of the Financial Analysts Federation – and the bid-ask spread. They find that bid-ask spreads decrease as information quality increases. Frost, Gordon and Hayes (2002) extend the analysis to compare liquidity across different equity markets and find that markets with strong disclosure systems also have the most liquidity.<sup>7</sup>

While most of the studies quoted above have looked at differences in spreads across stocks, Hasbrouck investigated why spreads change for the same stock at different points in time. He notes that large trades cause spreads to widen, relative to small trades, and hypothesizes that this is because large trades are more likely to contain information.

### *Market Microstructure and Bid-Ask Spreads*

Does the market in which a stock trades matter, when it comes to how big the bid-ask spread should be? Studies indicate that bid-ask spreads have historically been much higher on the NASDAQ than on the New York Stock Exchange, even after controlling for differences in the variables mentioned above – trading volume and price level. In fact, the bid-ask spreads of stocks drop when they switch from the NASDAQ to the NYSE.<sup>8</sup>

A 1994 study by Christie and Schultz provided one explanation for the

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and “Liquidity Effects of the Introduction of the S&P 500 Futures Contract on the Underlying Stocks” in *Journal of Business* (April 1993).

<sup>6</sup> Dey, M.K., and B. Radhakrishna, 2001, Institutional Trading, Trading Volume and Spread, Working Paper. They provide evidence of the link between institutional trading and the spread for stocks listed on the NYSE.

<sup>7</sup> Frost, C.A., E.A. Gordon and A.F. Hayers, 2002, Stock Exchange Disclosure and Market Liquidity: An Analysis of 50 International Exchanges, Working Paper, SSRN.

<sup>8</sup> See Barclay, M., 1997, Bid-Ask Spreads and the Avoidance of Odd-Eighth Quotes on Nasdaq: An Examination of Exchange Listings. *Journal of Financial Economics*, 45, 35-60.

phenomenon. They found that there were a disproportionately large number of  $1/4$  quotes and far too few  $1/8$  quotes.<sup>9</sup> They argued that dealers on the NASDAQ were colluding to set quotes too high and that investors were therefore paying the price with larger bid-ask spreads. This triggered an investigation by the Securities and Exchange Commission (SEC) which agreed that dealers were indeed engaged in anti-competitive behavior. Eventually, the exchange settled the lawsuit for more than a billion dollars. An alternative explanation is that the higher spreads on the NASDAQ, relative to the NYSE, can be explained by structural differences across the markets. Consider, for example, how limit orders are handled on the two exchanges. The specialists on the floor of the New York Stock Exchange are required to reflect in their bid-ask spread the limit prices, if they are better than their own quotes, and this has the effect of reducing the bid-ask spread. On the NASDAQ, limit orders do not affect the bid-ask quotes, and are executed only if prices move against the limit. You would expect larger bid-ask spreads as a consequence.<sup>10</sup>

In 2000, the New York Stock Exchange abandoned its historical practice of quoting prices in fractions ( $1/8$ ,  $1/4$ ... etc) and shifted to decimal prices. Since you can get finer gradations of prices in decimals, it was hypothesized that this should lead to lower bid-ask spreads. Studies since the shift indicate that there has been a decline in spreads on the smaller, less liquid stocks but no discernible impact on the more liquid listings.

### *The Price Impact*

Most investors assume that trading costs become smaller as portfolios become larger. While this is true for brokerage commissions, it is not always the case for the other components of trading costs. There is one component where larger investors bear more substantial costs than do smaller investors and that is in the impact that trading has on

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<sup>9</sup> If  $1/8$  and  $1/4$  quotes are equally likely to show up, roughly half of all quotes should end with an eighth ( $1/8$ ,  $3/8$ ,  $5/8$  or  $7/8$ ) and half should end with a quarter ( $1/4$ ,  $1/2$ ,  $3/4$ ).

<sup>10</sup> K. Chung, B. Van Ness, and R. Van Ness, 2001, "Can the Treatment of Limit Orders Reconcile the Differences in Trading Costs between NYSE and Nasdaq Issues?". *The Journal of Financial and Quantitative Analysis*, vol. 36, no. 2, 267-286.. While they find that the treatment of limit orders does lower the bid-ask spread on the NYSE, they conclude that collusion among dealers still leads to higher spreads on the NASDAQ.

prices. If the basic idea behind successful investing is to buy low and sell high, pushing the price up as you buy and then down as you sell reduces the profits from investing.

*Why is there a price impact?*

There are two reasons for the price impact, when investors trade. The first is that markets are not completely liquid. A large trade can create an imbalance between buy and sell orders, and the only way in which this imbalance can be resolved is with a price change. This price change that arises from lack of liquidity, will generally be temporary and will be reversed as liquidity returns to the market.

The second reason for the price impact is informational. A large trade attracts the attention of other investors in that asset market because it might be motivated by new information that the trader possesses. Notwithstanding claims to the contrary, investors usually assume, with good reason, that an investor buying a large block is buying in advance of good news and that an investor selling a large block has come into possession of some negative news about the company. This price effect will generally not be temporary, especially when we look at a large number of stocks where such large trades are made. While investors are likely to be wrong a fair proportion of the time on the informational value of large block trades, there is reason to believe that they will be right almost as often.

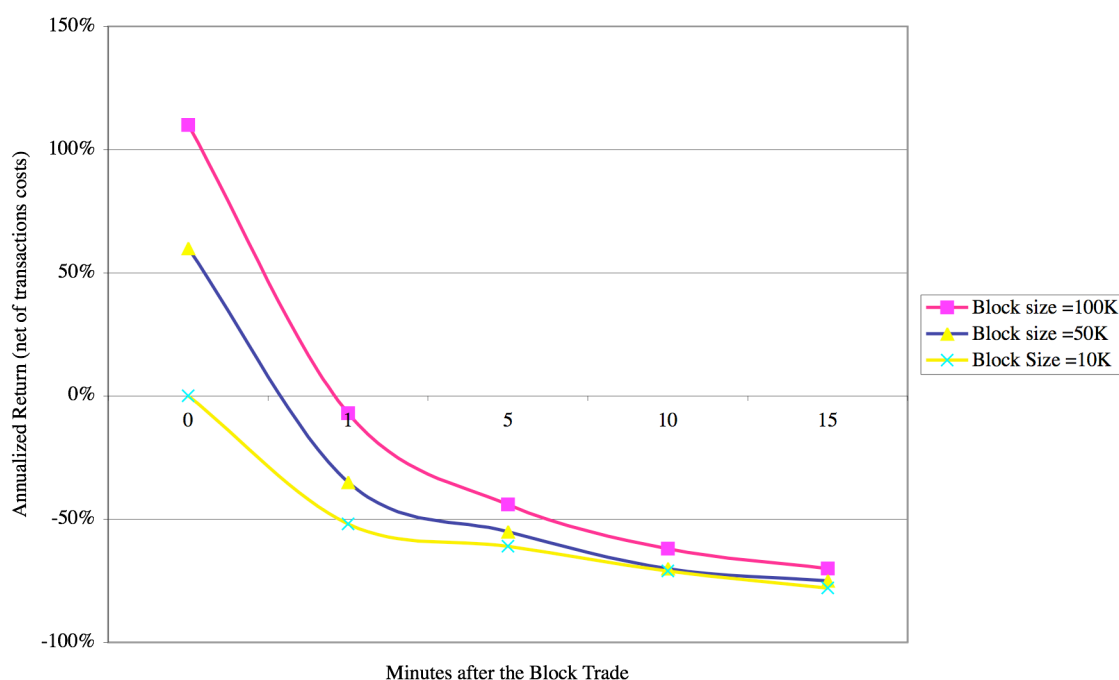
*How large is the price impact?*

There is conflicting evidence on how much impact large trades have on stock prices. On the one hand, studies of block trades on the exchange floor seem to suggest that markets are liquid and that the price impact of trading is small and is reversed quickly. These studies, however, have generally looked at heavily traded stocks at the New York Stock exchange. On the other hand, there are others who argue that the price impact is likely to be large, especially for smaller and less liquid stocks.

Studies of the price reaction to large block trades on the floor of the exchange conclude that prices adjust within a few minutes to such trades. An early study examined the speed of the price reaction by looking at the returns an investor could make by buying

stock right around the block trade and selling later<sup>11</sup>. They estimated the returns after transactions as a function of how many minutes after the block trade you traded, and found that only trades made within a minute of the block trade had a chance of making excess returns. (See Figure 2) Put another way, prices adjusted to the liquidity effects of the block trade within five minutes of the block. While this may be understated because of the fact that these were block trades on large stocks on the NYSE, it is still fairly strong evidence of the capacity of markets to adjust quickly to imbalances between demand and supply.

Figure 2: Annualized Returns from buying after block trades



Source: Dann, Mayers and Rabb (1977)

This study suffers from a sampling bias - it looks at large block trades in liquid stocks on the exchange floor. Studies that look at smaller, less liquid stocks find that the price impact tends to be larger and the adjustment back to the correct price is slower than it is for the more liquid stocks.<sup>12</sup> There are other interesting facts about block trades that

<sup>11</sup> Dann, L.Y., D. Mayers, and R. J. Rabb (1977) ,Trading Rules, Large Blocks and the Speed of Price Adjustment, *The Journal of Financial Economics*, 4, 3-22.

<sup>12</sup> Joel Haasbrouck looked at a detailed data set that contained information on quotes, trades and spreads of stocks listed on the NYSE and came to this conclusion.

have emerged from other studies. First, while stock prices go up on block buys and go down on block sells, they are far more likely to bounce back after sell trades. In other words, when prices go up after a block buy, they are more likely to stay up.<sup>13</sup> A study by Spierdijk, Nijman, and van Soest (2002) that looked at both liquid and illiquid stocks on the NYSE also finds a tendency on the part of markets to overshoot. When a block buy is made, the price seems to go up too much and it can take several days for it to revert back to a normal level for illiquid stocks.

These studies, while they establish a price impact, also suffer from another selection bias, insofar as they look only at actual executions. The true cost of market impact arises from those trades that would have been done in the absence of a market impact but were not because of the perception that it would be large. In one of few studies of how large this cost could be, Thomas Loeb collected bid and ask prices from specialists and market makers, at a point in time, for a variety of block sizes. Thus, the differences in the spreads as the block size increases can be viewed as an expected price impact from these trades. Table 1 summarizes his findings across stocks, classified by market capitalization:

*Table 1: Round-Trip Transactions Costs as a Function of Market Capitalization and Block Size*

	<i>Dollar Value of Block (\$ thousands)</i>								
<i>Sector</i>	<i>5</i>	<i>25</i>	<i>250</i>	<i>500</i>	<i>1000</i>	<i>2500</i>	<i>5000</i>	<i>10000</i>	<i>20000</i>
Smallest	17.30%	27.30%	43.80%						
2	8.90%	12.00%	23.80%	33.40%					
3	5.00%	7.60%	18.80%	25.90%	30.00%				
4	4.30%	5.80%	9.60%	16.90%	25.40%	31.50%			
5	2.80%	3.90%	5.90%	8.10%	11.50%	15.70%	25.70%		
6	1.80%	2.10%	3.20%	4.40%	5.60%	7.90%	11.00%	16.20%	
7	1.90%	2.00%	3.10%	4.00%	5.60%	7.70%	10.40%	14.30%	20.00%
8	1.90%	1.90%	2.70%	3.30%	4.60%	6.20%	8.90%	13.60%	18.10%
Largest	1.10%	1.20%	1.30%	1.71%	2.10%	2.80%	4.10%	5.90%	8.00%

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<sup>13</sup> See Holthausen, R. W., R. W. Leftwich, and D. Mayers, 1990, Large-Block Transactions, the Speed of Response, and Temporary and Permanent Stock-Price Effects," *Journal of Financial Economics*, 26, 71-95. and Keim, D. B., and A. Madhavan, 1995, Anatomy of the Trading Process: Empirical Evidence on the Behavior of Institutional Trades," *Journal of Financial Economics*, 37, 371-398.

The sectors refer to market capitalization, and show the negative relationship between size and price impact. Note, however the effect of increasing block sizes on expected price impact, within each sector; larger trades elicit much larger price impact than do smaller trades.

### *Determinants of the Price Impact*

Looking at the evidence, the variables that determine that price impact of trading seem to be the same variables that drive the bid-ask spread. That should not be surprising. The price impact and the bid-ask spread are both a function of the liquidity of the market. The inventory costs and adverse selection problems are likely to be largest for stocks where small trades can move the market significantly.

Breen, Hodrick and Korajczyk (2000) studied both the magnitude of the price impact and its determinants by looking at stocks listed on U.S. exchanges.<sup>14</sup> They find that increasing the turnover by 0.1% in a 5-minute interval can create a price impact of 2.65% for NYSE and AMEX firms and about 1.85% for NASDAQ stocks. Comparing the price impact across firms, they find evidence of the following:

1. The price impact of a trade of a given number of shares is smaller for larger market cap firms than for smaller firms. However, the price impact of a trade of the same percentage magnitude (as a percent of market cap) is greater for larger market cap firms than for smaller firms.
2. The price impact of a trade is smaller for firms with high trading volume in the previous quarter and for firms that have positive momentum (i.e, stock price has gone up in the six months prior to the trade).
3. The price impact of a trade is smaller for firms with high institutional holdings (as a percent of outstanding stock) than for a firm with lower institutional holdings.

### *The Opportunity Cost of Waiting*

The final component of trading costs is the opportunity cost of waiting. An investor could reduce the bid-ask spread and price impact costs of trading by breaking up

large blocks into small blocks and trading over a longer period. If, in fact, there was no cost to waiting, even a large investor could break up trades into small lots and buy or sell large quantities without affecting the price or the spread significantly. There is, however, a cost to waiting. In particular, the price of an asset that an investor wants to buy because he or she believes that it is undervalued may rise while the investor waits to trade, and this, in turn, can lead to one of two consequences. One is that the investor does eventually buy, but at a much higher price, reducing expected profits from the investment. The other is that the price rises so much that the asset is no longer under valued and the investor does not trade at all. A similar calculus applies when an investor wants to sell an asset that he or she thinks is overvalued.

The cost of waiting will depend in great part on the probability that the investor assigns that the price will rise (fall) while he or she waits to buy (sell). We would argue that this probability should be a function of why the investor thinks the asset is under or over valued. In particular, the following factors should affect this probability:

1. *Is the valuation assessment based upon private information or is based upon public information?* Private information tends to have a short shelf life in financial markets, and the risks of sitting on private information are much greater than the risks of waiting when the valuation assessment is based upon information that is already public. Thus, the cost of waiting is much larger when the strategy is to buy on the rumors (or information) of a possible takeover than it would be in a strategy of buying low PE ratio stocks.
2. *How active is the market for information?* Building on the first point, the risks of waiting, when one has valuable information, is much greater in markets where there are other investors actively searching for the same information. Again, in practical terms, the costs of waiting might be greater when there are dozens of analysts following the target stock than when there are few other investors paying attention to the stock.
3. *How long term or short term is the strategy?* While this generalization does not always hold, short-term strategies are much likely to be affected by the cost of

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<sup>14</sup> Breen, W.A., L.S. Hodrick and R.A. Korjczyk, 2000, Predicting Equity Liquidity, Working Paper, Kellogg Graduate School of Management.

waiting than longer term strategies. Some of this can be attributed to the fact that short term strategies are more likely to be motivated by private information, whereas long term strategies are more likely to be motivated by views on value.

4. *Is the investment strategy a “contrarian” or “momentum” strategy?* In a contrarian strategy, where investors are investing against the prevailing tide (buying when others are selling or selling when others are buying), the cost of waiting is likely to be smaller precisely because of this behavior. In contrast, the cost of waiting in a “momentum” strategy are likely to be higher since the investor is buying when other investors are buying and selling when others are selling.

In summary, the cost of waiting is likely to be greatest for short-term investment strategies, based upon private information or momentum, and in markets with active information gathering. It will be less of an issue for long-term investment strategies based upon public information and for contrarian strategies.

### ***The Costs of trading Non-traded Assets***

If the cost of trading stocks can be substantial, it should be even more significant for assets that are not traded regularly such as real assets or equity positions in private companies.

- Real assets can range from gold to real estate to fine art and the transactions costs associated with trading these assets can also vary substantially. The smallest transactions costs are associated with precious commodities – gold, silver or diamonds – since they tend to come in standardized units. With residential real estate, the commission that you have to pay a real estate broker or salesperson can be 5-6% of the value of the asset. With commercial real estate, commissions may be smaller for larger transactions, but they will be well in excess of commissions on financial assets. With fine art or collectibles, the commissions become even higher. If you sell a valuable painting through one of the auction houses, you may have to pay 15-20% of the value of the painting as a commission. Why are the costs so high? The first reason is that there are far fewer intermediaries in real asset businesses than there are in the stock or bond markets. The second is that real estate and fine art are not standardized



products. In other words, one Picasso can be very different from another, and you often need the help of experts to judge value. This adds to the cost in the process.

- The trading costs associated with buying and selling a private business can range from substantial to prohibitive, depending upon the size of the business, the composition of its assets and its profitability. There are relatively few potential buyers and the search costs (associated with finding these buyers) will be high. Later in this paper, we will put the conventional practice of applying 20-30% illiquidity discounts to the values of private businesses under the microscope.
- The difficulties associated with selling private businesses can spill over into smaller equity stakes in these businesses. Thus, private equity investors and venture capitalists have to consider the potential illiquidity of their private company investments when considering how much they should pay for them (and what stake they should demand in private businesses in return).

In summary, the costs of trading assets that are usually not traded are likely to be substantial.

### **The Cost of Illiquidity: Theory**

The notion that investors will pay less for illiquid assets than for otherwise similar liquid assets is neither new nor revolutionary. Over the last two decades researchers have examined the effect of illiquidity on price using three different approaches. In the first, the value of an asset is reduced by the present value of expected future transactions costs, thus creating a discount on value. In the second, the required rate of return on an asset is adjusted to reflect its illiquidity, with higher required rates of return (and lower values) for less liquid assets. In the third, the loss of liquidity is valued as an option, where the holder of the illiquid asset is assumed to lose the option to sell the asset when it has a high price. All three arrive at the conclusion that an illiquid asset should trade at a lower price than an otherwise similar liquid asset.

### ***An Illiquidity Discount on Value***

Assume that you are an investor trying to determine how much you should pay for an asset. In making this determination, you have to consider the cashflows that the asset

will generate for you and how risky these cashflows are to arrive at an estimate of intrinsic value. You will also have to consider how much it will cost you to sell this asset when you decide to divest it in the future. In fact, if the investor buying it from you builds in a similar estimate of transactions cost she will face when she sells it, the value of the asset today should reflect the expected value of all future transactions cost to all future holders of the asset. This is the argument that Amihud and Mendelson used in 1986, when they suggested that the price of an asset would embed the present value of the costs associated with expected transactions costs in the future.<sup>15</sup> In their model, the bid-ask spread is used as the measure of transactions costs and small spreads can translate into big illiquidity discounts on value. The magnitude of the discount will be a function of investor holding periods and turnover ratios, with shorter holding periods and higher turnover associated with bigger discounts. Vayanos (1998) argues that the effect of changes in transactions costs on asset prices is much smaller than estimated by Amihud and Mendelson because investors adjust holding periods to reflect transactions costs. In fact, he argues that the price of a stock can actually increase as its transactions costs increase, especially for more frequently traded stocks; the increase in holding periods can offset the transactions costs increase.<sup>16</sup>

Jarrow and Subramanian (2001) present an alternate model for estimating the illiquidity discount on value. They model the discount as the difference between the market value of an asset and its value when liquidated and argue that the discount should be larger when there are execution lags in liquidation. They derive optimal trading rules and the magnitude of the illiquidity discount for investors with power utility functions. Lo, Mamaysky and Wang (2001) assume fixed transactions costs and conclude, like Amihud and Mendelson, that small trading costs can create significant illiquidity discounts and that these discounts are influenced heavily by the risk aversion of investors.<sup>17</sup>

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<sup>15</sup> Amihud, Y. and H. Mendelson, 1986, Asset Pricing and the Bid-ask Spread, *Journal of Financial Economics*, v 17, 223-250.

<sup>16</sup> Vayanos, D., 1998, Transactions Costs and Asset Prices: A Dynamic Equilibrium Model, *Journal of Financial Economics*, v11, 1-58.

<sup>17</sup> Lo, A.W., H. Mamaysky and J. Wang, 2000, Asset Prices and Trading Volume under Fixed Transactions Costs, Working Paper, Yale International Center of Finance.

In summary, the papers that develop theoretical models for illiquidity discounts all link them to expected transactions costs on assets but require investor holding periods as an input for estimating the magnitude of the discount. The discount for any given transaction costs will be smaller if investors have long time horizons than if they have short time horizons.

### ***Illiquidity and Discount Rates***

In conventional asset pricing models, the required rate of return for an asset is a function of its exposure to market risk. Thus, in the CAPM, the cost of equity is a function of the beta of an asset, whereas in the APM or multi-factor model, the cost of equity is determined by the asset's exposure to multiple sources of market risk. There is little in these models that allow for illiquidity. Consequently, the required rate of return will be the same for liquid and illiquid assets with similar market risk exposure. In recent years, there have been attempts to expand these models to allow for illiquidity risk in one of two ways. The first are theoretical models that build in a market premium for illiquidity that affects all assets and measures of illiquidity for individual assets. Differences in the latter will cause required rates of return to vary across companies with different degrees of liquidity. The second are purely empirical multi-factor models that attempt to explain differences in returns across stocks over long time periods, with a measure of illiquidity such as trading volume or the bid-ask spread considered one of the factors.

The earliest theoretical discussions of how best to incorporate illiquidity into asset pricing models occurred in the 1970s. Mayers (1972, 1973, 1976) extended the capital asset pricing model to consider non-traded assets as well as human capital.<sup>18</sup> The resulting models did not make explicit adjustments for illiquidity, though. In a more recent attempt to incorporate illiquidity into expected return models, Acharya and Pedersen (2003) examine how assets are priced with liquidity risk and make a critical

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<sup>18</sup> Mayers, D., 1972, Nonmarketable assets and capital market equilibrium under uncertainty, in M.C. Jensen, *Studies in the Theory of Capital Markets* (Praeger, New York, NY); Mayers, D., 1973, Nonmarketable assets and the determination of capital asset prices in the absence of a riskless asset, *Journal of Business*, v46, 258-267; Mayers, D., 1976, Nonmarketable assets, market segmentation and the level of asset prices, *Journal of Financial and Quantitative Analysis*, v11, 1-12.

point. It is not just how illiquid an asset is that matters but when it is illiquid. In particular, an asset that is illiquid when the market itself is illiquid (which usually coincides with down markets and economic recessions) should be viewed much more negatively (with a resulting higher expected return) than an asset that is illiquid when the market is liquid. Thus the liquidity beta of an asset will reflect the covariance of the asset's liquidity with market liquidity. Acharya and Pedersen estimate that illiquid stocks have annualized risk premiums about 1.1% higher than liquid stocks, and that 80% of this premium can be explained by the covariance between a stock's illiquidity and overall market illiquidity. Pastor and Stambaugh (2002) also concluded that it is not a stock's liquidity per se that matters but its relationship to overall market liquidity. Over the 34-year period that they examined stock returns, they concluded that stocks whose returns are more sensitive to market liquidity have annual returns that are 7.5% higher than stocks whose returns have low sensitivity to market liquidity, after adjusting for the standard size, value and momentum factors.

The difficulties associated with modeling liquidity and arriving at usable models have lead many researchers to consider more practical ways of incorporating illiquidity into expected returns. Building on the work done on multi-factor models in the 1980s and proxy models the 1990s, they looked for ways of measuring liquidity and including these measures in models that explained differences in stock returns over long time periods. Amihud and Mendelson (1989) examined whether adding bid-ask spreads to betas helped better explain differences in returns across stocks in the U.S.<sup>19</sup> In their sample of NYSE stocks from 1961-1980, they concluded that every 1% increase in the bid-ask spread (as a percent of the stock price) increased the annual expected return by 0.24-0.26%. Eleswarapu (1997) confirmed this finding by showing a positive relationship between returns and spreads for Nasdaq stocks.<sup>20</sup> Other studies have used trading volume, turnover ratios (dollar trading volume/ market value of equity) and illiquidity ratios as proxies for

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<sup>19</sup> Amihud, Y. and . Mendelson, 1989, the Effects of Beta, Bid-Ask Spread, Residual Risk and Size on Stock Returns, *Journal of Finance*, v 44, 479-486.

<sup>20</sup> Eleswarapu, V.R. 1997, Cost of transacting and expected returns in the Nasdaq Market, *Journal of Finance*, 52 (5), 2113-2127. There are other studies that find a weaker or no relationship between stock returns and bid-ask spreads. Chalmers and Kadec use the amortized spread and find no relationship between spreads and returns for NYSE stocks. Chalmers, J.M.R. and G.B. Kadlec, 1998, An Empirical examination of the Amortized Spread, *Journal of Financial Economics*, 48 (2), 159-188.

illiquidity with consistent results. Brennan and Subrahmanyam(1996) break transactions costs down into fixed and variable costs and find evidence of a significant effect on returns due to the variable cost of trading after controlling for factors such as firm size and the market to book ratio.<sup>21</sup> Brennan, Chordia and Schwartz (1998) find that dollar trading volume and stock returns are negatively correlated, after adjusting for other sources of market risk.<sup>22</sup> Datar, Nair and Radcliffe (1998) use the turnover ratio as a proxy for liquidity. After controlling for size and the market to book ratio, they conclude that liquidity plays a significant role in explaining differences in returns, with more illiquid stocks (in the 90th percentile of the turnover ratio) having annual returns that are about 3.25% higher than liquid stocks (in the 10<sup>th</sup> percentile of the turnover ratio). In addition, they conclude that every 1% increase in the turnover ratio reduces annual returns by approximately 0.54%.<sup>23</sup> Amihud (2002) developed a measure of illiquidity by dividing the absolute price change by the average daily trading volume for the stock to estimate an illiquidity ratio and concluded that stock returns are positively correlated with this measure.<sup>24</sup> Nguyen, Mishra and Prakash (2004) conclude that stocks with higher turnover ratios do have lower expected returns. They also find that market capitalization and price to book ratios, two widely used proxies that have been shown to explain differences in stock returns, do not proxy for illiquidity.

In summary, both the theoretical models and the empirical results suggest that we should adjust discount rates for illiquidity, with the former focusing on systematic liquidity as the key factor and the latter using proxies such as bid-ask spreads and turnover ratios to measure liquidity. Both approaches also seem to indicate that the adjustment will vary across time and will be dependent upon a market wide demand for liquidity. Thus, for any given level of illiquidity, the expected premium added on to

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<sup>21</sup> Brennan, M. J. and A. Subrahmayam, 1996, "Market microstructure and Asset Pricing: On the Compensation for Illiquidity in Stock Returns," *Journal of Financial Economics* 41, 441-464.

<sup>22</sup> Brennan, M. J., T. Chordia, and A. Subrahmanyam, 1998. Alternative factor specifications, security characteristics and the cross-section of expected stock returns, *Journal of Financial Economics*, 49, 345–373.

<sup>23</sup> Datar, V.T., N. Y. Naik and R. Radcliffe, 1998, "Liquidity and stock returns: An alternative test," *Journal of Financial Markets* 1, 203-219.

<sup>24</sup> Amihud, Y., 2002, Illiquidity and stock returns: Cross-section and time-series effects, *Journal of Financial Markets* 5, 31-56.

discount rates will be much greater in periods when the market values liquidity more and smaller in periods when it values it less.

### ***Illiquidity as an Option***

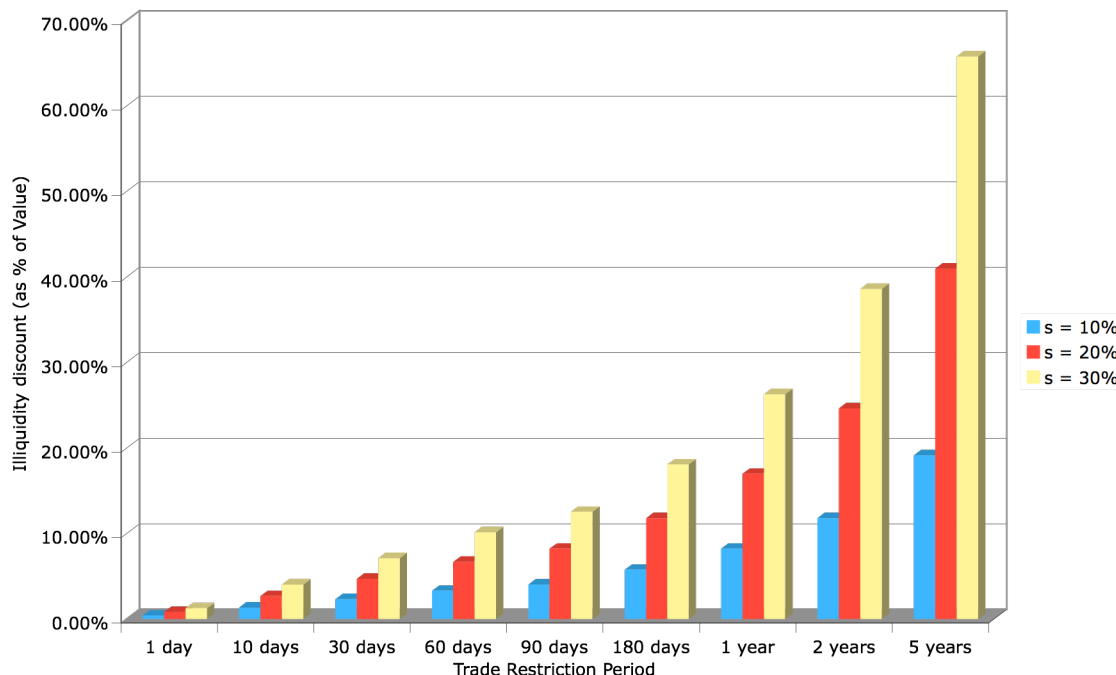
What is the value of liquidity? Put differently, when does an investor feel the loss of liquidity most strongly when holding an asset? There are some who would argue that the value of liquidity lies in being able to sell an asset, when it is most overpriced; the cost of illiquidity is not being able to do this. In the special case, where the owner of an asset has the information to know when this overpricing occurs, the value of illiquidity can be considered an option.

Longstaff (1995) presents an upper bound for the option by considering an investor with perfect market timing abilities who owns an asset on which she is not allowed to trade for a period ( $t$ ). In the absence of trading restrictions, this investor would sell at the maximum price that an asset reaches during the time period and the value of the look-back option estimated using this maximum price should be the outer bound for the value of illiquidity.<sup>25</sup> Using this approach, Longstaff estimates how much marketability would be worth as a percent of the value of an asset for different illiquidity periods and asset volatilities. The results are graphed in figure 3:

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<sup>25</sup> Longstaff, F.A., 1995, How much can marketability affect security values? *Journal of Finance*, v 50, 1767-1774.

Figure 3: Upper bounds on Marketability Discount - Option Pricing Model



It is worth emphasizing that these are upper bounds on the value of illiquidity since it is based upon the assumption of a perfect market timer. To the extent that investors are unsure about when an asset has reached its maximum price, the value of illiquidity will be lower than these estimates. The more general lessons will still apply. The cost of illiquidity, stated as a percent of firm value, will be greater for more volatile assets and will increase with the length of the period for which trading is restricted.

### The Cost of Illiquidity: Empirical Evidence

If we accept the proposition that illiquidity has a cost, the next question becomes an empirical one. How big is this cost and what causes it to vary across time and across assets? The evidence on the prevalence and the cost of illiquidity is spread over a number of asset classes. In this section, we will begin by considering the price attached to illiquidity in the bond market and then move on to the equity market. In the final part of the section, we will look at the illiquidity effects on private equity investments and real assets.

## **Bonds**

There are wide differences in liquidity across bonds issued by different entities, and across maturities, for bonds issued by the same entity. These differences in liquidity offer us an opportunity to examine whether investors price liquidity and if so, how much, by comparing the yields of liquid bonds with otherwise similar illiquid bonds. Studies of bond market liquidity have looked at the treasury bond, corporate bond and subordinated bond markets.

- *Treasury Bills/Bonds*: Amihud and Mendelson (1991) compared the yields on treasury bonds with less than six months left to maturity with treasury bills that have the same maturity.<sup>26</sup> They concluded that the yield on the less liquid treasury bond was 0.43% higher on an annualized basis than the yield on the more liquid treasury bill, a difference that they attributed to illiquidity. A subsequent study by Kamara (1994) confirmed their finding and concluded that the yield difference was 0.37%.<sup>27</sup> Strebulaev (2002) contests their finding, noting that the tax treatment on bonds varies from the tax treatment of treasury bills and that this may explain the difference in yields. He compares treasury notes maturing on the same date and concludes that they trade at essentially identical prices, notwithstanding big differences in liquidity.<sup>28</sup>
- *Corporate bonds*: Chen, Lesmond and Wei (2005) compared over 4000 corporate bonds in both investment grade and speculative categories, and concluded that illiquid bonds had much higher yield spreads than liquid bonds. To measure liquidity, they used multiple measures including the bid-ask spread, the occurrence of zero returns in the time series<sup>29</sup> and the LOT measure (which incorporates the bid-ask spread, opportunity costs and price impact). Not surprisingly, they find that liquidity decreases as they move from higher bond ratings to lower ones and increases as they move from short to long maturities. Comparing yields on these corporate bonds, they

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<sup>26</sup> Amihud, Y., and H. Mendelson, 1991, Liquidity, Maturity and the Yield on U.S. Treasury Securities, *Journal of Finance*, 46, 1411-1425.

<sup>27</sup> Kamara, A., 1994, Liquidity, Taxes, and Short-Term Yields, *Journal of Financial and Quantitative Analysis*, 29, 403-417.

<sup>28</sup> Strebulaev, I., 2002, Liquidity and Asset Pricing: Evidence from the US Treasuries Market, Working Paper, London Business School.

<sup>29</sup> When an asset does not trade during a period, the return will be zero during the period. Counting the number of zero return periods can provide one proxy for illiquidity.



conclude that the yield increases 0.21% for every 1% increase in transactions costs for investment grade bonds, whereas the yield increases 0.82% for every 1% increase in transactions costs for speculative bonds.<sup>30</sup>

- *Subordinated bonds*: A study of 211 subordinated bonds issued by 22 large banks in the United States concluded that more illiquid bonds trade at higher default spreads than otherwise similar liquid bonds.<sup>31</sup> They find that bonds that have not traded within the last six months have a default spread that is about 0.20% higher than a traded bond, and that this spread widens out to 0.64% when the bond has not traded in the last two years.

Looking across the studies, the consensus finding is that liquidity matters for all bonds, but that it matters more with risky bonds than with safer bonds. This may explain why the prevalence of a liquidity premium in the government bond market is debatable but not in the corporate bond market.

### ***Publicly Traded Stocks***

If liquidity becomes more of an issue with riskier bonds than with safer bonds, it stands to reason that it should be a bigger factor in the equity market (where there are more sources of risk) than the bond market. Studies of illiquidity in the equity market have run the gamut ranging from examining differences in liquidity across the broad cross section of stocks and how they translate into differences in expected returns to more focused studies that try to find a subset of stocks where illiquidity is an issue and try to measure how investors react to that illiquidity.

#### *a. Equities as a class*

It can be reasonably argued that the costs associated with trading equities are larger than the costs associated with trading treasury bonds or bills. It follows therefore

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<sup>30</sup> Chen, L., D.A. Lesmond and J. Wei, 2005, Corporate Yield Spreads and Bond Liquidity, Working Paper, SSRN.

<sup>31</sup> Bianchi, C., D. Hancock and L. Kawano, 2004, Does Trading Frequency affect Subordinated Debt Spreads? Working Paper, Federal Reserve Bank, Washington D.C. To measure liquidity, they consider whether a “generic price” is available on Bloomberg for a bond. Since a generic price is available only when a bond trades, it becomes a proxy for liquidity with more liquid bonds having more generic prices listed for them.

that some of the risk premium attributed to equity has to reflect these additional transactions costs. Jones (2002), for instance, examines bid-ask spreads and transactions costs for the Dow Jones stocks from 1900 to 2000 and concludes that the transactions costs are about 1% lower today than they were in the early 1990s and that this may account for the lower equity risk premium in recent years.<sup>32</sup> He also presents evidence that increases in the bid-ask spread and lower turnover are harbingers of higher stock returns in the future, which he takes as evidence that illiquidity is a factor behind both the magnitude of and changes in the equity risk premiums. His research is in line with others who have argued that variations in liquidity (and the associated costs) over time may explain a portion of the shifts in the equity risk premium from period to period.

#### *b. Cross Sectional Differences*

Some stocks are more liquid than others and studies have looked at the consequences of these differences in liquidity for returns. The consensus conclusion is that investors demand higher returns when investing in more illiquid stocks. Put another way, investors are willing to pay higher prices for more liquid investments relative to less liquid investments. In our earlier discussion of adjusting discount rates for illiquidity, we pointed to evidence that some of the return variation across stocks can be explained by differences in illiquidity.

There has been other research that seems to establish a connection between stock price movements and liquidity.

- Cox and Petersen (1994) examined U.S. stocks that had one-day price declines of more than 10% and the subsequent price reversal the day after. They concluded that a large component of the reversal could be explained by the bid-ask spread and that the price reversal could therefore be viewed as compensation for illiquidity.<sup>33</sup> Avramov, Chordia and Goyal (2005) also find a strong relationship

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<sup>32</sup> This becomes clear when we look at forward-looking or implied equity risk premiums rather than historical risk premiums. The premiums during the 1990s averaged about 3%, whereas there were more than 5% prior to 1960.

<sup>33</sup> Cox, D.R. and D.R. Peterson (1994), 'Stock returns following large one-day declines: Evidence on short-term reversals and longer-term performance', *Journal of Finance*, 48,255-267.

between short run price reversals and illiquidity.<sup>34</sup> The largest price reversals are in the most illiquid stocks, which would also indicate that contrarian investment strategies, which try to take advantage of these price reversals, will be saddled with higher transactions costs.

- Temporal anomalies such as the weekend effect and the January effect are most accentuated for illiquid stocks. Eleswarapu and Reinganum (1993) note that stocks with low liquidity and high bid-ask spreads earn most of their excess returns in January.<sup>35</sup> The high transactions costs associated with trading on these stocks may explain why these anomalies continue to have the staying power that they do.
- Ellul and Pagano (2002) related the underpricing of 337 British initial public offerings to the illiquidity of the issues after the offerings, and found evidence that the less liquid shares are expected to be and the less predictable the liquidity, the greater the under pricing.<sup>36</sup>
- While it would be foolhardy to attribute all of the well documented excess returns<sup>37</sup> that have been associated with owning small market capitalization and low price to book stocks to illiquidity, smaller and more distressed companies (which tend to trade at low price to book ratios) are more illiquid than the rest of the market.

The interplay between illiquidity and so many observed inefficiencies in the market suggests that it plays a key role in how investors price stocks and the returns that we observe in the aftermath. It may also explain why there are so many ways of making excess returns on paper and so few in practice.

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<sup>34</sup> Avramov, Chordia and Goyal, 2005, Liquidity and Autocorrelations in Individual Stock Returns, Working Paper, SSRN.

<sup>35</sup> Eleswarapu, V, and M. Reinganum, "The Seasonal Behaviour of the Liquidity Premium in Asset Pricing", *Journal of Financial Economics*, 1993, 34, 281-305.

<sup>36</sup> Ellul, A. and M. Pagano, 2002, IPO Underpricing and After-market Liquidity, Working Paper, SSRN.

<sup>37</sup> Fama, E.F. and K.R. French, 1992, The Cross-Section of Expected Returns, *Journal of Finance*, v47, 427-466.

### *c. Controlled Differences*

Studies that compare stocks with different liquidity can always be faulted for not controlling for other factors. After all, companies with more liquid stocks tend to have larger market capitalization and lower risk. Consequently, the cleanest tests for illiquidity are those that compare stocks with different degrees of liquidity issued by the same company. Differences in stock prices can then be attributed purely to liquidity.

#### *1. Restricted Stock and Private Placements*

Much of the evidence on illiquidity discounts comes from examining “restricted stock” issued by publicly traded firms. Restricted securities are securities issued by a publicly traded company, not registered with the SEC, and sold through private placements to investors under SEC Rule 144. They cannot be resold in the open market for a one-year holding period<sup>38</sup>, and limited amounts can be sold after that. When this stock is issued, the issue price is set much lower than the prevailing market price, which is observable, and the difference can be viewed as a discount for illiquidity. The results of two of the earliest and most quoted studies that have looked at the magnitude of this discount are summarized below:

- Maher examined restricted stock purchases made by four mutual funds in the period 1969-73 and concluded that they traded an average discount of 35.43% on publicly traded stock in the same companies.<sup>39</sup>
- Silber examined restricted stock issues from 1981 to 1988 and found that the median discount for restricted stock is 33.75%.<sup>40</sup> He also noted that the discount was larger for smaller and less healthy firm, and for bigger blocks of shares.

Other studies confirm these findings of a substantial discount, with discounts ranging from 30-35%. One recent study by Johnson (1999) did find a smaller discount of 20%.<sup>41</sup>

These studies of restricted stock have been used by practitioners to justify large marketability discounts but there are reasons to be skeptical. First, these studies are based

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<sup>38</sup> The holding period was two years prior to 1997 and has been reduced to one year since.

<sup>39</sup> Maher, J.M., 1976, Discounts for Lack of Marketability for Closely Held Business Interests, *Taxes*, 54, 562-571.

<sup>40</sup> Silber, W.L., 1991, Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices, *Financial Analysts Journal*, v47, 60-64.

<sup>41</sup> B. A. Johnson, 1999, Quantitative Support for Discounts for Lack of Marketability, *Business Valuation Review*, v16, 152-55.

upon small sample sizes, spread out over long time periods, and the standard errors in the estimates are substantial. Second, most firms do not make restricted stock issues and the firms that do make these issues tend to be smaller, riskier and less healthy than the typical firm. This selection bias may be skewing the observed discount. Third, the investors with whom equity is privately placed may be providing other services to the firm, for which the discount is compensation.

One way of isolating the service difference would be to compare unregistered private placements, which represent the restricted stock issues, to registered private placements of equity by companies. Since only the former have restrictions on marketability, the difference in discounts between the two may be a better measure of the illiquidity discount. Wruck (1989) made this comparison and estimated a difference of 17.6% in average discounts and only 10.4% in the median discount between the two types of placements.<sup>42</sup> Hertz and Smith (1993) expanded on this comparison of restricted stock and registered private placements by looking at 106 private placements of equity from 1980 to 1987.<sup>43</sup> They concluded that while the median discount across all private placements was 13.26%, the discount was 13.5% higher for restricted stock than for registered stock. Bajaj, Dennis, Ferris and Sarin (2001) looked at 88 private placements from 1990 to 1997 and report median discounts of 9.85% for registered private placements and 28.13% for restricted stocks. After controlling for differences across the firms making these issues, they attribute only 7.23% to the marketability discount.<sup>44</sup>

It should be noted that these studies also pinpoint the selection bias inherent in focusing on firms that make private placements. Hedrick and Smith compare firms making private placements to those making public issues and note that firms making private placements tend to be smaller and riskier than other firms, and are usually listed

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<sup>42</sup> Wruck, K.H., 1989, Equity Ownership Concentration and Firm Value: Evidence from Private Equity Financings, *Journal of Financial Economics*, 23, 3-28. She concluded that a significant portion of the discount could be attributed to control changes at the firms.

<sup>43</sup> Hertz, M. and R.L. Smith, 1993, Market Discounts and Shareholder Gains from Placing Equity Privately, *Journal of Finance*, v48, 459-486.

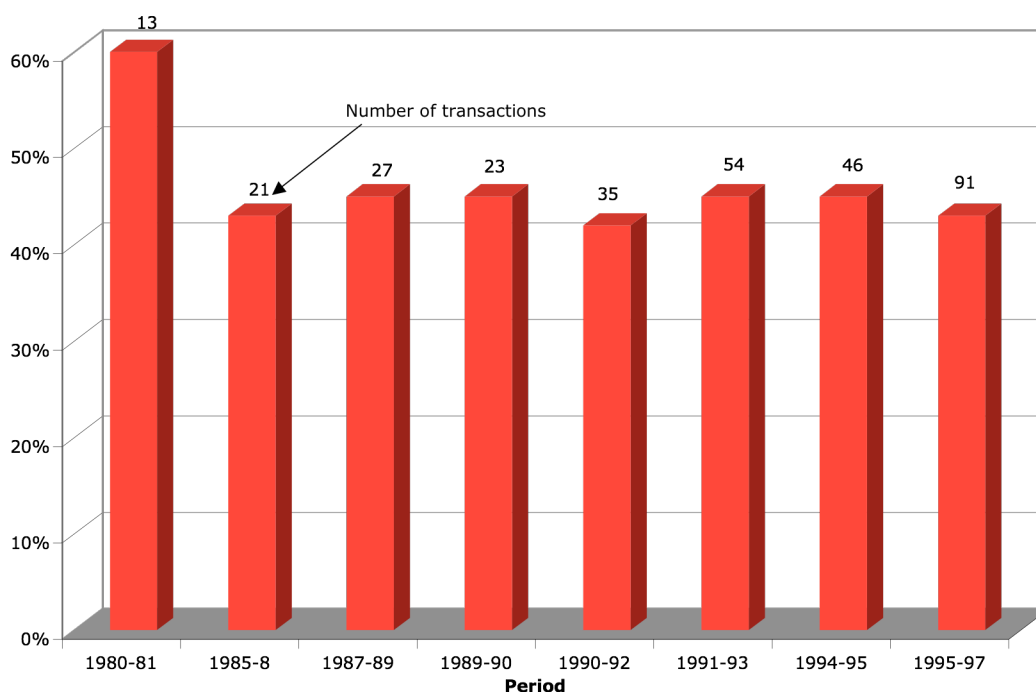
<sup>44</sup> Bajaj, M., D.J. Dennis, S.P. Ferris and A. Sarin, 2001, Firm Value and Marketability Discounts, *Journal of Corporate Law*, v27.

on the OTC market. Many of these firms are also closely held. Thus, the discounts estimated from these small samples have to be considered with caution.

## 2. Initial Public Offerings

An alternative way of computing the illiquidity discount is to compare the initial public offering stock prices of companies to the prices on transactions involving these same shares prior to the initial public offering. The difference, it is argued, can be viewed as a discount for illiquidity. Emory (1996) compared stock prices in transactions in the five months prior to an IPO to the IPO price and reported a discount of about 45% for private offerings. Figure 4 reports the discount and the sample size by year:<sup>45</sup>

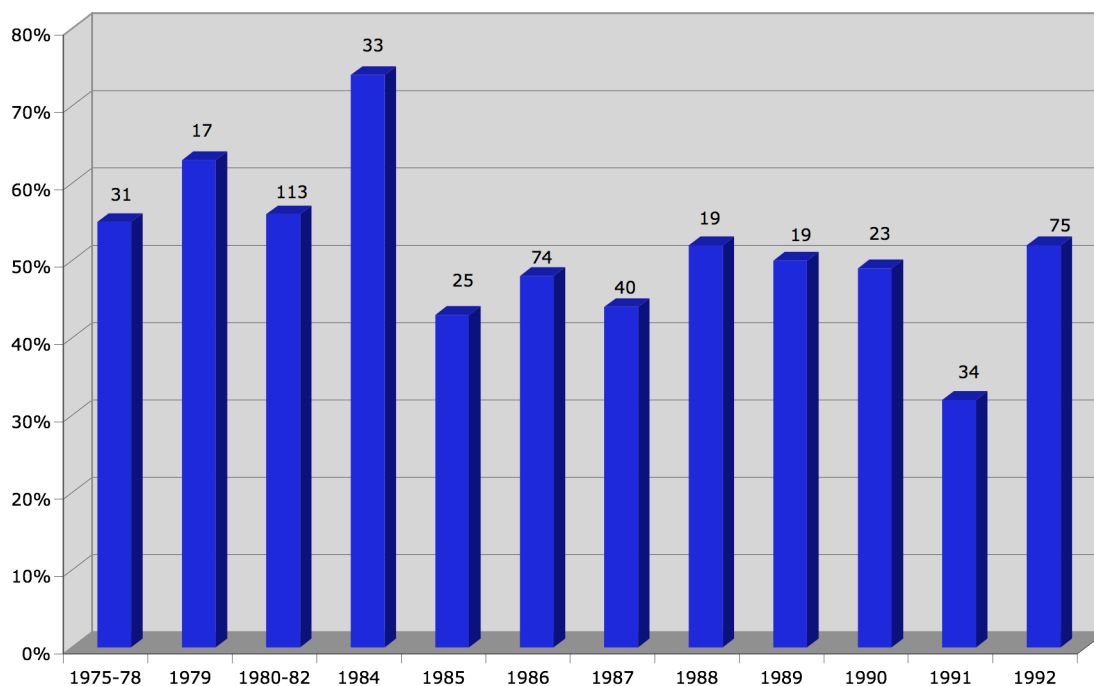
Figure 4: Discount on IPOs



Williamette Associates extended this study to look at transactions in the three years prior to initial public offerings, adjusted for changes in the PE ratio between the time of the transaction and the IPO and reported discounts ranging from 32 to 75%. Figure 5 summarizes their findings:

<sup>45</sup> This study is reported in Pratt, S., R. Reilly and R.P. Schwiehs, 1997, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, McGraw-Hill.

Figure 5: Discount on IPOs- Three years prior to IPO



The size of the discount is striking in both of these studies. It is difficult to see why an investor would be willing to accept a 40% discount on estimated value if an initial public offering is forthcoming. It seems likely that what these studies conclude is a marketability discount is reflective of other factors.

### 3. Share Classes

Some companies have multiple classes of shares in the same market, with some classes being more liquid than others. If there are no other differences (in voting rights or dividends, for instance) across the classes, the difference in prices can be attributed to liquidity. One candidate for study is the Chinese market, where most companies have Restricted Institutional Shares (RIS) which are almost completely illiquid<sup>46</sup> and common shares which are traded on the exchange. Chen and Xiang (2001) compare the market prices of the traded common stock in 258 Chinese companies with the auction and private placement prices of the RIS shares and conclude that the discount on the latter is 78% for

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<sup>46</sup> Restricted Institutional Shares have to be transacted through private placements. Starting in August 2000, the Chinese Government has also allowed for auctions of these shares, where it is presumably a little easier to find a potential buyer.

auctions and almost 86% for private placements. This astoundingly high discount, which they attributed to illiquidity, does vary across firms, with smaller discounts at larger, less volatile firms. In a different vein, researchers have compared the stock prices of Class A and Class B shares of Chinese companies. The former are open only to Chinese investors, whereas the latter can be bought by both domestic and foreign investors. While they both offer the same claims on the cashflows, Class B shares trade at a significant discount on Class A shares. The differences, though, seem to be only partially attributable to differences in liquidity and seem more due to differential information.<sup>47</sup>

#### *d. Options and Futures*

As derivative securities, the values of options and futures are bound to their underlying assets by arbitrage restrictions. The effect of illiquidity on option and futures values has been studied in two contexts. The first is when the derivative securities are illiquid but the underlying asset is liquid. Brenner, Eldor and Hauser (2001) studied non-traded currency options and concluded that they traded at a discount of approximately 21%, relative to otherwise similar liquid options.<sup>48</sup> This issue is of particular relevance in the valuation of employee stock options that are offered as compensation at publicly traded companies. Since the options cannot be traded, there is evidence that employees will often exercise options well before they expire in order to gain access to the tradable underlying stock<sup>49</sup>. In fact, Hull and White (2001) bring this tendency towards early exercise into the valuation of employee options by assuming that exercise will occur if the price appreciates by a pre-specified percentage.<sup>50</sup> The second is when the underlying asset is itself illiquid and there are options on the asset. In this case, any illiquidity discount that applies to the underlying asset will also reduce the value of any options on that asset.

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<sup>47</sup> Wang, S.S. and L. Jiang, 2003, Location of Trade, Ownership Restrictions, and Market Illiquidity: Examining Chinese A- and H-Shares, Working Paper, Hong Kong Polytechnic University. Chan, K., A.J. Menkveld and Z. Yang, 2002, Evidence of the Foreign Share Discount Puzzle in China: Liquidity or Information Asymmetry, Working Paper, National Center for Economic Research, Tsinghua University.

<sup>48</sup> Brenner, M., R.Eldor, and S.Hauser, 2001, The price of options illiquidity, *Journal of Finance*, 26, 789-806.

<sup>49</sup> Huddart, S. and M. Lang. 1996. "Employee stock options exercises: An empirical analysis." *Journal of Accounting and Economics*, vol. 21, no. 1:5-43.



### ***Private Equity***

Private equity and venture capital investors often provide capital to private businesses in exchange for a share of the ownership in these businesses. Implicit in these transactions must be the recognition that these investments are not liquid. If private equity investors value liquidity, they will discount the value of the private business for this illiquidity and demand a larger share of the ownership of illiquid businesses for the same investment. Looking at the returns earned by private equity investors, relative to the returns earned by those investing in publicly traded companies, should provide a measure of how much value they attach to illiquidity.

Ljungquist and Richardson (2003) estimate that private equity investors earn excess returns of 5 to 8%, relative to the public equity market, and that this generates about 24% in risk-adjusted additional value to a private equity investor over 10 years. They interpret it to represent compensation for holding an illiquid investment for 10 years. Das, Jagannathan and Sarin (2003) take a more direct approach to estimating private company discounts by looking at how venture capitalists value businesses (and the returns they earn) at different stages of the life cycle. They conclude that the private company discount is only 11% for late stage investments but can be as high as 80% for early stage businesses. The perils of concluding that these discounts are for marketability are manifold. In addition to illiquidity, private equity investors often are not diversified and some of the additional return may represent a premium for this non-diversification. In addition, private equity investors also exercise some or even significant control over the firms they invest in, resulting in a higher payoff.

### ***Real Assets***

If illiquidity is a problem with financial assets, it should be doubly so when investing in real assets. After all, selling a real asset can often be far more difficult and expensive (in terms of transactions costs) than selling a financial asset. While it is difficult to quantify the illiquidity discount in most real asset markets, there have been attempts to do so in the real estate market. Krainer, Spiegel and Yamori (2004) attempted

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<sup>50</sup> Hull, J. and A. White, 2004, How to Value Employee Stock Options, Financial Analysts Journal, v60, 114-119.

to measure illiquidity in the Japanese real estate market by measuring how quickly real estate prices adjusted after the 1990 stock market crash. They present a model and supportive evidence that illiquidity increases after a price decline and that it increases more for assets with more predictable cashflows. In their tests, the prices of commercial real estate (with its lower variance cash flows) declined further and faster than residential real estate in Japan after 1990.<sup>51</sup>

### **Dealing with Illiquidity in Valuation**

Both the theory and the empirical evidence suggest that illiquidity matters and that investors attach a lower price to assets that are more illiquid than to otherwise similar assets that are liquid. The question that we face when valuing assets then is to how best show this illiquidity. In this section, we consider three alternatives. The first is to value an asset or business as if it were a liquid investment, and to then apply an illiquidity discount to that value. The second is to adjust the discount rate used in a discounted cashflow valuation for the illiquidity of the asset; more illiquid assets will have higher discount rates. The third is through relative valuation, by valuing an asset based upon how assets of similar liquidity have been priced in transactions. In this section, we will consider all three.

#### ***Illiquidity Discounts on Value***

In conventional valuation, there is little scope for showing the effect of illiquidity. The cashflows are expected cashflows, the discount rate is usually reflective of the risk in the cashflows and the present value we obtain is the value for a liquid business. With publicly traded firms, we then use this value, making the implicit assumption that illiquidity is not a large enough problem to factor into valuation. In private company valuations, analysts have been less willing (with good reason) to make this assumption. The standard practice in many private company valuations is to apply an illiquidity discount to this value. But how large should this discount be and how can we best estimate in? This is a very difficult question to answer empirically because the discount

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<sup>51</sup> Krainer, J., M.M. Spiegel and N. Yamori, 2004, Asset Price Declines and Real Estate Liquidity: Evidence from Japanese Land Values, Working Paper, Federal Reserve Bank of San Francisco.

in private company valuations itself cannot be observed. Even if we were able to obtain the terms of all private firm transactions, note that what is reported is the price at which private firms are bought and sold. The value of these firms is not reported and the illiquidity discount is the difference between the value and the price. In this section, we will consider four approaches that are in use – a fixed discount (with marginal and subjective adjustments for individual firm differences), a firm-specific discount based upon a firm’s characteristics, a discount obtained by estimating a synthetic bid-ask spread for an asset and an option-based illiquidity discount.

#### *a. Fixed Discount*

The standard practice in many private company valuations is to either use a fixed illiquidity discount for all firms or, at best, to have a range for the discount, with the analyst’s subjective judgment determining where in the range a particular company’s discount should fall. The evidence for this practice can be seen in both the handbooks most widely used in private company valuation and in the court cases where these valuations are often cited. The genesis for these fixed discounts seems to be in the early studies of restricted stock that we noted in the last section. These studies found that restricted (and therefore illiquid) stocks traded at discounts of 25-35%, relative to their unrestricted counterparts, and private company appraisers have used discounts of the same magnitude in their valuations.<sup>52</sup> Since many of these valuations are for tax court, we can see the trail of “restricted stock” based discounts littering the footnotes of dozens of cases in the last three decades.<sup>53</sup>

As we noted in the last section, some researchers have argued that these discounts are too large because of the sampling bias inherent in using restricted stock and that they should be replaced with smaller discounts. In recent years, the courts have begun to look favorably at these arguments. In a 2003 case<sup>54</sup>, the Internal Revenue Service, often at the

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<sup>52</sup> In recent years, some appraisers have shifted to using the discounts on stocks in IPOs in the years prior to the offering. The discount is similar in magnitude to the restricted stock discount.

<sup>53</sup> As an example, in one widely cited tax court case (McCord versus Commissioner, 2003), the expert for the taxpayer used a discount of 35% that he backed up with four restricted stock studies.

<sup>54</sup> The court case was McCord versus Commissioner. In the case, the taxpayer’s expert argued for a discount of 35% based upon the restricted stock studies. The IRS argued for a discount of 7%, on the basis

short end of the illiquidity discount argument, was able to convince the judge that the conventional restricted stock discount was too large and to accept a smaller discount.

### *b. Firm-specific Discount*

Much of the theoretical and empirical discussion in this paper supports the view that illiquidity discounts should vary across assets and business. In particular, with a private company, you would expect the illiquidity discount to be a function of the size and the type of assets that the company owns. In this section, we will consider the determinants of the illiquidity discount and practical ways of estimating it.

### *Determinants of Illiquidity Discounts*

With any asset, the illiquidity discount should be a function of the number of potential buyers for the asset and the ease with which that asset can be sold. Thus, the illiquidity discount should be relatively small for an asset with a large number of potential buyers (such as real estate) than for an asset with a relatively small number of buyers (an expensive collectible). With private businesses, the illiquidity discount is likely to vary across both firms and buyers, which renders rules of thumb useless. Let us consider first some of the factors that may cause the discount to vary across firms.

1. *Liquidity of assets owned by the firm:* The fact that a private firm is difficult to sell may be rendered moot if its assets are liquid and can be sold with no significant loss in value. A private firm with significant holdings of cash and marketable securities should have a lower illiquidity discount than one with factories or other assets for which there are relatively few buyers.
2. *Financial Health and Cash flows of the firm:* A private firm that is financially healthy should be easier to sell than one that is not healthy. In particular, a firm with strong earnings and positive cash flows should be subject to a smaller illiquidity discount than one with losses and negative cash flows.
3. *Possibility of going public in the future:* The greater the likelihood that a private firm can go public in the future, the lower should be the illiquidity discount attached to its value. In effect, the probability of going public is built into the valuation of the private firm. To illustrate, the owner of a private e-commerce firm in 1998 or 1999

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that a big portion of the observed discount in restricted stock and IPO studies reflects factors other than

would not have had to apply much of a illiquidity discount to his firm's value, if at all, because of the ease with which it could have been taken public in those years.

*4. Size of the Firm:* If we state the illiquidity discount as a percent of the value of the firm, it should become smaller as the size of the firm increases. In other words, the illiquidity discount should be smaller as a percent of firm value for private firms like Cargill and Koch Industries, which are worth billions of dollars, than it should be for a small firm worth \$5 million.

*5. Control Component:* Investing in a private firm is decidedly more attractive when you acquire a controlling stake with your investment. A reasonable argument can be made that a 51% stake in a private business should be more liquid than a 49% stake in the same business.<sup>55</sup>

The illiquidity discount is also likely to vary across potential buyers because the desire for liquidity varies among investors. It is likely that those buyers who have deep pockets, longer time horizons and see little or no need to cash out their equity positions will attach much lower illiquidity discounts to value, for similar firms, than buyers that do not possess these characteristics. The illiquidity discount is also likely to vary across time, as the market-wide desire for liquidity ebbs and flows. In other words, the illiquidity discount attached to the same business will change over time even for the same buyer.

### *Estimating Firm-Specific Illiquidity Discount*

While it is easy to convince skeptics that the illiquidity discount should vary across companies, it is much more difficult to get consensus on how to estimate the illiquidity discount for an individual company. In this section, we revert back to the basis for the fixed discount studies and look and look for clues on why discounts vary across companies and how to incorporate these differences into illiquidity discounts.

#### *i. Restricted Stock Studies*

Earlier in the paper, we looked at studies of the discount in restricted stock. One of the papers that we referenced by Silber (1991) examined factors that explained differences in discounts across different restricted stock by relating the size of the

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liquidity. The court ultimately decided on an illiquidity discount of 20%.

<sup>55</sup> For more on the value of control, see the companion paper on the value of control. (<http://www.damodaran.com>: Look under research/papers)

discount to observable firm characteristics including revenues and the size of the restricted stock offering. He reported the following regression.

$$\text{LN(RPRS)} = 4.33 + 0.036 \text{ LN(REV)} - 0.142 \text{ LN(RBRT)} + 0.174 \text{ DERN} + 0.332 \text{ DCUST}$$

where,

RPRS = Restricted Stock Price/ Unrestricted stock price = 1 – illiquidity discount

REV = Revenues of the private firm (in millions of dollars)

RBRT = Restricted Block relative to Total Common Stock (in % )

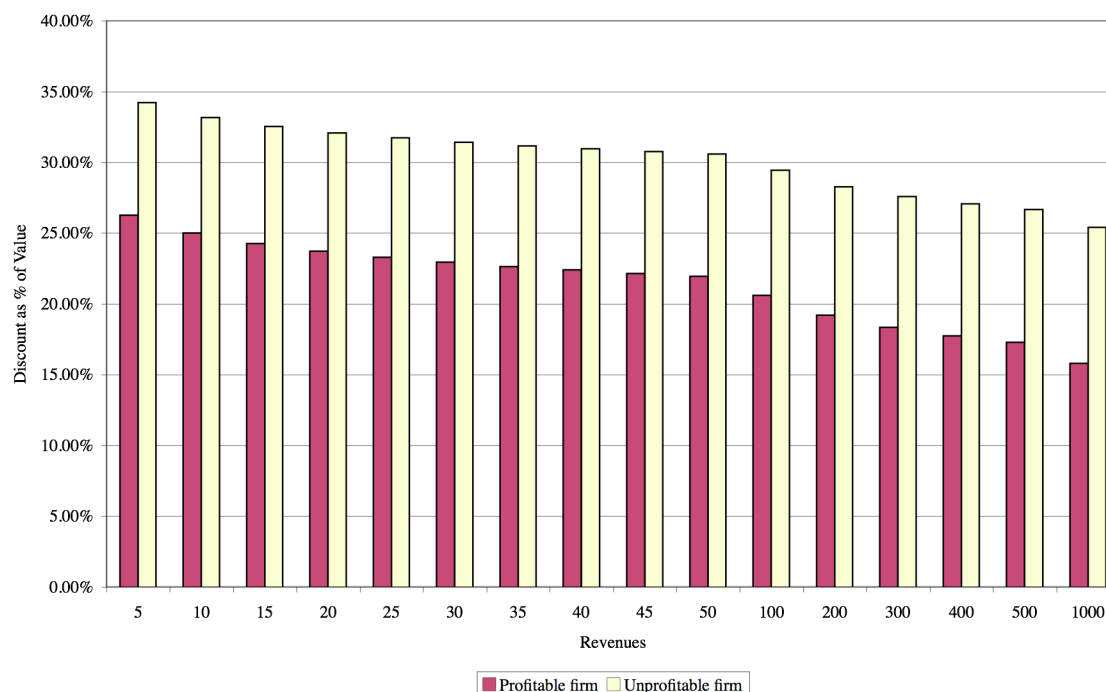
DERN = 1 if earnings are positive; 0 if earnings are negative;

DCUST = 1 if there is a customer relationship with the investor; 0 otherwise;

The illiquidity discount tends to be smaller for firms with higher revenues, decreases as the block offering decreases and is lower when earnings are positive and when the investor has a customer relationship with the firm. These findings are consistent with some of the determinants that we identified in the previous section for the illiquidity premium. In particular, the discounts tend to be smaller for larger firms (at least as measured by revenues) and for healthy firms (with positive earnings being the measure of financial health). This would suggest that the conventional practice of using constant discounts across private firms is wrong and that we should be adjusting for differences across firms.

Consider again the regression that Silber presents on restricted stock. Not only does it yield a result specific to restricted stock, but it also provides a measure of how much lower the discount should be as a function of revenues. A firm with revenue of \$20 million should have an illiquidity discount that is 1.19% lower than a firm with revenues of \$10 million. Thus, we could establish a benchmark discount for a profitable firm with specified revenues (say \$10 million) and adjust this benchmark discount for individual firms that have revenues much higher or lower than this number. The regression can also be used to differentiate between profitable and unprofitable firms. Figure 6 presents the difference in illiquidity discounts across both profitable and unprofitable firms with different revenues, using a benchmark discount of 25% for a firm with positive earnings and \$10 million revenues.

Figure 6: Illiquidity Discounts: Base Discount of 25% for profitable firm with \$ 10 million in revenues



There are clearly dangers associated with extending a regression run on a small number of restricted stocks to estimating discounts for private firms, but it does provide at least a road map for adjusting discount factors.

## ii. Private Placements

Just as Silber considered fundamental factors that cause restricted stock discounts to vary across firms, Bajaj et al. (referenced earlier) considered various fundamental factors that may cause illiquidity discounts to vary across firms in private placements. Their regression, run across 88 private placements between 1990 and 1995 is summarized below:

$$\text{DISC} = 4.91\% + 0.40 \text{ SHISS} - 0.08 \text{ Z} - 7.23 \text{ DREG} + 3.13 \text{ SDEV} \quad R^2 = 35.38\%$$

(0.89)   (1.99)        (2.51)   (2.21)        (3.92)

Where

DISC = Discount on the Market Price

SHISS = Private Placement as percent of outstanding shares

Z = Altman Z-Score (for distress)

DREG = 1 if registered; 0 if unregistered (restricted stock)

SDEV = Standard deviation of returns

Other things remaining equal, the discount is larger for larger private placements (as a percent of outstanding stocks) by risky and distressed firms and smaller for safer firms. As noted before, the discount is larger for restricted stock than for registered stock. Hedrick and Smith (also referenced earlier) ran a similar regression with 106 private placements between 1980 and 1987 and also found larger private placement discounts at more distressed, riskier and smaller firms.

These regressions are a little more difficult to adapt for use with private company valuations since they are composite regressions that include registered private placements (where there is no illiquidity). However, the results reinforce the Silber regression findings that troubled or distressed firms should have larger illiquidity discounts than healthy firms.

There are legitimate criticisms that can be mounted against the regression approach. The first is that the R squared of these regressions is moderate (30-40%) and that the estimates will have large standard errors associated with them. The second is that the regression coefficients are unstable and likely to change over time. While both criticisms are valid, they really can be mounted against any cross sectional regression and cannot be used to justify a constant discount for all firms. After all, these regressions clearly reject the hypothesis that the discount is the same across all firms.

### *c. Synthetic Bid-ask Spread*

The biggest limitation of using studies based upon restricted stock or private placements is that the samples are small. We would be able to make far more precise estimates if we could obtain a large sample of firms with illiquidity discounts. We would argue that such a sample exists, if we consider the fact that an asset that is publicly traded is not completely liquid. In fact, liquidity varies widely across publicly traded stock. A small company listed over-the-counter is much less liquid than a company listed on the New York Stock Exchange which in turn is much less liquid than a large capitalization company that is widely held. If, as we argued earlier, the bid-ask spread is a measure of the illiquidity of a stock, we can compute the spread as a percent of the market price and relate it to a company's fundamentals. While the bid-ask spread might only be a quarter



or half a dollar, it looms as a much larger cost when it is stated as a percent of the price per unit. For a stock that is trading at \$2, with a bid-ask spread of 1/4, this cost is 12.5%. For higher price and very liquid stocks, the illiquidity discount may be less than 0.5% of the price, but it is not zero. What relevance does this have for illiquidity discounts on private companies? Think of equity in a private company as a stock that never trades. On the continuum described above, you would expect the bid-ask spread to be high for such a stock and this would essentially measure the illiquidity discount.

To make estimates of the illiquidity discounts using the bid-ask spread as the measure, you would need to relate the bid-ask spreads of publicly traded stocks to variables that can be measured for a private business. For instance, you could regress the bid-ask spread against the revenues of the firm and a dummy variable, reflecting whether the firm is profitable or not, and extend the regression done on restricted stocks to a much larger sample. You could even consider the trading volume for publicly traded stocks as an independent variable and set it to zero for a private firm. Using data from the end of 2000, for instance, we regressed the bid-ask spread against annual revenues, a dummy variable for positive earnings (DERN: 0 if negative and 1 if positive), cash as a percent of firm value and trading volume.

$$\text{Spread} = 0.145 - 0.0022 \ln (\text{Annual Revenues}) - 0.015 (\text{DERN}) - 0.016 (\text{Cash/Firm Value}) - 0.11 (\$ \text{ Monthly trading volume/ Firm Value})$$

Plugging in the corresponding values – with a trading volume of zero – for a private firm should yield an estimate of the synthetic bid-ask spread for the firm. This synthetic spread can be used as a measure of the illiquidity discount on the firm.

#### *d. Option-Based Discount*

In an earlier section, we examined an option-pricing based approach, which allowed you to estimate an upper bound for the illiquidity discount, by assuming an investor with perfect market timing skills. There have been attempts to extend option pricing models to valuing illiquidity, with mixed results. In one widely used variation, liquidity is modeled as a put option for the period when an investor is restricted from trading. Thus, the illiquidity discount on value for an asset where the owner is restricted

from trading for 2 years will be modeled as a 2-year at-the-money put option.<sup>56</sup> There are several flaws, both intuitive and conceptual, with this approach. The first is that liquidity does not give you the right to sell a stock at today's market price anytime over the next 2 years. What it does give you is the right to sell at the prevailing market price anytime over the next 2 years.<sup>57</sup> The second (and smaller) problem is that option pricing models are based upon continuous price movements and arbitrage and it is difficult to see how these assumptions will hold up for an illiquid asset.

The value of liquidity ultimately has to derive from the investor being able to sell at some pre-determined price during the non-trading period rather than being forced to hold until the end of the period. The look-back option approach that assumes a perfect market timer, explained earlier in the paper, assumes that the sale would have occurred at the high price and allows us to estimate an upper bound on the value. Can we use option pricing models to value illiquidity without assuming perfect market timing. Consider one alternative. Assume that you have a disciplined investor who always sells investments, when the price rises 25% above the original buying price. Not being able to trade on this investment for a period (say, 2 years) undercuts this discipline and it can be argued that the value of illiquidity is the produce of the value of the put option (estimated using a strike price set 25% above the purchase price and a 2 year life) and the probability that the stock price will rise 25% or more over the next 2 years.

If you decide to apply option pricing models to value illiquidity in private businesses, the value of the underlying asset (which is the private business) and the standard deviation in that value will be required inputs. While estimating them for a private business is more difficult to do than for a publicly traded firm, we can always use industry averages.

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<sup>56</sup> In a 1993 study, David Chaffe used this approach to estimate illiquidity discounts ranging from 28-49% for an asset, using the Black Scholes option pricing model and volatilities ranging from 60 to 90% for the underlying asset.

<sup>57</sup> There is a simple way to illustrate that this put option has nothing to do with liquidity. Assume that you own stock in a liquid, publicly traded company and that the current stock price is \$ 50. A 2-year put option on this stock with a strike price of \$ 50 will have substantial value, even though the underlying stock is completely liquid. The value has nothing to do with liquidity but is a price you are willing to pay for insurance.

*Illustration 1: Estimating the illiquidity discount for a private firm*

Kristin Kandy is a privately owned candy manufacturing business that generated \$ 400,000 million in pre-tax operating income on \$ 3 million in revenues in the most recent financial year. Based upon its expected growth rate of 6.36% for the next 5 years and 4% thereafter, you have estimated a value of \$1.796 million for the firm today, without any adjustments for illiquidity (See figure 8 for the valuation). In valuing Kristin Kandy, we have adopted an estimation process very similar to the one we would have adopted for a publicly traded firm, with one key exception. In estimating the cost of capital, we used a beta based upon total risk rather than the more conventional beta based upon market risk alone.<sup>58</sup>

Cost of equity based upon total beta = 4.5% + 2.94 (4%) = 16.26%

Cost of equity based upon market beta = 4.5% + 0.98 (4%) = 8.42%

The resulting higher costs of equity (and capital) generate much lower values for the firm but this value discount is for non-diversification and not for illiquidity. We can estimate the illiquidity discount for Kristin Kandy, using all of the approaches described in the section above. In making these estimates, we used the following benchmarks:

- We computed the Silber regression discount using a base discount of 15% for a healthy firm with \$ 10 million in revenues. The difference in illiquidity discount for a firm with \$ 10 million in revenues and a firm with a firm with \$ 3 million in revenues in the Silber regression is 2.17%. Adding this on to the base discount of 15% yields a total discount of 17.17%.
  - The synthetic bid-ask spread was computed using the spread regression presented earlier and the inputs for Kristin Kandy (revenues = \$3 million, positive earnings, cash/ firm value = 6.56% and no trading)
- $$\text{Spread} = 0.145 - 0.0022 \ln (\text{Annual Revenues}) - 0.015 (\text{DERN}) - 0.016 (\text{Cash/Firm Value}) - 0.11 (\$ \text{ Monthly trading volume/ Firm Value}) = 0.145 - 0.0022 \ln (3) - 0.015 (1) - 0.016 (0.0696) - 0.11 (0) = 0.1265 \text{ or } 12.65\%$$

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<sup>58</sup> The rationale we use is that the owner of a private business is not diversified and has her entire wealth tied up in this business. Consequently, she is exposed to all of the risk in the company and not just the non-diversifiable risk.

- To value illiquidity as an option, we chose arbitrary values for illustrative purposes of an upper limit on the price (at which you would have sold) of 20% above the current value, an industry average standard deviation of 25% and a 1-year trading restriction.

The resulting option has the following parameters:

$S$  = Estimated value of equity = \$1,796 million;  $K = 1,796 (1.20) = \$2,155$  million;  $t = 1$ ; Riskless rate = 4.5% and  $\sigma = 25\%$

The value of liquidity is the product of the value of the option, based on the parameters listed above, and the probability that the stock price would increase by more than 20% over the next year.

Value of liquidity = Value of option to sell at 20% above the current stock price \* Probability that stock price will increase by more than 20% over next year = \$354 million \* 0.4405 = \$156 million

The option value was estimated using the Black-Scholes model. The latter was estimated, using the expected return of 16.26% for the equity<sup>59</sup> and the standard deviation of 25%, to be 44.05%.<sup>60</sup> Dividing the value of liquidity by the estimated value of equity for Kristin Kandy of \$1,796 million yields an illiquidity discount of 8.67%.

The resulting values are provided in table 2 below:

*Table 2: Illiquidity Discounts for Kristin Kandy*

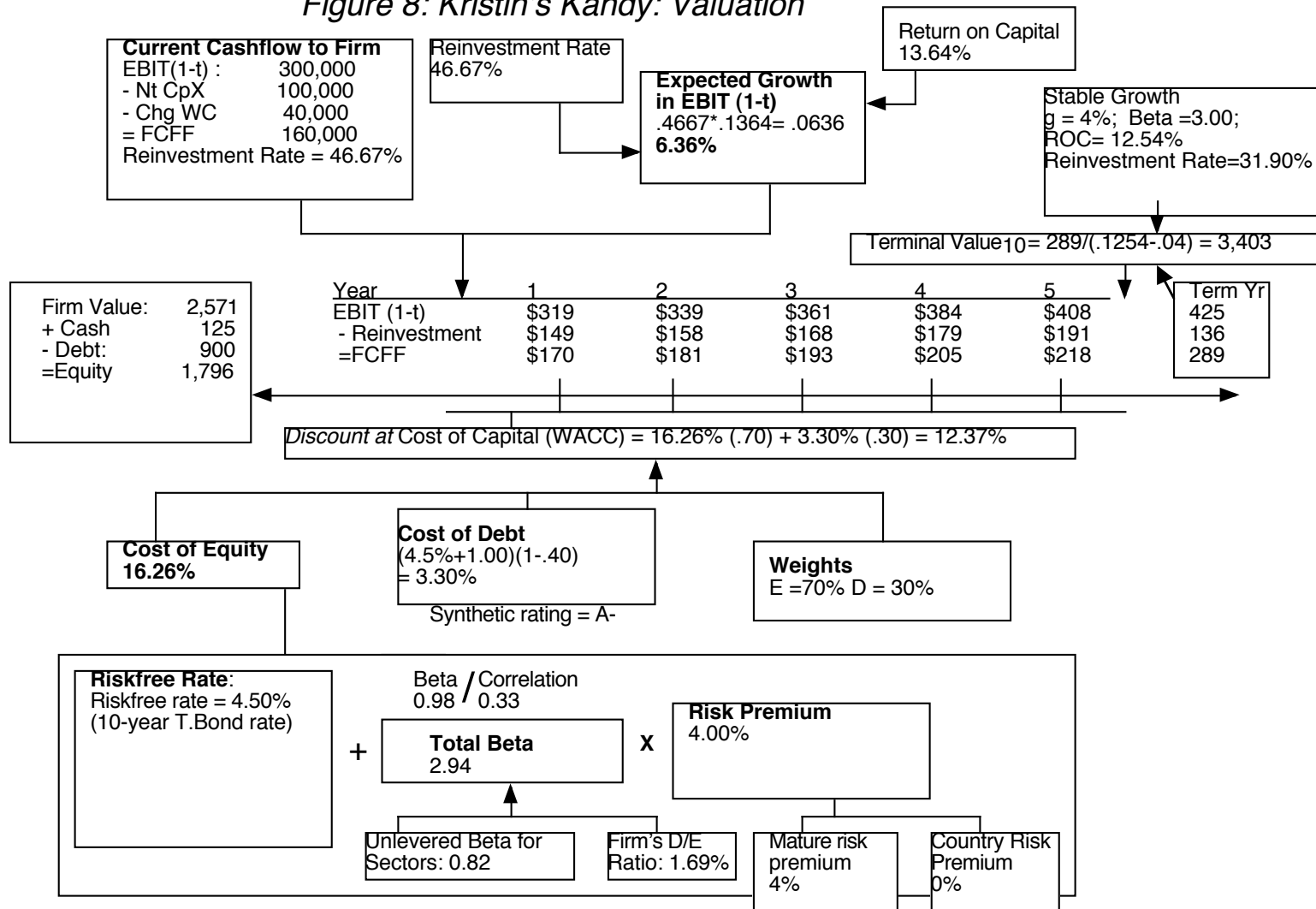
<i>Approach</i>	<i>Estimated Discount</i>	<i>Liquidity Adjusted Value</i>
Fixed Discount- Restricted Stock	25.00%	\$1,347.00
Fixed Discount- Restricted Stock vs Registered Placements	15.00%	\$1,526.60
15% base discount adjusted for Revenues/Health (Silber)	17.17%	\$1,487.63
Synthetic Spread	12.65%	\$1,570.42
Option Based approach (20% upside; Industry variance of 25%; 1 year trading restriction)	8.67%	\$1,640.24

<sup>59</sup> The cost of equity for Kristin Kandy of 16.26% is used as the expected return on the equity.

<sup>60</sup> For simplicity, we assumed a normal distribution for returns and computed the cumulative probability that returns would be greater than 20% over the next year. ( $Z = (20-12)/25 = 0.32$ ,  $N(Z) = 0.6255$ )

If we ignore the pure restricted stock discount of 25% as an over estimate and the option based approach, the illiquidity discounts fall within a fairly tight range (12.65% to 17.17%).

Figure 8: Kristin's Kandy: Valuation



### ***Adjusting Discount Rates for Illiquidity***

The other approach to dealing with illiquidity is to adjust the discount rate used in discounted cashflow valuation for illiquidity. In practical terms, this amounts to adding an illiquidity premium to the discount rate and deriving a lower value for the same set of expected cashflows. Earlier, we presented asset pricing models that attempt to incorporate illiquidity risk but they are not specific about how we should go about estimating the additional premium (other than saying that it should be larger for investments which are illiquid when the market is illiquid). There are three practical solutions to the estimation problem:

1. Add a constant illiquidity premium to the discount rate for all illiquid assets to reflect the higher returns earned historically by less liquid (but still traded) investments, relative to the rest of the market. This is akin to another very common adjustment made to discount rates in practice, which is the small stock premium. The costs of equity for smaller companies are often augmented by 3-3.5% reflecting the excess returns earned by smaller cap companies over very long periods. The same historical data that we rely on for the small stock premium can provide us with an estimate of an “illiquidity premium”.

- Practitioners attribute all or a significant portion of the small stock premium reported by Ibbotson Associates to illiquidity and add it on as an illiquidity premium. Note, though, that even the smallest stocks listed in their sample are several magnitudes larger than the typical private company and perhaps more liquid.
- An alternative estimate of the premium emerges from studies that look at venture capital returns over long period. Using data from 1984-2004, Venture Economics, estimated that the returns to venture capital investors have been about 4% higher than the returns on traded stocks.<sup>61</sup> We could attribute this difference to illiquidity and add it on as the “illiquidity premium” for all private companies.

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<sup>61</sup> The sample of several hundred venture capital funds earned an annual average return of 15.7% over the period whereas the annual average return was 11.7% on the S&P 500 over the same period. They did not adjust for risk. Broken down into classes, venture capital investments in early stage companies earned 19.9% whereas investments in late stage ventures earned only 13.7%.

The key is to avoid double counting the cost of illiquidity since some of the small stock premium may be compensation for the illiquidity of small cap companies.

2. *Add a firm-specific illiquidity premium, reflecting the illiquidity of the asset being valued:* For liquidity premiums that vary across companies, we have to estimate a measure of how exposed companies are to liquidity risk. In other words, we need liquidity betas or their equivalent for individual companies. Drawing on the work done on the liquidity based capital asset pricing model, these liquidity betas should reflect not only the magnitude of trading volume on an investment but how that trading volume varies with the market trading volume over time. It may be possible to do this for some real assets (such as real estate) where there are transactions from time to time, but may be impossible to do for unique private businesses.

3. *Relate the observed illiquidity premium on traded assets to specific characteristics of those assets.* Thus healthier firms with more liquid holdings should have a smaller liquidity premium added on to the discount rate than distressed firms with non-marketable assets. While this can be done subjectively, it would make more sense to have a solid quantitative basis for the adjustment.

The three different approaches to adjusting discount rates are similar to the approaches used to estimate illiquidity discounts on value. The constant liquidity premium approach mirrors the fixed liquidity discount whereas the firm-specific liquidity premium approaches resemble the approaches used to adjust the illiquidity discount for individual firms. In fact, we could build regression models that relate expected returns on stocks to measures of illiquidity and use these regressions to forecast discount rates for private firms.

There are practitioners who have tried to develop models that incorporate illiquidity. One widely publicized model is called the Quantitative Marketability Discount Model (QMDM).<sup>62</sup> The QMDM allows analysts to adjust the discount rate for illiquidity factors, though the adjustment is subjective, and then values illiquidity as a percent of firm value for different holding periods. To illustrate how the model works,

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<sup>62</sup> The model was developed by Chris Mercer, a principal at Mercer Capital. A fuller discussion of the model is available in Mercer, Z. C., 2004, *The Integrated Theory of Business Valuation*, Peabody Publishing.



consider a firm with an expected cash flow next period of \$ 1.00. Assume that the appropriate discount rate, based upon fundamental risk but before adjusting for liquidity risk is 9% and that the expected growth in the cashflows in perpetuity is 4%. This firm would have an intrinsic value of \$ 20.<sup>63</sup> In the QMDM, the analyst would adjust the discount rate for illiquidity (assume that he would add 3% to the discount rate to arrive at a required return of 12%), specify a holding period (say, 5 years) and the percent of the available cashflows that will be paid out (say 60%). The new firm value would then be computed as follows:

$$\begin{aligned}\text{New Firm Value} &= \text{PV of cashflows during holding period} + \text{PV of terminal value} \\ &= \text{PV of \$ 0.60 growing 4\% a year for 5 years} + 20 (1.04)^5 / (1.12)^5 \\ &= \$ 16.13\end{aligned}$$

The first term is the present value of annual cashflows during the holding period- \$ 0.60 (60% of \$ 1) growing at 4% a year for the next 5 year- and the second term is the present value of the terminal value (\$ 20 growing at 4% a year for the next 5 years), all discounted back at the liquidity adjusted discount rate of 12%. Comparing the estimated value (\$16.13) to the unadjusted value (\$20) yields an illiquidity discount of 19.35%.

While the QMDM model is well intentioned, it fails on three levels. First, the cashflow that does not get paid out over the next 5 years is assumed to be wasted by the controlling stockholders for private benefits that do not accrue to the business.<sup>64</sup> If this is indeed the case, the firm value should have been computed at \$ 12 initially, rather than \$ 20.<sup>65</sup> Second, the illiquidity discount computed in the model is a consequence of both control and illiquidity. While Mercer makes the reasonable point that the two are interrelated, one can very easily exist without the other. In other words, you can have a completely liquid investment with absolutely no control over how a firm is run, as is often the case with stock in a large publicly traded company. The fact that you can sell your stock at any time will not protect you from management or controlling stockholder

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<sup>63</sup> Intrinsic value = \$ 1 / (.09-.04) = \$ 20

<sup>64</sup> If the cash is held back in the firm (rather than wasted), it will add on to the terminal value and the value of the firm should not be affected.

<sup>65</sup> The model seems to assume that the firm will revert back to being optimally run at the end of the illiquidity period. There is no reason why this should happen. If you did not expect it to happen, the value of the firm would be based upon \$ 0.60 in cashflow, growing at 4% a year in perpetuity:

Value of firm = \$0.60 / (.09-.04) = \$ 12

actions since the price you sell at will reflect management foibles. Third, for a model that claims to quantify non-marketability, the QMDM is surprisingly elusive on the adjustment made to the discount rate for illiquidity, other than to note that it can be backed out of observed illiquidity discounts in restricted stock studies.

*Illustration 2: Estimating the illiquidity adjusted discount rate for a private firm*

Earlier in the paper, we applied various estimates of the illiquidity discount to the estimated value of \$1.796 billion to arrive liquidity-adjusted values. As an alternative, we could have adjusted the discount rate that we used to value Kristin Kandy to reflect the illiquidity.

- Adding an illiquidity premium of 4% (based upon the premium earned across all venture capital investments) to the cost of equity yields a cost of equity of 20.26% and a cost of capital of 15.17%. Using this higher cost of capital lowers the value of equity in the firm to \$1.531 million, about 15.78% lower than the original estimated.<sup>66</sup>
- Allowing for the fact that Kristin Kandy is an established business that is profitable would allow us to lower the illiquidity premium to 2% (based upon late stage venture capital investments). This will lower the cost of equity to 18.26%, the cost of capital to 13.77% and result in a value of equity of \$1.658 million. The resulting illiquidity discount is 7.66%.

Two general points should be made about adjusting discount rates for illiquidity. The first is that small adjustments to the discount rate will translate into large illiquidity discounts. The second is that the length of the period that we make the illiquidity adjustment for will affect the magnitude of the discount. If we increase discount rates for illiquidity in perpetuity rather than the 5 years that we used in both calculations above, the resulting discounts will be much larger (31.77% for the 4% illiquidity premium and 17.66% for the 2% illiquidity premium).

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<sup>66</sup> The higher cost of capital was used only for the first 5 years. Extending into perpetuity reduces the value of equity to \$1.225 million, a decline of 31.77%.

## ***Relative Valuation***

The valuation adjustments that we have talked about so far are structured around intrinsic valuation, where we try to estimate the value of a business based upon its cashflows and a risk-adjusted discounted rate. In practice, most valuations of both private and publicly traded companies are relative valuations, where we value businesses, based upon how similar assets are priced. In this section, we consider two ways of incorporating illiquidity into relative valuation.

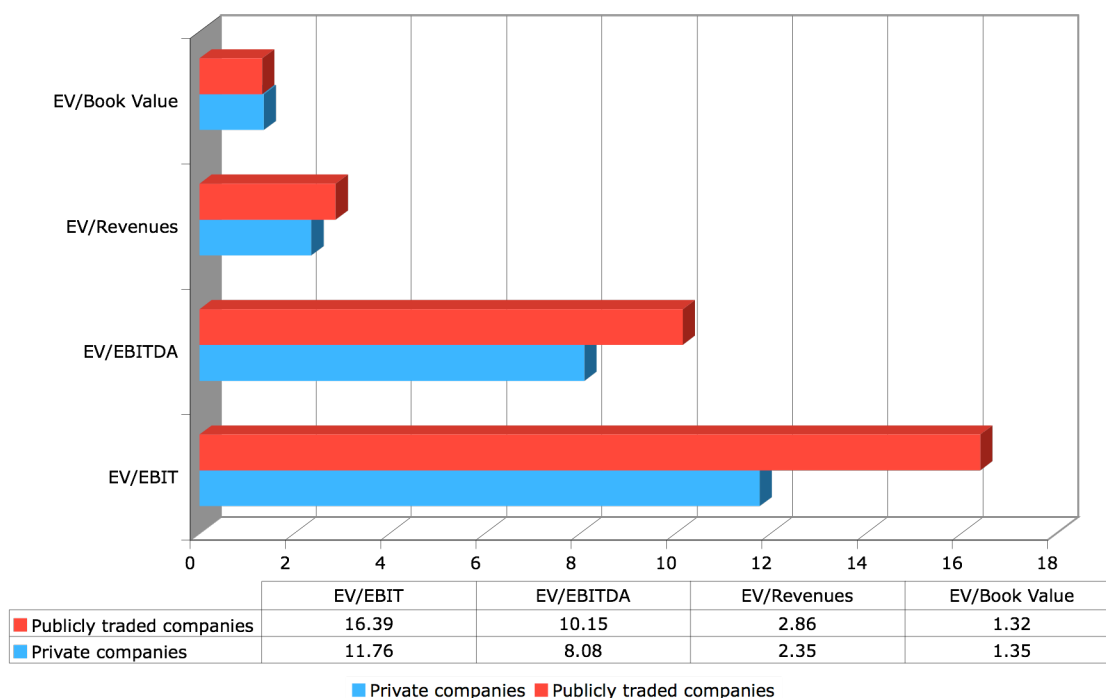
### ***1. Relative Valuation with Illiquid Assets***

The simplest way of incorporating illiquidity into relative valuation is to value a company based upon the pricing of other companies of similar liquidity (or illiquidity). The key to using this approach is in finding these comparable companies. Koeplin, Sarin and Shapiro (2000) provide an illustration of this approach by comparing the multiples paid for 84 private companies that were acquisition targets to the multiples of earnings paid for 198 “similar” publicly traded firms between 1984 and 1998.<sup>67</sup> Figure 9 shows the average multiples of earnings, book value and sales for private and public firms:

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<sup>67</sup> The multiples they used were all based upon enterprise value (market value of equity + debt – cash) in the numerator. They compared enterprise value to EBIT, EBITDA, Sales and the Book Value of Capital.

Figure 9: Private versus Public Acquisitions



Note that, with the exception of revenue multiples, the private companies were acquired at multiples about 20-30% lower than those paid for publicly traded firms; the discount was larger (40-50%) for foreign private firms. They do note that notwithstanding their attempts to get a controlled sample, the private companies in their sample were smaller and had higher growth rates than the publicly traded companies.

Can you value a private company (asset) then by looking at the pricing of similar private companies (assets)? In some cases, you can, if the following conditions hold:

- There are a number of private businesses that are similar in their fundamental characteristics (growth, risk and cashflows) to the private business being valued.
- There are a large enough number of transactions involving these private businesses (assets) and information on transactions prices is widely available.
- The transactions prices can be related to some fundamental measure of company performance (like earnings, book value and sales) and these measures are computed with uniformity across the different companies.
- Other information encapsulating the risk and growth characteristics of the businesses that were bought is also easily available.

With real assets (art, collectibles and real estate), relative valuation is widely used since these conditions hold for the most part. All one has to do is check on Ebay to see that the owners of valuable baseball cards or Tiffany lamps price them based upon recent transactions involving the same assets. The prices paid for residential property have generally been based on the prices at which similar properties have sold at in the recent past. With commercial real estate, the values of office buildings can be estimated as a multiple of expected rental income or square footage, since this information is available on other recent transactions.

With private businesses, it becomes more difficult to use this approach for several reasons. The first is that the sphere of comparable businesses becomes much smaller and transactions occur far less frequently. Even when they do occur, transactions prices might not be made public or may reflect other considerations (such as tax planning). Finally, there are wide differences in accounting standards and practices, skewing standardizing measures of such as revenues and earnings. Notwithstanding these limitations, you do see relative valuation widely used in some areas of private business valuations (restaurant franchises and medical practices, for instance), with rules of thumb on multiples of revenues and earnings used to determine estimated values. In fact, transactions data on private businesses is now widely available from sources such as the Institute of Business Appraisers (IBA), BIZCOMPS and Pratt's Stats.

## *2. Relative Valuation with Illiquidity Discount*

In most private company valuations, it is difficult to get a subset of comparable private businesses where there have been recent transactions. Analysts often have to use a subset of publicly traded firms as comparable firms, derive a multiple of revenues or earnings from these firms and then modify this multiple to value their private business. The key question centers on how to adjust a multiple derived from publicly traded firms for use with a private company. There are two alternatives:

- Use an illiquidity discount, estimated using the same approaches described in the prior section, to adjust the multiple: For instance, an analyst who believes that a fixed illiquidity discount of 25% is appropriate for all private businesses would then reduce the public multiple by 25% for private company valuations. An analyst who believes

that multiples should be different for different firms would adjust the discount to reflect the firm's size and financial health and apply this discount to public multiples.

- Instead of estimating a mean or median multiples for publicly traded firms, relate the multiples of these firms to the fundamentals of the firms (including size, growth, risk and a measure of illiquidity). The resulting regression can then be used to estimate the multiple for a private business.

With both approaches, you are valuing a private firm by looking at how publicly traded firms are priced. To the extent that these publicly traded firms are priced correctly by the market, the resulting valuation will be reasonable. If the market is over pricing or under pricing the comparable companies, you will build in the over or under valuation into your valuation as well.

*Illustration 3: Estimating illiquidity in a relative valuation*

Consider again the valuation of Kristin Kandy. We defined comparable firms to be publicly traded food processing companies, with a market cap less than \$ 300 million. The resulting sample of 38 companies is provided in table 3, with enterprise value to sales ratios, operating margins and turnover ratios:

*Table 3: Food Processing Companies: Revenue Multiples and Fundamentals*

<i>Company Name</i>	<i>Ticker Symbol</i>	<i>EV/Sales</i>	<i>Operating Margin</i>	<i>Turnover Ratio</i>
Gardenburger Inc	GBUR	0.62	0.03	0.65
Paradise Inc	PARF	0.33	0.05	0.38
Armanino Foods Dist	ARMF	0.59	0.06	0.37
Vita Food Prods	VSF	0.57	0.02	0.13
Yocream Intl Inc	YOCM	0.53	0.07	0.70
Allergy Research Group Inc	ALRG	0.72	0.15	0.16
Unimark Group Inc	UNMG	0.55	0.02	0.14
Tofutti Brands	TOF	0.81	0.05	0.10
Advanced Nutraceuticals Inc	ANII	1.13	0.20	0.26
Sterling Sugars Inc	SSUG	0.96	0.15	0.23
Spectrum Organic Products Inc	SPOP	0.75	0.02	0.20
Northland Cranberries Inc	NRCNA	0.66	0.10	0.07
Scheid Vineyards	SVIN	1.77	0.25	0.26
Medifast Inc	MED	1.41	0.16	0.74
Galaxy Nutritional Foods Inc.	GXY	1.44	0.09	0.17
Natrol Inc	NTOL	0.51	0.06	0.15
Monterey Gourmet Foods Inc	PSTA	0.76	0.01	0.34
ML Macadamia Orchards LP	NUT	3.64	0.08	0.39

Golden Enterprises	GLDC	0.50	0.02	0.12
Natural Alternatives Intl Inc.	NAII	0.59	0.08	1.31
Rica Foods Inc	RCF	0.80	0.06	0.06
Tasty Baking	TBC	0.52	0.06	0.39
Scope Industries	SCPJ	0.75	0.17	0.15
Bridgford Foods	BRID	0.47	0.03	0.07
Poore Brothers	SNAK	1.12	0.10	0.70
High Liner Foods Inc	HLF.TO	0.42	0.07	0.23
Seneca Foods 'A'	SENEA	0.38	0.07	0.05
LIFEWAY FOODS	LWAY	5.78	0.22	1.95
Seneca Foods 'B'	SENEB	0.38	0.07	0.13
FPI Limited	FPL.TO	0.40	0.04	0.14
Rocky Mountain Choc Factory	RMCF	6.24	0.22	1.65
Calavo Growers Inc.	CVGW	0.50	0.04	0.13
MGP Ingredients Inc.	MGPI	0.55	0.07	1.86
Hanover Foods Corporation /PA/	HNFSA	0.58	0.09	0.09
Omega Protein	OME	1.16	0.15	0.22
Reliv Intl	RELV	1.70	0.11	0.47
Agricore United	AU.TO	0.27	0.04	0.27
Sanfilippo John B.	JBSS	0.50	0.10	1.80

Regressing EV/Sales ratios for these firms against operating margins and turnover ratios yields the following:

$$\text{EV/Sales} = 0.11 + 10.78 \text{ EBIT/Sales} + 0.89 \text{ Turnover Ratio} - 0.67 \text{ Beta} \quad R^2 = 45.04\%$$

(0.27)    (3.81)                      (2.81)                      (1.06)

(T statistics in brackets below coefficients)

Kristin Kandy has a pre-tax operating margin of 25%, a zero turnover ratio (to reflect its status as a private company) and a beta (total) of 2.94. This generates an expected EV/Sales ratio of 0.296.

$$\text{EV/Sales} = 0.11 + 10.78 (.25) + 0.89 (0) - 0.67 (2.94) = 0.835$$

Multiplying this by Kristin Kandy's revenues of \$2 million in the most recent financial year generates an estimated value for the firm of \$1.67 million. This value is already adjusted for illiquidity and it is comparable to the estimates of value obtained from the discounted cashflow approaches in illustrations 1 and 2.

### The Consequences of Illiquidity

Illiquidity has consequences for almost every aspect of finance. The question of whether a company should go public may ultimately represent a trade off between the

control (associated with being the owner of a private business) with the liquidity of becoming a publicly traded firm. Investors, be they portfolio managers, private equity investors or venture capitalists will have to modify how they invest and what they invest in, based upon liquidity, and performance evaluation and risk management tools have to grapple with illiquidity. Basic corporate finance measures (such as the cost of capital) may have to be adjusted to reflect illiquidity, and investment, financing and dividend decisions will undoubtedly be affected by a firm's perception of its own liquidity (or lack thereof).

### ***Going Public (Private)***

The question of whether a growing and successful private company should go public does involve trade offs. It is true that publicly traded firms have more access to capital and provide more liquidity to their owners. It is also true that the owners of private businesses have far more control on how much information they reveal to markets and how their businesses get run. This trade off between illiquidity and control will determine whether firms will go public in the first place.

Given that going public allows investors to trade on a firm's equity, and in effect reduce the illiquidity discount on value, we can draw the following conclusions about the incentives to go public in different sectors and variations over time:

- Researchers who track initial public offerings have noted the phenomenon of hot and cold periods in public offerings. In some years, there are dozens of public offerings and in others, almost none. If, as we noted earlier, the market price of illiquidity varies over time, you would expect more public offerings by small companies when the market premium for illiquidity is smallest (leading to higher values for these companies), which also happens to coincide with market upswings.
- It is also worth noting that public offerings in periods are often clustered in a few sectors, though the sectors themselves may vary across time. One possible explanation (of many) for this clustering is that you are more likely to see companies go public in sectors where the illiquidity discount is largest. There are both theoretical and empirical reasons for believing this is most likely to occur in volatile sectors. The empirical evidence is supportive of this hypothesis.



What about publicly traded companies that go private, as is often the case with management or leveraged buyouts? These companies are also making a trade off, but they are trading off more control for less liquidity. In making this trade off, though, note that most of these “going private” transactions are done with an eye on going public again in the near future. Thus, the illiquidity here is for a limited period and should have a lower cost than the permanent illiquidity associated with being a private business.

### ***Portfolio Management***

If illiquidity represents a drag on value, investors have to examine its consequences when choosing investments and developing trading strategies as well as when evaluating portfolio performance. Consider the consequences for investment choices first. If, as the evidence seems to indicate, less liquid stocks generate higher expected returns over time to compensate for illiquidity, investors with long time horizons will be able to generate excess returns by directing their money towards these investments. The higher returns that these investments will more than cover the cost of illiquidity for these investors. The magnitude of the excess returns will depend upon the relative numbers of investors with long and short time horizons in the markets, with the returns being largest when long term investors are scarce. In contrast, investors with shorter time horizons should focus their portfolios more on more liquid investments. Generalizing, we would expect investment strategies that combine high turnover and an emphasis on risky, small market cap stocks to under perform the market.<sup>68</sup> Extending this analysis to venture capital and private equity investments, illiquidity should be an even bigger factor in investment choices. Investors in these investments, when negotiating for a share of the business that they should receive in exchange for supplying funds, have to consider how much to discount the value for illiquidity.

There are consequences for performance evaluation as well. With publicly traded stocks, we generally use market prices to measure returns and these prices should reflect the consequences of illiquidity directly. In other words, a portfolio manager who invests

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<sup>68</sup> There is evidence in studies of mutual funds that support this proposition. There is negative correlation between turnover ratios at mutual funds and excess returns, but the correlation is strongest for small, high growth mutual funds.

primarily in less liquid stocks will not gain an advantage over one who invests in more liquid stocks. With private equity and venture capital funds, where the assets are not traded and the valuations are generated internally (by the fund managers), the stated value of a portfolio may be misleading if illiquidity is not explicitly factored into the value. In general, this will lead to returns being overstated at funds with more illiquid investments and the magnitude of the misstatement will be greater in periods of overall market illiquidity (when liquidity commands a greater premium).<sup>69</sup>

### ***Corporate Finance***

There are two levels at which illiquidity can affect corporate financial decisions. The first relates to the liquidity of the securities (stocks and/or bonds) issued by a firm to raise capital. The second is centered on the liquidity of the assets owned by a firm.

The liquidity or lack thereof of the securities issued by a firm can have significant consequences for almost every aspect of corporate finance.

- If we accept the proposition that the cost of equity includes a premium for illiquidity, less liquid firms will have higher costs of equity (and capital) than more liquid firms. There is also some evidence that they face higher issuance costs in raising capital. Using 2,387 seasoned equity offerings from 1993-2000, Butler, Grullon and Weston (2002) find that, after controlling for other factors, investment banks charge lower fees to firms with more liquid stocks. They also find that the time to complete a seasoned equity offering declines with the level of market liquidity.<sup>70</sup>
- Turning to the investment decision, the perception of illiquidity can have consequences for the types of investments that a firm will take. In general, firms with illiquid securities will be less willing to invest in long-term projects with significant negative cash flows in the early years, even if these projects are good projects,

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<sup>69</sup> This was clearly visible in the after-math of the dot-com bust in 2001 and 2002, when venture capital funds reported much lower negative returns than would have been expected given the collapse in the public market. Some of this understatement can be attributed to value smoothing at the funds but some can be explained by the failure to consider the greater cost of illiquidity in these periods.

<sup>70</sup> Stock Market Liquidity and the Cost of Raising Capital, Butler, A.W., G. Grullon and J.P. Weston, Working Paper, SSRN.

because of the concern that they will be unable to fund these cashflows. In general, the illiquidity

- Firms with liquid securities can also afford to pay more in dividends and retain less cash, knowing that they can always raise fresh capital (with low transactions costs) to fund shortfalls.
- Finally, the liquidity or lack thereof of securities can have consequences for how management at companies get compensated. In recent years, firms have increasingly used options and restricted stock to compensate management. To the extent that the underlying stock is illiquid, the options will be worth less and the firm presumably has to offer more options or pay cash to generate the same equivalent compensation.<sup>71</sup>

What about asset liquidity? A firm with liquid assets may make very different decisions than a firm with illiquid assets. In general, firms with liquid assets can borrow more (on the belief that they can always sell some of their assets in the event of a cash shortfall) and be more flexible when it comes to both dividend and investment policy for the same reasons. In a study of L.A. Gear, a firm that saw its equity value drop from \$ 1 billion in 1989 to zero in 1998, DeAngelo, DeAngelo and Wruck (2000) concluded that asset liquidity can give managers substantial discretion, especially when the firm is in financial distress.<sup>72</sup>

## Conclusion

Illiquidity matters. Investors are generally willing to pay higher prices for more liquid assets than for otherwise similar liquid assets. While this proposition is widely accepted, there is substantial debate about how to measure illiquidity and to incorporate it into value. In this paper, we began by relating illiquidity to transactions costs; less liquid investments have higher costs of transacting, especially if we defined these costs broadly to include the bid-ask spread and a price impact. We then looked at the empirical evidence on how much markets value liquidity. Considering a broad array of investments,

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<sup>71</sup> Firms that switched to restricted stock to compensate management (as Microsoft did in 2004) are faced with the illiquidity question much more directly. The employees receiving this stock will incorporate the illiquidity of this stock into its valuation.

<sup>72</sup> DeAngelo, H., L. DeAngelo, and K.H. Wruck, 2000, Asset Liquidity, Debt Covenants and Managerial Discretion in Financial Distress: The Collapse of L.A. Gear, Working Paper, SSRN.

from government bonds to private equity, the consensus conclusion that we draw is that illiquid investments trade at lower prices than liquid investments and generate higher returns. The magnitude of the illiquidity discount varies across investments, with riskier investments bearing larger illiquidity discounts, and across time, with the discounts being greatest when the overall market itself is least liquid.

In the next part of the paper, we consider different ways of incorporating illiquidity into value. In discounted cash flow valuations, we can either value an asset or business first as a liquid asset and then apply an illiquidity discount, or adjust the discount rate to reflect illiquidity (by adding a premium for illiquid investments). In either case, the adjustment should reflect firm specific factors and be larger for some assets (risky and troubled firms) than for others. In relative valuation, we can attempt to bypass the estimation issue by finding transactions prices on similar illiquid assets and using this information to price the asset in question. If this is not possible, we have to rely on adjusting the relative value for illiquidity in much the same way as we adjust discounted cash flow valuations.

The question of how illiquidity affects value has consequences for both investors and financial managers. For investors, it pinpoints the importance of finding an investment strategy that matches time horizon; less liquid investments are much better suited for long term investors. For financial managers, the perceived liquidity of the firm's securities and its assets can affect investment, financing and dividend policy. In general, firms with less liquid assets and securities will tend to be more conservative when it comes to investing in long-term projects and paying dividends.