2. Futures and Forward Markets
2.1. Institutions

1. When a new trade is completed, what are the possible effects on the open interest?
(a) What happens if one party enters a long position and the other party enters a short position?

The open interest increases by one contract.

(b) What happens if one party closes a long position and the other party closes a short position?

The open interest decreases by one contract.

(c) What happens if one party enters a long (short) position and the other party closes a long (short) position?

The open interest does not change.

2. Futures and Forward Markets
2.1. Institutions

2. Today is March 15, 1998. The CU Business School just hired Professor N. Professor N will receive her first pay check in September for the amount of USD 5000. She will use this amount to offset her student loan balance in Canada. Professor N wants to hedge the foreign exchange risk. In the newspaper today, the spot exchange rate is quoted at USD 0.72 / CAD and the September forward rate is quoted at USD 0.7 / CAD.
(a) Does Professor N do better by hedging or by not hedging if the September spot rate turns out to be USD 0.65 / CAD? Compute Professor N's CAD amount credited against her student loan balance under the two alternatives.

hedge: $USD\ 5000 \times \frac{CAD\ 1}{USD\ 0.7} = CAD\ 7142.86$
do not hedge: $USD\ 5000 \times \frac{CAD\ 1}{USD\ 0.65} = CAD\ 7692.31$
Professor N does better by not hedging.

(b) Does Professor N do better by hedging or by not hedging if the September spot rate turns out to be USD 0.75 / CAD? Compute Professor N's CAD amount credited against her student loan balance under the two alternatives.

hedge: $USD\ 5000 \times \frac{CAD\ 1}{USD\ 0.7} = CAD\ 7142.86$
do not hedge: $USD\ 5000 \times \frac{CAD\ 1}{USD\ 0.75} = CAD\ 6666.67$
Professor N does better by hedging.

(c) Does Professor N take a long or short forward position in CAD?

Buying $CAD \equiv$ *long position in* CAD.

2. Futures and Forward Markets
2.1. Institutions

3. A company enters into two short heating oil futures contracts. Each contract is for the delivery of 42,000 gallons. The current futures price is 60 cents per gallon, the initial margin is $5000 per contract, and the maintenance margin is $3000 per contract.
(a) What price change would lead to a margin call?

$$\begin{aligned} 42,000(\$0.60 - F) &= -\$2000 \\ 25,200 - 42,000F &= -2000 \\ F &= \tfrac{27,200}{42,000} \\ &= 0.6476 \end{aligned}$$

i.e., a price change of $+\$0.0476$.

(b) What price change would lead to a $500 withdrawal from the margin account?

$$\begin{aligned} 42,000(\$0.60 - F) &= \$250 \\ 25,200 - 42,000F &= 250 \\ F &= \tfrac{24,950}{42,000} \\ &= 0.5940 \end{aligned}$$

i.e., a price change of $-\$0.0060$.

2. Futures and Forward Markets
2.1. Institutions

4. (Baby Hull 2.4) A company enters into a short futures contract to sell 5000 bushels of wheat for 250 cents per bushel. The initial margin is \$3000 and the maintenance margin is \$2000. What price change would lead to a margin call?

There is a margin call if \$1000 is lost on the contract.

$$\begin{aligned} 5000(-F + \$2.50) &= -\$1000 \\ 5000F - 12,500 &= 1000 \\ F &= \tfrac{13,500}{5000} \\ &= 2.70 \end{aligned}$$

This will happen if the futures price rises to \$2.70 per bushel.

Under what circumstances could \$1500 be withdrawn from the margin account?

$$\begin{aligned} 5000(-F + \$2.50) &= \$1500 \\ 5000F - 12,500 &= -1500 \\ F &= \tfrac{11,000}{5000} \\ &= 2.20 \end{aligned}$$

\$1500 can be withdrawn if the futures price falls to \$2.20 per bushel.

2. Futures and Forward Markets
2.1. Institutions

5. (Baby Hull 2.5, Papa Hull 2.12) An investor enters into two long futures contracts on frozen orange juice. Each contract is for the delivery of 15 000 pounds. The current futures price is 160 cents per pound, the initial margin is \$6000 per contract, and the maintenance margin is \$4500 per contract. What price change would lead to a margin call?

There is a margin call if \$1500 is lost on one contract.

$$\begin{aligned} 15,000(F - \$1.60) &= -\$1500 \\ 15,000F - 24,000 &= -1500 \\ F &= \tfrac{22,500}{15,000} \\ &= 1.50 \end{aligned}$$

This happens if the futures price falls to \$1.50 per pound.

Under what circumstances could \$2000 be withdrawn from the margin account?

\$2000 can be withdrawn from the margin account if the value of one contract rises by \$1000.

$$\begin{aligned} 15,000(F - \$1.60) &= \$1000 \\ 15,000F - 24,000 &= 1000 \\ F &= \tfrac{25,000}{15,000} \\ &= 1.6667 \end{aligned}$$

This happens if the futures price rises to \$1.6667 per pound.

2. Futures and Forward Markets
2.1. Institutions

6. (Papa Hull 2.10) Suppose that you enter into a short futures contract to sell July silver for \$5.20 per ounce on the New York Commodity Exchange. The size of the contract is 5000 ounces. The initial margin is \$4000 and the maintenance margin is \$3000. What change in the futures price will lead to a margin call?

$$\begin{aligned} 5,000(\$5.20 - F) &= -\$1000 \\ 26,000 - 5000F &= -1000 \\ F &= \tfrac{27{,}000}{5000} \\ &= 5.40 \end{aligned}$$

i.e., a price change of +\$0.20 per ounce.

What happens if you do not meet the margin call?

If you don't meet the margin call, your position gets liquidated.

2. Futures and Forward Markets
2.1. Institutions

7. (Baby Hull 2.9, Papa Hull 2.13) At the end of one day a clearinghouse member is long 100 contracts, and the settlement price is \$50 000 per contract. The original margin is \$2000 per contract. On the following day the member becomes responsible for clearing an additional 20 long contracts, entered into at a price of \$51 000 per contract. The settlement price at the end of this day is \$51 200. How much does the member have to add to its margin account with the exchange clearinghouse?

Nothing. It is the clearinghouse that must give the member \$84,000.

From the clearinghouse to the member:
100 old contracts with (\$51,200-\$50,000) settlement each.

From the member to the clearinghouse:
20 new contracts with \$2000 original margin each.

From the clearinghouse to the member:
20 new contracts with (\$51,200-\$51,000) settlement each.

2. Futures and Forward Markets
2.2. Pricing

1. The IBM stock is expected to pay a \$1.50 dividend in one month and every three months thereafter. The stock price is \$90 and the (continuously compounded) risk-free interest rate is four percent per annum for all maturities. You just entered into a long position in a nine-month forward contract on the stock.
(a) What is the delivery price specified in the contract?

$$\begin{aligned} I &= 1.5e^{-0.04\frac{1}{12}} + 1.5e^{-0.04\frac{4}{12}} + 1.5e^{-0.04\frac{7}{12}} \\ &= 1.4950 + 1.4801 + 1.4654 \\ &= 4.4405 \end{aligned}$$

$$\begin{aligned} K = F &= (S - I)e^{rT} \\ &= (90 - 4.4405)e^{0.04\frac{9}{12}} \\ &= 88.1652 \end{aligned}$$

(b) What is the initial value of your long position in the contract?

$$\begin{aligned} f &= (F - K)e^{-rT} \\ &= 0 \end{aligned}$$

(c) Three months later, the stock price is \$92 and the interest rate is still four percent per annum for all maturities. What is the value of your long position in the forward contract?

$$\begin{aligned} I &= 1.5e^{-0.04\frac{1}{12}} + 1.5e^{-0.04\frac{4}{12}} \\ &= 1.4950 + 1.4801 \\ &= 2.9751 \end{aligned}$$

2. Futures and Forward Markets
2.2. Pricing

1. (c) (Continued)

$$\begin{aligned} F &= (S-I)e^{rT} \\ &= (92-2.9751)e^{0.04\frac{6}{12}} \\ &= 90.8233 \end{aligned}$$

$$\begin{aligned} f &= (F-K)e^{-rT} \\ &= (90.8233-88.1652)e^{-0.04\frac{6}{12}} \\ &= 2.6055 \end{aligned}$$

2. Futures and Forward Markets
2.2. Pricing

2. The U.S. (annually compounded) risk-free interest rate is 4.50 percent per annum. You observe the following futures quotes in the Wall Street Journal today:

delivery	settle ($/Mexican peso)
Mar	.12632
June	.11990
Sep	.11410
Dec	.10950

(a) As you can see, futures prices decrease as the time to maturity increases. How do we call such a market?

inverted

(b) Based on this type of market, do you expect the Mexican (continuously compounded) risk-free interest rate to be higher or lower than the U.S. (continuously compounded) risk-free interest rate?

$$\begin{aligned} F &= Se^{(r-r_f)T} \\ \frac{\partial F}{\partial T} &= Se^{(r-r_f)T}(r-r_f) \\ &= F(r-r_f) < 0 \iff r < r_f \end{aligned}$$

(c) What is the Mexican (continuously compounded) risk-free interest rate?

$$\begin{aligned} \left(1+\frac{r_m}{m}\right)^{mT} &= e^{r_c T} \\ \left(1+\frac{0.045}{1}\right)^{1} &= e^{r_c} \\ r_c &= \log 1.045 = 0.044 \end{aligned}$$

2. Futures and Forward Markets
2.2. Pricing

2. (c) (Continued)

$$\begin{aligned} \frac{\partial F}{\partial T} &= F(r - r_f) \\ \frac{0.10950 - 0.12632}{0.75} &= 0.12632(0.044 - r_f) \\ r_f &= 0.2215 \end{aligned}$$

2. Futures and Forward Markets
2.2. Pricing

3. You use copper in your production process. On March 1, 1999, you bought a few tons of copper because you anticipate the possibility of a shortage in the next five months. You paid 88 cents per pound while the six-month futures price was quoted at 85 cents per pound. The (continuously compounded) risk-free interest rate is five percent per annum and the storage cost, as a proportion of the spot price, is two percent per annum.
(a) What is the convenience yield?

$$\begin{aligned} F &= Se^{(r+u-y)T} \\ 0.85 &= 0.88e^{(0.05+0.02-y)\frac{6}{12}} \\ \log\frac{0.85}{0.88} &= (0.07-y)\frac{6}{12} \\ y &= 0.1394 \end{aligned}$$

(b) Suppose that you also take a short position in the six-month forward contract to sell some of that copper in September 99. The New York Mercantile Exchange specifies that you can make delivery anytime during that month. Will you choose to deliver on the first day or the last day of September?

$$\begin{aligned} \frac{\partial F}{\partial T} &= Se^{(r+u-y)T}(r+u-y) \\ &= F(r+u-y) < 0 \end{aligned}$$

since $r+u-y=-0.0694$.

The value of your short position $f=(K-F)e^{-rT}$ *is larger when* F *is small. You thus choose to deliver on the last day.*

2. Futures and Forward Markets
2.2. Pricing

4. (Baby Hull 3.4, Papa Hull 3.11) A one-year long forward contract on a nondividend-paying stock is entered into when the stock price is \$40 and the risk-free rate of interest is 10 percent per annum with continuous compounding.
(a) What are the forward price and the initial value of the contract?

$$\begin{aligned} F &= Se^{rT} \\ &= 40e^{0.1\frac{12}{12}} \\ &= 44.21 \end{aligned}$$

$$\begin{aligned} f &= (F-K)e^{-rT} \\ &= 0 \end{aligned}$$

(b) Six months later, the price of the stock is \$45 and the risk-free interest rate is still 10 percent. What are the forward price and the value of the forward contract?

$$\begin{aligned} F &= Se^{rT} \\ &= 45e^{0.1\frac{6}{12}} \\ &= 47.31 \end{aligned}$$

$$\begin{aligned} f &= (F-K)e^{-rT} \\ &= (47.31-44.21)e^{-0.1\frac{6}{12}} \\ &= 2.95 \end{aligned}$$

2. Futures and Forward Markets
2.2. Pricing

5. (Baby Hull 3.5) A stock is expected to pay a dividend of $1 per share in two months and in five months. The stock price is $50 and the risk-free rate of interest is 8 percent per annum with continuous compounding for all maturities. An investor has just taken a short position in a six-month forward contract on the stock.
(a) What are the forward price and the initial value of the forward contract?

$$\begin{aligned} I &= 1e^{-0.08\frac{2}{12}} + 1e^{-0.08\frac{5}{12}} \\ &= 0.9868 + 0.9672 \\ &= 1.9540 \end{aligned}$$

$$\begin{aligned} F &= (S - I)e^{rT} \\ &= (50 - 1.9540)e^{0.08\frac{6}{12}} \\ &= 50.01 \end{aligned}$$

$$\begin{aligned} f &= (K - F)e^{-rT} \\ &= 0 \end{aligned}$$

(b) Three months later, the price of the stock is $48 and the risk-free rate of interest is still 8 percent per annum. What are the forward price and the value of the short position in the forward contract?

$$\begin{aligned} I &= 1e^{-0.08\frac{2}{12}} \\ &= 0.9868 \end{aligned}$$

$$\begin{aligned} F &= (S - I)e^{rT} \\ &= (48 - 0.9868)e^{0.08\frac{3}{12}} \\ &= 47.96 \end{aligned}$$

2. Futures and Forward Markets
2.2. Pricing

5. (b) (Continued)

$$\begin{aligned} f &= (K - F)e^{-rT} \\ &= (50.01 - 47.96)e^{-0.08\frac{3}{12}} \\ &= 2.01 \end{aligned}$$

2. Futures and Forward Markets
2.2. Pricing

6. (Baby Hull 3.7, Papa Hull 3.13) Assume that the risk-free interest rate is 9 percent per annum with continuous compounding and that the dividend yield on a stock index varies throughout the year. In February, May, August, and November, it is 5 percent per annum. In other months, it is 2 percent per annum. Suppose that the value of the index on July 31, 1997 is 300. What is the futures price for a contract deliverable on December 31, 1997?

average dividend yield $\frac{5+2+2+5+2)}{5} = 3.2\%$

$$\begin{aligned} F &= Se^{(r-q)T} \\ &= 300e^{(0.09-0.032)\frac{5}{12}} \\ &= 307.34 \end{aligned}$$

2. Futures and Forward Markets
2.2. Pricing

7. (Baby Hull 3.9, Papa Hull 3.15) Estimate the difference between risk-free rates of interest in Japan and the United States with the following information:

delivery	settle ($/yen)
Dec	.9267
Mar	.9393
June	.9519

$$\begin{aligned} F &= Se^{(r-r_f)T} \\ \frac{\partial F}{\partial T} &= Se^{(r-r_f)T}(r-r_f) \\ &= F(r-r_f) \\ \frac{0.9519-0.9267}{0.5} &= 0.9267(r-r_f) \\ (r-r_f) &= 0.054 \end{aligned}$$

2. Futures and Forward Markets
2.3. Hedging Strategies

1. You want to hedge the exposure of your stock portfolio to market risk (where the market is proxied by the S&P 500 index) over the next four months. The value of your portfolio is \$50,000,000 and its β is 1.2. You use five-month S&P 500 futures contract to hedge your portfolio. One futures contract delivers 500 times the index value. The value of the S&P 500 index is \$375 and its dividend yield is three percent per annum. The risk-free interest rate is six percent per annum.
(a) How many contracts do you short?

$$\begin{aligned} F &= Se^{(r-q)T} \\ &= 375e^{(0.06-0.03)\frac{5}{12}} \\ &= 379.72 \end{aligned}$$

$$\begin{aligned} N &= \beta\frac{N_S S}{Q_F F} \\ &= 1.2\frac{50{,}000{,}000}{379.72\times 500} \\ &= 316.02 \end{aligned}$$

You short 316 contracts.

After four months, the price of the S&P 500 index drops to \$300.
(b) What is the value of your futures position?

$$\begin{aligned} F &= Se^{(r-q)T} \\ &= 300e^{(0.06-0.03)\frac{1}{12}} \\ &= 300.75 \end{aligned}$$

$$\begin{aligned} f &= 316\times 500(-300.75+379.72)e^{-0.06\frac{1}{12}} \\ &= 12,415,029.41 \end{aligned}$$

2. Futures and Forward Markets
2.3. Hedging Strategies

1. (c) Using the CAPM equation $E[r_p] - r = \beta(E[r_m] - r)$ where r_p is the portfolio rate of return, r is the riskfree rate, and r_m is the market rate of return, what is the expected value of your portfolio? (Simplify your calculations by continuing to use continously compounded rates in the CAPM equation.)

$$\begin{aligned} r = & \quad 0.06\tfrac{4}{12} = 0.02 \\ q = & \quad 0.03\tfrac{4}{12} = 0.01 \\ E[r_m] = & \quad \tfrac{300-375}{375} + 0.01 = -0.19 \end{aligned}$$

$$\begin{aligned} E[r_p] = & \quad r + \beta(E[r_m] - r) \\ = & \quad 0.02 + 1.2(-0.19 - 0.02) \\ = & \quad -0.232 \end{aligned}$$

The expected value is $50,000,000(1 + E[r_p]) = 38,400,000$

(d) What is the expected value of your total position? How close is it to the perfect hedge?

total position: $12,415,029.41 + 38,400,000 = 50,815,029.41$

perfect hedge: $50,000,000e^{0.06\frac{4}{12}} = 51,010,067$

There is a difference of $195,037.59$ *or 0.3% of the initial portfolio value.*

2. Futures and Forward Markets
2.3. Hedging Strategies

2. It is September 15. A company has a portfolio of stocks worth \$5 million. The beta of the portfolio is 0.7. The company would like to use December S&P500 futures contract to change the β of the portfolio to 0.5 during the period of September 15 to November 15. The S&P 500 futures price (per unit of the index) is currently \$200 and one contract is for the delivery of 500 times the index. How many contracts should the company enter into? Should the company take a long or a short position in these contracts?

$$\begin{aligned} N &= (\beta_F - \beta)\frac{N_S S}{Q_F F} \\ &= (0.5 - 0.7)\frac{5{,}000{,}000}{200 \times 500} \\ &= -10 \end{aligned}$$

The company should short 10 contracts.

2. Futures and Forward Markets
2.3. Hedging Strategies

3. The standard deviation of monthly changes in the spot price of wheat is 0.8 (in cents per bushel). The standard deviation of monthly changes in the futures price of wheat is 0.9. The correlation between futures price changes and spot price changes is 0.5. It is now August 1. A wheat farmer is committed to sell 100,000 bushels on September 15. The farmer wants to use October wheat futures contracts to hedge the risk. Each contract is for the delivery of 5000 bushels of wheat. How many contracts should the farmer enter into?

$$\begin{aligned} h &= \rho\frac{\sigma_S}{\sigma_F} \\ &= 0.5\frac{0.8}{0.9} \\ &= 0.\bar{4} \end{aligned}$$

$$\begin{aligned} N &= \frac{hN_S}{Q_F} \\ &= \frac{0.\bar{4}100{,}000}{5000} \\ &= 8.\bar{8} \end{aligned}$$

The farmer should short 9 contracts.

2. Futures and Forward Markets
2.3. Hedging Strategies

4. The following table gives data on monthly changes in the spot price and the futures price for a certain commodity. Use the data to calculate the minimum variance hedge ratio.

Spot Price Change	Futures Price Change
-0.35	-0.44
$+0.79$	$+0.60$
$+0.04$	-0.06

$$\begin{aligned}
\sum x &= \quad 0.48 \\
\sum y &= \quad 0.1 \\
\sum x^2 &= \quad 0.7482 \\
\sum y^2 &= \quad 0.5572 \\
\sum xy &= \quad 0.6256
\end{aligned}$$

$$\begin{aligned}
\sigma_x &= \sqrt{\tfrac{0.7482}{2} - \tfrac{0.48^2}{3\times 2}} = 0.5794 \\
\sigma_y &= \sqrt{\tfrac{0.5572}{2} - \tfrac{0.1^2}{3\times 2}} = 0.5262 \\
\rho &= \frac{3(0.6256)-(0.48)(0.1)}{\sqrt{\{3(0.7482)-0.48^2\}\{3(0.5572)-0.1^2\}}} = 0.9997
\end{aligned}$$

$h = \rho\frac{\sigma_S}{\sigma_F} = 1.1007$

2. Futures and Forward Markets
2.3. Hedging Strategies

5. (Baby Hull 4.9, Papa Hull 2.25) The standard deviation of monthly changes in the spot price of live cattle is (in cents per pound) 1.2. The standard deviation of monthly changes in the futures price of live cattle for the closest contract is 1.4. The correlation between the futures price changes and the spot price changes is 0.7. It is now October 15. A beef producer is committed to purchasing 200,000 pounds of live cattle on November 15. The producer wants to use the December live-cattle futures contracts to hedge its risk. Each contract is for the delivery of 40,000 pounds of cattle. What strategy should the beef producer follow?

$$\begin{aligned} h &= \rho\frac{\sigma_S}{\sigma_F} \\ &= 0.7\frac{1.2}{1.4} \\ &= 0.6 \end{aligned}$$

$$\begin{aligned} N &= \frac{hN_S}{Q_F} \\ &= \frac{0.6200{,}000}{40{,}000} \\ &= 3 \end{aligned}$$

The beef producer should long 3 contracts.

2. Futures and Forward Markets
2.3. Hedging Strategies

6. (Baby Hull 4.11) On July 1 an investor holds 50,000 shares of a certain stock. The market price is \$30 per share. The investor is interested in hedging against movements in the market over the next two months and decides to use the December NYSE Index futures contract. The futures price is currently 150 and one contract is for the delivery of \$500 times the index. The beta of the stock is 1.3. What strategy should the investor follow?

$$\begin{aligned} N &= \beta \frac{N_S S}{Q_F F} \\ &= 1.3 \frac{50{,}000 \times 30}{500 \times 150} \\ &= 26 \end{aligned}$$

The investor should short 26 contracts.

2. Futures and Forward Markets
2.3. Hedging Strategies

7. (Baby Hull 4.12) It is July 16. A company has a portfolio of stocks worth \$10 million. The beta of the portfolio is 1.0. The company would like to use the CME December futures contract on the S&P 500 to change the beta of the portfolio to 0.5 during the period July 16 to November 16. The contract's futures price is currently 400. What should the company do?

$$\begin{aligned} N &= (\beta_F - \beta)\frac{N_S S}{Q_F F} \\ &= (0.5 - 1)\frac{10{,}000{,}000}{500 \times 400} \\ &= -25 \end{aligned}$$

The company should short 25 contracts.

2. Futures and Forward Markets
2.3. Hedging Strategies

8. (Baby Hull 4.13) Suppose that in the previous problem, the company would like to change the beta of the portfolio to 1.5. What should it do?

$$\begin{aligned} N &= (\beta_F - \beta)\frac{N_S S}{Q_F F} \\ &= (1.5 - 1)\frac{10{,}000{,}000}{500 \times 400} \\ &= 25 \end{aligned}$$

The company should long 25 contracts.

3. Swaps

1. Compute forward rates using the following zero rates:

Year to Maturity	Zero Rate
1	0.060
2	0.058
3	0.055

There are three forward rates:
(1) *the rate from year one to year two,*
(2) *the rate from year two to year three, and*
(3) *the rate from year one to year three.*

$$r_f = \frac{r_2 t_2 - r_1 t_1}{t_2 - t_1}$$

$$(1)\ r_f = \frac{0.058 \times 2 - 0.060 \times 1}{2-1} = 0.056$$

$$(2)\ r_f = \frac{0.055 \times 3 - 0.058 \times 2}{3-2} = 0.049$$

$$(3)\ r_f = \frac{0.055 \times 3 - 0.060 \times 1}{3-1} = 0.0525$$

3. Swaps

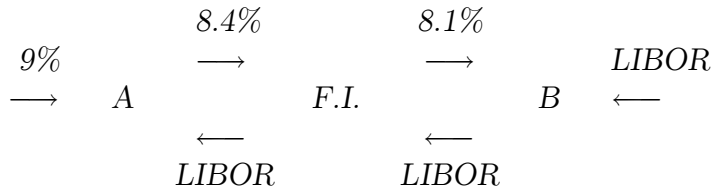
2. Company A would like to receive investment income at a floating rate, while Company B would like to receive investment income at a fixed rate. The rates offered on the fixed and floating markets are:

Company	Fixed Rate	Floating Rate
A	0.090	LIBOR
B	0.075	LIBOR

(a) What are the gains from a swap?

0.015

(b) The financial institution charges a 0.003 spread to arrange a swap for Company A and Company B. Design a swap that will appear equally attractive to the two companies.



A receives: *9%*
LIBOR
-8.4%
LIBOR+0.6%

B receives: *LIBOR*
-LIBOR
8.1%
7.5%+0.6%

3. Swaps

3. Under the terms of a currency swap, a financial institution receives seven percent per annum on a $10 million principal and pays four percent per annum on a principal of 30 million German marks. The dollar risk-free interest rate is eight percent, the mark risk-free interest rate is six percent, and a dollar can be currently exchanged for 2.5 marks. Payments are exchanged every year. The swap will last for two more years. Compute the value of the swap to the financial institution.

Value with bonds:

$$\begin{aligned}
B &= 0.7e^{-0.08\times 1} + 10.7e^{-0.08\times 2} \\
&= \$9,764,100 \\
B_f &= 1.2e^{-0.06\times 1} + 31.2e^{-0.06\times 2} \\
&= DM28,802,000 \\
V^{swap} &= B - B_f S \\
&= \$9,764,100 - DM28,802,000 \times \tfrac{\$1}{DM2.5} \\
&= -\$1,756,700
\end{aligned}$$

Value with forwards:

$$\begin{aligned}
F &= Se^{(r-r_f)T} \\
F_{1yr} &= \tfrac{1}{2.5}e^{(0.08-0.06)\times 1} \\
&= 0.4081 \\
F_{2yr} &= \tfrac{1}{2.5}e^{(0.08-0.06)\times 2} \\
&= 0.4163 \\
V_1 &= (0.07 \times 10 - 0.04 \times 30 \times 0.4081)e^{-0.08\times 1} \\
&= \$194,100 \\
V_2 &= (1.07 \times 10 - 1.04 \times 30 \times 0.4163)e^{-0.08\times 2} \\
&= -\$1,950,200 \\
V^{swap} &= V_1 + V_2 = -\$1,756,100
\end{aligned}$$

3. Swaps

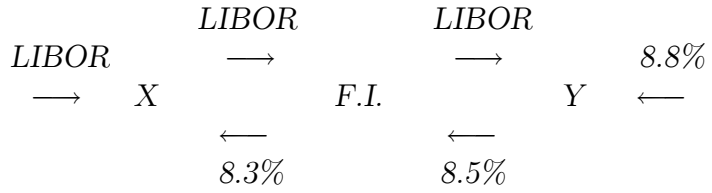
4. (Baby Hull 6.1, Papa Hull 5.8) Companies X and Y have been offered the following rates per annum on a $5 million 10-year investment:

Company	Fixed Rate	Floating Rate
X	8%	LIBOR
Y	8.8%	LIBOR

Company X requires a fixed-rate investment; company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and that will appear equally attrative to X and Y.

gains from swap: $|0.8\% - 0| = 0.8\%$

net gains $\frac{0.8\%-0.2\%}{2} = 0.3\%$



X receives: *LIBOR*
8.3%
-LIBOR

8.3% > 8% w/o swap

Y receives: *8.8%*
LIBOR
-8.5%

LIBOR +0.3% > LIBOR w/o swap

3. Swaps

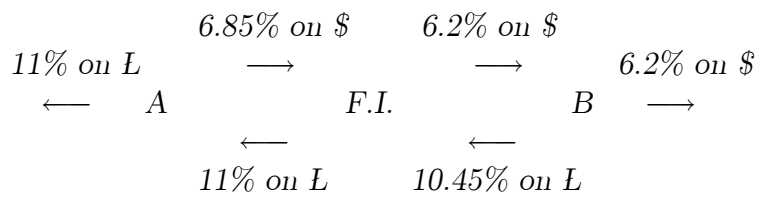
5. (Baby Hull 6.2, Papa Hull 5.18) Company A, a British manufacturer, wishes to borrow U.S. dollars at a fixed rate of interest. Company B, a U.S. multinational, wishes to borrow sterling at a fixed rate of interest. They have been quoted the following rates per annum (adjusted for tax effects):

Company	Sterling	U.S. Dollars
A	11%	7%
B	10.6%	6.2%

Design a swap that will net a bank, acting as intermediary, 10 basis points per annum and that will produce a gain of 15 basis points per annum for each of the two companies.

gains from swap: $|0.4\% - 0.8\%| = 0.4\%$

net gains $\frac{0.4\%-0.1\%}{2} = 0.15\%$



A pays: *11% on Ł*
6.85% on $
-11% on Ł
6.85% on $ < 7% w/o swap

B pays: *6.2% on $*
10.45% on Ł
-6.2% on $
10.45% on Ł< 10.6% w/o swap

3. Swaps

6. (Baby Hull 6.3, Papa Hull 5.19) Under the terms of an interest-rate swap, a financial institution has agreed to pay 10 percent per annum and to receive three-month LIBOR in return on a notional principal of $100 million with payments being exchanged every three months. The swap has a remaining life of 14 months. The average bid-ask fixed rate currently being swapped for three-month LIBOR is 12 percent per annum for all maturities. The three-month LIBOR one month ago was 11.8 percent per annum. All rates are compounded quarterly. What is the value of the swap?

Value with bonds:

$$\begin{aligned} e^{r_c T} &= \left(1 + \tfrac{r_m}{m}\right)^{mT} \\ e^{r_c} &= \left(1 + \tfrac{0.12}{4}\right)^{4} \\ r_c &= 4 \log 1.03 \\ &= 0.1182 \end{aligned}$$

$Q = \$100M$

$k = 0.1 \times \frac{3}{12} \times \$100M = \$2.5M$

$k^* = 0.118 \times \frac{3}{12} \times \$100M = \$2.95M$

$$\begin{aligned} B_{fx} &= \sum_{i=1}^{n} k e^{-r_i t_i} + Q e^{-r_n t_n} \\ &= 2.5e^{-0.1182\frac{2}{12}} + 2.5e^{-0.1182\frac{5}{12}} + 2.5e^{-0.1182\frac{8}{12}} + 2.5e^{-0.1182\frac{11}{12}} + 102.5e^{-0.1182\frac{14}{12}} \\ &= \$98.68M \\ B_{fl} &= (k^* + Q)e^{-r_1 t_1} \\ &= 102.95e^{-0.1182\frac{2}{12}} \\ &= \$100.96M \\ V^{swap} &= B_{fl} - B_{fx} \\ &= \$2.26M \end{aligned}$$

6. (Continued)

Value with FRAs:

$$\begin{aligned}
V_1 &= \tfrac{3}{12} \times \$100M(0.118 - 0.1)e^{-0.1182\frac{2}{12}} = \$441,200 \\
V_2 &= \tfrac{3}{12} \times \$100M(0.12 - 0.1)e^{-0.1182\frac{5}{12}} = \$476,000 \\
V_3 &= \tfrac{3}{12} \times \$100M(0.12 - 0.1)e^{-0.1182\frac{8}{12}} = \$462,100 \\
V_4 &= \tfrac{3}{12} \times \$100M(0.12 - 0.1)e^{-0.1182\frac{11}{12}} = \$448,700 \\
V_5 &= \tfrac{3}{12} \times \$100M(0.12 - 0.1)e^{-0.1182\frac{14}{12}} = \$435,600 \\
V^{swap} &= V_1 + V_2 + V_3 + V_4 + V_5 = \$2.26M
\end{aligned}$$

4. Options Markets
4.1. Institutions

1. (Baby Hull 7.1) Suppose that a European call option to buy a share for \$50 costs \$2.50 and is held until maturity.

$X = \$50$
$c = \$2.50$

Under what circumstances will the holder of the option make a profit?

The holder of the option makes a profit if the share price S_T is above \$52.50 at maturity.

$$\begin{aligned} \max\{S_T - X, 0\} - c &> 0 \\ S_T - 50 - 2.50 &> 0 \\ S_T &> 52.50 \end{aligned}$$

Under what circumstances will the option be exercised?

The option is exercised if the stock price at maturity is above \$50.

$$\begin{aligned} \max\{S_T - X, 0\} &> 0 \\ S_T - 50 &> 0 \\ S_T &> 50 \end{aligned}$$

Draw a diagram illustrating how the profit from a long position in the option depends on the stock price at maturity of the option.

(You get the picture, don't you?)

4. Options Markets
4.1. Institutions

2. (Baby Hull 7.2) Suppose that a European put option to sell a share for \$60 costs \$4 and is held until maturity.

$X = \$60$
$p = \$4$

Under what circumstances will the seller of the option make a profit?

The seller of the option makes a profit if the share price S_T is above \$56 at maturity.

$$\begin{aligned} -\max\{X - S_T, 0\} + p &> 0 \\ -60 + S_T + 4 &> 0 \\ S_T &> 56 \end{aligned}$$

Under what circumstances will the option be exercised?

The option is exercised if the stock price at maturity is below \$60.

$$\begin{aligned} \max\{X - S_T, 0\} &> 0 \\ 60 - S_T &> 0 \\ S_T &< 60 \end{aligned}$$

Draw a diagram illustrating how the profit from a short position in the option depends on the stock price at maturity of the option.

(Ask me if you don't get it.)

4. Options Markets
4.1. Institutions

3. (Baby Hull 7.10, Papa Hull 6.7) Consider an exhange-traded call option contract to buy 500 shares with a strike price of \$40 and maturity in four months.

$N = 500$
$X = 40$

Explain how the terms of the option contract change when there is:
(a) a ten percent stock dividend

A 10% stock dividend is equivalent as a $n = 110$ *for* $m = 100$ *stock split.*
The strike price changes to $\frac{mX}{n} = \frac{100 \times 40}{110} = 36.36$.
The number of options changes to $\frac{nN}{m} = \frac{110 \times 500}{100} = 550$.

(b) a ten percent cash dividend

The terms of an option contract are not adjusted for cash dividends.

(c) a four-for-one stock split

$n = 4$ *for* $m = 1$ *split*
The strike price changes to $\frac{mX}{n} = \frac{1 \times 40}{4} = 10$.
The number of options changes to $\frac{nN}{m} = \frac{4 \times 500}{1} = 2000$.

4. Options Markets
4.2. Basic Properties

1. (Baby Hull 8.1, Papa Hull 7.10) What is a lower bound for the price of a six-month call option on a non-dividend-paying stock when the stock price is \$80, the strike price is \$75, and the risk-free interest rate is 10 percent per annum?

$T = \frac{6}{12}$
$D = 0$
$S = 80$
$X = 75$
$r = 0.1$

$$\begin{aligned} c \geq & \quad S - Xe^{-rT} \\ \geq & \quad 80 - 75e^{-0.1\frac{6}{12}} \\ \geq & \quad 8.66 \end{aligned}$$

4. Options Markets
4.2. Basic Properties

2. (Baby Hull 8.2, Papa Hull 7.11) What is a lower bound for the price of a two-month European put option on a non-dividend-paying stock when the stock price is \$58, the strike price is \$65, and the risk-free interest rate is 5 percent per annum?

$T = \frac{2}{12}$
$D = 0$
$S = 58$
$X = 65$
$r = 0.05$

$$\begin{aligned} p \geq & \quad Xe^{-rT} - S \\ \geq & \quad 65e^{-0.05\frac{2}{12}} - 58 \\ \geq & \quad 6.46 \end{aligned}$$

4. Options Markets
4.2. Basic Properties

3. (Baby Hull 8.3, Papa Hull 7.12) A four-month European call option on a dividend-paying stock is currently selling for \$5. The stock price is \$64, the strike price is \$60, and a dividend of \$0.80 is expected in one month. The risk-free interest rate is 12 percent per annum for all maturities. What opportunities are there for an arbitrageur?

$T = \frac{4}{12}$
$c = 5$
$S = 64$
$X = 60$
$r = 0.12$
$D = 0.8e^{-0.12\frac{1}{12}} = 0.79$

$$\begin{aligned} c &\geq S - D - Xe^{-rT} \\ 5 &\geq 64 - 0.79 - 60e^{-0.12\frac{4}{12}} \\ 5 &\geq 5.56 \end{aligned}$$

	0	$\frac{1}{12}$	$\frac{4}{12}$
buy call	-5		$+\max\{S_T - 60, 0\}$
sell stock	$+64$		$-S_T$
	-0.79	$+0.8$	
invest	$-60e^{-0.12\frac{4}{12}}$		$+60$
	$+0.56$		*if* $S_T > 60$, *profit*$_T = S_T - 60 - S_T + 60 = 0$
	upfront profit!		*if* $S_T < 60$, *profit*$_T = 0 - S_T + 60 > 0$

4. Options Markets
4.2. Basic Properties

4. (Baby Hull 8.4, Papa Hull 7.13) A one-month European put option on a non-dividend-paying stock is currently selling for \$2.50. The stock price is \$47, the strike price is \$50, and the risk-free interest rate is 6 percent per annum. What opportunities are there for an arbitrageur?

$T = \frac{1}{12}$
$D = 0$
$p = 2.50$
$S = 47$
$X = 50$
$r = 0.06$

$$\begin{aligned} p \geq & \quad Xe^{-rT} - S \\ 2.50 \geq & \quad 50e^{-0.06\frac{1}{12}} - 47 \\ 2.50 \geq & \quad 2.75 \end{aligned}$$

	0	$\frac{1}{12}$
buy put	-2.50	$+\max\{50 - S_T, 0\}$
buy stock	-47	$+S_T$
borrow	$+49.50$	$-49.50e^{0.06\frac{1}{12}} = -49.75$
	0	*if* $S_T < 50$, $profit_T = 50 - S_T + S_T - 49.75 = 0.25$
		if $S_T > 50$, $profit_T = -49.75 + 0 - S_T > 0.25$

4. Options Markets
4.2. Basic Properties

5. (Baby Hull 8.6, Papa Hull 7.15) The price of a European call which expires in six months and has a strike price of \$30 is \$2. The underlying stock price is \$29, and a dividend of \$0.50 is expected in two months and again in five months. The term structure is flat, with all risk-free interest rates being 10 percent. What is the price of a European put option that expires in six months and has a strike price of \$30?

$c = 2$
$T = \frac{6}{12}$
$X = 30$
$S = 29$
$r = 0.1$
$D = 0.5e^{-0.1\frac{2}{12}} + 0.5e^{-0.1\frac{5}{12}} = 0.97$

$$\begin{aligned} c + D + Xe^{-rT} &= p + S \\ 2 + 0.97 + 30e^{-0.1\frac{6}{12}} &= p + 29 \\ p &= 2.51 \end{aligned}$$

4. Options Markets
4.2. Basic Properties

6. (Baby Hull 8.7, Papa Hull 7.16) Explain carefully the arbitrage opportunities in the previous problem if the European put price is \$3.

if $p = 3 > 2.51$, then there exists an arbitrage opportunity.

	0	$\frac{2}{12}$	$\frac{5}{12}$	$\frac{6}{12}$
buy call	-2			$+\max\{S_T - 30, 0\}$
invest	$-30e^{-0.1\frac{6}{12}}$			$+30$
write put	$+3$			$-\max\{30 - S_T, 0\}$
short share	$+29$			$-S_T$
	-0.97	*+0.5*	*+0.5*	
	$+0.49$			*if $S_T > 30$, profit$_T = 0$*
	upfront profit!			*if $S_T < 30$, profit$_T = 0$*

4. Options Markets
4.2. Basic Properties

7. (Baby Hull 8.8, Papa Hull 7.17) The price of an American call on a non-dividend-paying stock is \$4. The stock price is \$31, the strike price is \$30, and the expiration date is in three months. The risk-free interest rate is 8 percent. Derive upper and lower bounds for the price of an American put on the same stock with the same strike price and expiration date.

$C = 4$
$D = 0$
$S = 31$
$X = 30$
$T = \frac{3}{12}$
$r = 0.08$

$$
\begin{aligned}
S - X < C - P < S - Xe^{-rT} \\
31 - 30 < 4 - P < 31 - 30e^{-0.08\frac{3}{12}} \\
1 < 4 - P < 1.59 \\
-3 < -P < -2.41 \\
3 > P > 2.41
\end{aligned}
$$

4. Options Markets
4.3. Trading Strategies

1. (Baby Hull 9.3, Papa Hull 8.10) Suppose that put options on a stock with strike prices \$30 and \$35 cost \$4 and \$7 respectively. Construct a table that shows the profit and payoff. How can the options be used to create
(a) a bull spread?

bull spread: buy a \$30 put and write a \$35 put
upfront flow: $-4+7=+3$

stock price	*payoff* $= +\max\{30-S_T,0\}-\max\{35-S_T,0\}$	*profit* =*payoff*+3
$S_T \geq 35$	$+0-0=0$	$0+3=3$
$30 \leq S_T < 35$	$+0-(35-S_T)=S_T-35$	$(S_T-35)+3=S_T-32$
$S_T < 30$	$+(30-S_T)-(35-S_T)=-5$	$-5+3=-2$

(b) a bear spread?

bear spread: buy a \$35 put and write a \$30 put
upfront flow: $-7+4=-3$

stock price	*payoff* $= +\max\{35-S_T,0\}-\max\{30-S_T,0\}$	*profit* =*payoff*−3
$S_T \geq 35$	$+0-0=0$	$0-3=-3$
$30 \leq S_T < 35$	$+(35-S_T)-0=35-S_T$	$(35-S_T)-3=32-S_T$
$S_T < 30$	$+(35-S_T)-(30-S_T)=5$	$5-3=2$

4. Options Markets
4.3. Trading Strategies

2. (Baby Hull 9.4, Papa Hull 8.20) Three put options on a stock have the same expiration date and strike prices of \$55, \$60, and \$65. The market prices are \$3, \$5, and \$8 respectively. Explain how a butterfly spread can be created. Construct a table showing the profit from the strategy.

butterfly spread: buy a \$55 put, write two \$60 put, and buy a \$65 put
upfront flow: $-3 + 2(5) - 8 = -1$

stock price	*payoff* $= +\max\{55 - S_T, 0\} - 2\max\{60 - S_T, 0\} + \max\{65 - S_T, 0\}$	*profit* =*payoff*-1
$S_T \geq 65$	$+0 - 2(0) + 0 = 0$	$0 - 1 = -1$
$60 \leq S_T < 65$	$+0 - 2(0) + (65 - S_T) = 65 - S_T$	$(65 - S_T) - 1 = 64 - S_T$
$55 \leq S_T < 60$	$+0 - 2(60 - S_T) + (65 - S_T) = S_T - 55$	$(S_T - 55) - 1 = S_T - 56$
$S_T < 55$	$+(55 - S_T) - 2(60 - S_T) + (65 - S_T) = 0$	$0 - 1 = -1$

For what range of stock prices would the butterfly spread lead to a loss?

The butterfly spread leads to a loss when the final stock price is greater than \$64 or less than \$56.

4. Options Markets
4.3. Trading Strategies

3. (Baby Hull 9.7, Papa Hull 8.12) A call with a strike price of \$60 costs \$6. A put with the same strike price and expiration date costs \$4. Construct a table that shows the profit from a straddle.

straddle: buy a call and a put
upfront flow: $-6-4=-10$

stock price	*payoff* $= +\max\{S_T-60,0\}+\max\{60-S_T,0\}$	*profit* =*payoff*-10
$S_T \geq 60$	$(S_T-60)+0=S_T-60$	$(S_T-60)-10=S_T-70$
$S_T < 60$	$0+(60-S_T)=60-S_T$	$(60-S_T)-10=50-S_T$

For what range of stock prices would the straddle lead to a loss?

The straddle leads to a loss when the final stock price is between \$50 and \$70.

4. Options Markets
4.4. Binomial Trees

1. (Baby Hull 10.1, Papa Hull 9.8) A stock price is currently \$50. It is known that at the end of two months it will be either \$53 or \$48. The risk-free interest rate is 10 percent per annum with continuous compounding. What is the value of a two-month European call option with a strike price of \$49?

$r = 0.1$
step= $\frac{2}{12}$
$X = 49$

53
$\max\{53 - 49, 0\} = 4$
↗
50
f?
↘
48
$\max\{48 - 49, 0\} = 0$

$$
\begin{aligned}
u &= \frac{53}{50} = 1.06 \\
d &= \frac{48}{50} = 0.96 \\
p &= \frac{e^{rT} - d}{u - d} = \frac{e^{0.1\frac{2}{12}} - 0.96}{1.06 - 0.96} = 0.5681 \\
1 - p &= 0.4319 \\
f &= (0.5681 \times 4 + 0.4319 \times 0)e^{-0.1\frac{2}{12}} = 2.23
\end{aligned}
$$

4. Options Markets
4.4. Binomial Trees

2. (Baby Hull 10.2, Papa Hull 9.9) A stock price is currently \$80. It is known that at the end of four months it will be either \$75 or \$85. The risk-free interest rate is 5 percent per annum with continuous compounding. What is the value of a four-month European put option with a strike price of \$80?

$r = 0.05$
step$= \frac{4}{12}$
$X = 80$

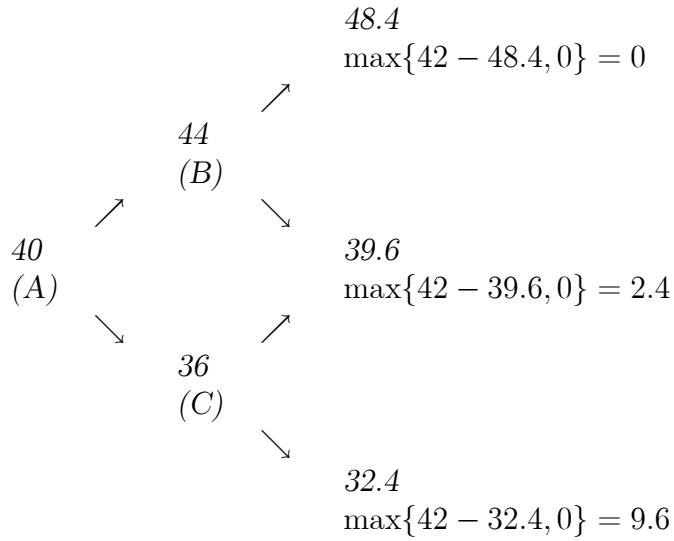
$$
\begin{array}{lll}
 & & \textit{85} \\
 & & \max\{80-85,0\} = 0 \\
 & \nearrow & \\
\textit{80} & & \\
\textit{f?} & & \\
 & \searrow & \\
 & & \textit{75} \\
 & & \max\{80-75,0\} = 5
\end{array}
$$

$$
\begin{aligned}
u &= \frac{85}{80} = 1.0625 \\
d &= \frac{75}{80} = 0.9375 \\
p &= \frac{e^{rT}-d}{u-d} = \frac{e^{0.05\frac{4}{12}}-0.9375}{1.0625-0.9375} = 0.6345 \\
1-p &= 0.3655 \\
f &= (0.6345 \times 0 + 0.3655 \times 5)e^{-0.05\frac{4}{12}} = 1.80
\end{aligned}
$$

4. Options Markets
4.4. Binomial Trees

3. (Baby Hull 10.7, Papa Hull 9.15) A stock price is currently \$40. Over each of the next two three-month periods it is expected to go up by 10 percent or down by 10 percent. The risk-free interest rate is 12 percent per annum with continuous compounding.
(a) What is the value of a six-month European put option with a strike price of \$42?

$r = 0.12$
$step = \frac{3}{12}$
$u = 1.1$
$d = 0.9$
$S = 40$
$X = 42$



$$
\begin{aligned}
p = & \quad \frac{e^{rT}-d}{u-d} = \frac{e^{0.12\frac{3}{12}}-0.9}{1.1-0.9} = 0.6523 \\
1-p = & \quad 0.3477 \\
f_{(B)} = & \quad (0.6523 \times 0 + 0.3477 \times 2.4)e^{-0.12\frac{3}{12}} = 0.81 \\
f_{(C)} = & \quad (0.6523 \times 2.4 + 0.3477 \times 9.6)e^{-0.12\frac{3}{12}} = 4.76 \\
f_{(A)} = & \quad (0.6523 \times 0.81 + 0.3477 \times 4.76)e^{-0.12\frac{3}{12}} = 2.12
\end{aligned}
$$

4. Options Markets
4.4. Binomial Trees

3. (Continued) (b) What is the value of a six-month American put option with a strike price of \$42?

- *Exercise at (B)? No*

  $\max\{42-44,0\} = 0 < 0.81$

- *Exercise at (C)? Yes*

  $\max\{42-36,0\} = 6 > 4.76$

  $f_{(A)} = (0.6523 \times 0.81 + 0.3477 \times 6)e^{-0.12\frac{3}{12}} = 2.54$

- *Exercise at (A)? NO*

  $\max\{42-40,0\} = 2 < 2.54$

4. Options Markets
4.5. Black-Scholes

1. (Baby Hull 11.9, Papa Hull 11.13) What is the price of a European call option on a non-dividend-paying stock when the stock price is \$52, the strike price is \$50, the risk-free interest rate is 12 percent per annum, the volatility is 30 percent per annum, and the time to maturity is three months?

$S = 52$
$X = 50$
$r = 0.12$
$\sigma = 0.3$
$T = \frac{3}{12}$

$$\begin{aligned} d_1 = & \frac{\log\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ = & \frac{\log\left(\frac{52}{50}\right) + \left(0.12 + \frac{0.3^2}{2}\right)\frac{3}{12}}{0.3\sqrt{\frac{3}{12}}} = 0.5365 \approx 0.54 \end{aligned}$$

$$\begin{aligned} d_2 = & d_1 - \sigma\sqrt{T} \\ = & 0.5365 - 0.3\sqrt{\frac{3}{12}} = 0.3865 \approx 0.39 \end{aligned}$$

$$\begin{aligned} N(d_1) = & N(0.54) \\ = & 0.7054 \end{aligned}$$

$$\begin{aligned} N(d_2) = & N(0.39) \\ = & 0.6517 \end{aligned}$$

$$\begin{aligned} c = & SN(d_1) - Xe^{-rT}N(d_2) \\ = & 52 \times 0.7054 - 50e^{-0.12\frac{3}{12}} \times 0.6517 \\ = & 5.06 \end{aligned}$$

4. Options Markets
4.5. Black-Scholes

2. (Baby Hull 11.10, Papa Hull 11.14) What is the price of a European put option on a non-dividend-paying stock when the stock price is \$69, the strike price is \$70, the risk-free interest rate is 5 percent per annum, the volatility is 35 percent per annum, and the time to maturity is six months?

$S = 69$
$X = 70$
$r = 0.05$
$\sigma = 0.35$
$T = \frac{6}{12}$

$$\begin{aligned} d_1 = & \frac{\log\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ = & \frac{\log\left(\frac{69}{70}\right) + \left(0.05 + \frac{0.35^2}{2}\right)\frac{6}{12}}{0.35\sqrt{\frac{6}{12}}} = 0.1666 \approx 0.17 \end{aligned}$$

$$\begin{aligned} d_2 = & d_1 - \sigma\sqrt{T} \\ = & 0.1666 - 0.35\sqrt{\frac{6}{12}} = -0.0809 \approx -0.08 \end{aligned}$$

$$\begin{aligned} N(-d_1) = & N(-0.17) \\ = & 0.4325 \end{aligned}$$

$$\begin{aligned} N(-d_2) = & N(0.08) \\ = & 0.5319 \end{aligned}$$

$$\begin{aligned} p = & Xe^{-rT}N(-d_2) - SN(-d_1) \\ = & 70e^{-0.05\frac{6}{12}} \times 0.5319 - 69 \times 0.4325 \\ = & 6.47 \end{aligned}$$

4. Options Markets
4.5. Black-Scholes

3. (Baby Hull 11.11, Papa Hull 11.27) Consider an option on a non-dividend-paying stock when the stock price is \$30, the exercise price is \$29, the risk-free interest rate is 5 percent per annum, the volatility is 25 percent per annum, and the time to maturity is four months.

$S = 30$
$X = 29$
$r = 0.05$
$\sigma = 0.25$
$T = \frac{4}{12}$

$$\begin{aligned} d_1 &= \frac{\log\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ &= \frac{\log\left(\frac{30}{29}\right) + \left(0.05 + \frac{0.25^2}{2}\right)\frac{4}{12}}{0.25\sqrt{\frac{4}{12}}} = 0.4225 \approx 0.42 \end{aligned}$$

$$\begin{aligned} d_2 &= d_1 - \sigma\sqrt{T} \\ &= 0.4225 - 0.25\sqrt{\frac{4}{12}} = 0.2782 \approx 0.28 \end{aligned}$$

$$\begin{aligned} N(d_1) &= N(0.42) \\ &= 0.6628 \end{aligned}$$

$$\begin{aligned} N(d_2) &= N(0.28) \\ &= 0.6103 \end{aligned}$$

$$\begin{aligned} N(-d_1) &= 1 - N(d_1) \\ &= 1 - 0.6628 \\ &= 0.3372 \end{aligned}$$

$$\begin{aligned} N(-d_2) &= 1 - N(d_2) \\ &= 1 - 0.6103 \\ &= 0.3897 \end{aligned}$$

4. Options Markets
4.5. Black-Scholes

3. (Continued)
(a) What is the price of the option if it is a European call?

$$\begin{aligned} c = & \quad SN(d_1) - Xe^{-rT}N(d_2) \\ = & \quad 30 \times 0.6628 - 29e^{-0.05\frac{4}{12}} \times 0.6103 \\ = & \quad 2.48 \end{aligned}$$

(b) What is the price of the option if it is an American call?

$C = c = 2.48$

(c) What is the price of the option if it is a European put?

$$\begin{aligned} p = & \quad Xe^{-rT}N(-d_2) - SN(-d_1) \\ = & \quad 29e^{-0.05\frac{4}{12}} \times 0.3897 - 30 \times 0.3372 \\ = & \quad 1.00 \end{aligned}$$

(d) Verify that put-call parity holds.

$$\begin{aligned} p + S = & \quad c + Xe^{-rT} \\ 1 + 30 = & \quad 2.48 + 29e^{-0.05\frac{4}{12}} \\ 31 = & \quad 31 \end{aligned}$$

4. Options Markets
4.5. Black-Scholes

4. (Baby Hull 11.12, Papa Hull 11.28) Assume that the stock in the previous problem is due to go ex-dividend in 1.5 months. The expected dividend is 50 cents.

$$D = 0.5e^{-0.05\frac{1.5}{12}} = 0.4969$$

$$\begin{aligned} d_1 = & \frac{\log\left(\frac{S-D}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ = & \frac{\log\left(\frac{30-0.4969}{29}\right)+\left(0.05+\frac{0.25^2}{2}\right)\frac{4}{12}}{0.25\sqrt{\frac{4}{12}}} = 0.3068 \approx 0.31 \end{aligned}$$

$$\begin{aligned} d_2 = & d_1 - \sigma\sqrt{T} \\ = & 0.3068 - 0.25\sqrt{\frac{4}{12}} = 0.1625 \approx 0.16 \end{aligned}$$

$$\begin{aligned} N(d_1) = & N(0.31) \\ = & 0.6217 \end{aligned}$$

$$\begin{aligned} N(d_2) = & N(0.16) \\ = & 0.5636 \end{aligned}$$

$$\begin{aligned} N(-d_1) = & 1 - N(d_1) \\ = & 1 - 0.6271 \\ = & 0.3783 \end{aligned}$$

$$\begin{aligned} N(-d_2) = & 1 - N(d_2) \\ = & 1 - 0.5636 \\ = & 0.4364 \end{aligned}$$

4. Options Markets
4.5. Black-Scholes

4. (Continued)
(a) What is the price of the option if it is a European call?

$$\begin{aligned} c = & \quad (S-D)N(d_1) - Xe^{-rT}N(d_2) \\ = & \quad (30 - 0.4969) \times 0.6217 - 29e^{-0.05\frac{4}{12}} \times 0.5636 \\ = & \quad 2.27 \end{aligned}$$

(b) What is the price of the option if it is a European put?

$$\begin{aligned} p = & \quad Xe^{-rT}N(-d_2) - (S-D)N(-d_1) \\ = & \quad 29e^{-0.05\frac{4}{12}} \times 0.4364 - (30 - 0.4969) \times 0.3783 \\ = & \quad 1.29 \end{aligned}$$

4. Options Markets
4.6. Applications

1. (Baby Hull 12.3, Papa Hull 12.8) Consider a stock index currently standing at 250. The dividend yield on the index is 4 percent per annum and the risk-free rate is 6 percent per annum. A three-month European call option on the index with a strike price of 245 is currently worth \$10. What is the value of a three-month put option on the index with a strike price of 245?

$S = 250$
$q = 0.04$
$r = 0.06$
$T = \frac{3}{12}$
$X = 245$
$c = 10$

$$\begin{aligned} p + Se^{-qT} &= c + Xe^{-rT} \\ p + 250e^{-0.04\frac{3}{12}} &= 10 + 245e^{-0.06\frac{3}{12}} \\ p &= 3.84 \end{aligned}$$

4. Options Markets
4.6. Applications

2. (Baby Hull 12.4, Papa Hull 12.15) The S&P index currently stands at 348 and has a volatility of 30 percent per annum. The risk-free rate of interest is 7 percent per annum and the index provides a dividend yield of 4 percent per annum. Calculate the value of a three-month European put with an exercise price of 350.

$S = 348$
$\sigma = 0.3$
$r = 0.07$
$q = 0.04$
$T = \frac{3}{12}$
$X = 350$

$$\begin{aligned} d_1 = & \frac{\log\left(\frac{S}{X}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ = & \frac{\log\left(\frac{348}{350}\right) + \left(0.07 - 0.04 + \frac{0.3^2}{2}\right)\frac{3}{12}}{0.3\sqrt{\frac{3}{12}}} = 0.0868 \approx 0.09 \end{aligned}$$

$$\begin{aligned} d_2 = & \quad d_1 - \sigma\sqrt{T} \\ = & \quad 0.0868 - 0.3\sqrt{\frac{3}{12}} = -0.0632 \approx -0.06 \end{aligned}$$

$$\begin{aligned} N(-d_1) = & \quad N(-0.09) \\ = & \quad 0.4641 \end{aligned}$$

$$\begin{aligned} N(-d_2) = & \quad N(0.06) \\ = & \quad 0.5239 \end{aligned}$$

$$\begin{aligned} p = & \quad Xe^{-rT}N(-d_2) - Se^{-qT}N(-d_1) \\ = & \quad 350e^{-0.07\frac{3}{12}} \times 0.5239 - 348e^{-0.04\frac{3}{12}} \times 0.4641 = 20.28 \end{aligned}$$

4. Options Markets
4.6. Applications

3. (Baby Hull 12.5, Papa Hull 12.42) Suppose that the spot price of the Canadian dollar is U.S.$0.75 and that the Canadian dollar/U.S. dollar exchange rate has a volatility of 4 percent per annum. The risk-free rates of interest in Canada and the United States are 9 percent and 7 percent per annum, respectively. Calculate the value of a European call option with an exercise price of $0.75 and an exercise date in 9 months.

$S = 0.75$
$\sigma = 0.04$
$r_f = 0.09$
$r = 0.07$
$X = 0.75$
$T = \frac{9}{12}$

$$\begin{aligned} d_1 &= \frac{\log\left(\frac{S}{X}\right) + \left(r - r_f + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ &= \frac{\log\left(\frac{0.75}{0.75}\right) + \left(0.07 - 0.09 + \frac{0.04^2}{2}\right)\frac{9}{12}}{0.04\sqrt{\frac{9}{12}}} = -0.4157 \approx -0.42 \end{aligned}$$

$$\begin{aligned} d_2 &= d_1 - \sigma\sqrt{T} \\ &= -0.4157 - 0.04\sqrt{\frac{9}{12}} = -0.4503 \approx -0.45 \end{aligned}$$

$$\begin{aligned} N(d_1) &= N(-0.42) \\ &= 0.3372 \end{aligned}$$

$$\begin{aligned} N(d_2) &= N(-0.45) \\ &= 0.3264 \end{aligned}$$

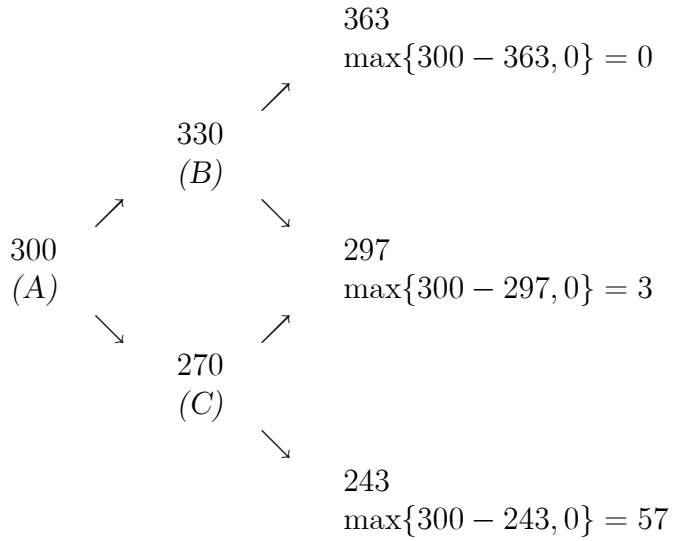
$$\begin{aligned} c &= Se - r_f T N(d_1) - Xe^{-rT} N(d_2) \\ &= 0.75e^{-0.09\frac{9}{12}} \times 0.3372 - 0.75e^{-0.07\frac{9}{12}} \times 0.3264 = 0.0041 \end{aligned}$$

4. Options Markets
4.6. Applications

4. (Baby Hull 12.8, Papa Hull 12.41) A stock index currently stands at 300. It is expected to increase or decrease by 10 percent over each of the next two time periods of three months. The risk-free interest rate is 8 percent and the dividend yield on the index is 3 percent. What is the value of a six-month put option on the index with a strike price of 300 if it is

(a) European?

$S = 300$
$u = 1.1$
$d = 0.9$
$step = \frac{3}{12}$
$r = 0.08$
$q = 0.03$
$X = 300$



$$p = \frac{e^{(r-q)T} - d}{u - d} = \frac{e^{(0.08-0.03)\frac{3}{12}} - 0.9}{1.1 - 0.9} = 0.5629$$

$$1 - p = 0.4371$$

4. Options Markets
4.6. Applications

4. (a) (Continued)

$$\begin{aligned} f_{(B)} = &\quad (0.5629 \times 0 + 0.4371 \times 3)e^{-0.08\frac{3}{12}} = 1.29 \\ f_{(C)} = &\quad (0.5629 \times 3 + 0.4371 \times 57)e^{-0.08\frac{3}{12}} = 26.08 \\ f_{(A)} = &\quad (0.5629 \times 1.29 + 0.4371 \times 26.08)e^{-0.08\frac{3}{12}} = 11.89 \end{aligned}$$

(b) American?

- *Exercise at (B)? No*
  $\max\{300 - 330, 0\} = 0 < 1.29$
- *Exercise at (C)? Yes*
  $\max\{300 - 270, 0\} = 30 > 26.08$
  $f_{(A)} = (0.5629 \times 1.29 + 0.4371 \times 30)e^{-0.08\frac{3}{12}} = 13.57$
- *Exercise at (A)? NO*
  $\max\{300 - 300, 0\} = 0 < 13.57$

4. Options Markets
4.6. Applications

5. (Baby Hull 12.12, Papa Hull 12.24) Suppose that a portfolio is worth \$60 million and the S&P 500 is at 300. If the value of the portfolio mirrors the value of the index, what options should be purchased to provide protection against the value of the portfolio falling below \$54 million in one year's time?

2000 put contracts should be bought with a strike of 270.

$$\begin{aligned} N &= \beta \frac{N_S S}{Q_{S\&P} S\&P} \\ &= 1 \frac{60{,}000{,}000}{100 \times 300} \\ &= 2000 \end{aligned}$$

$$\begin{aligned} N &= \beta \frac{N_S S}{Q_{S\&P} X} \\ 2000 &= 1 \frac{54{,}000{,}000}{100 \times X} \\ X &= 270 \end{aligned}$$

4. Options Markets
4.6. Applications

6. (Baby Hull 12.13, Papa Hull 12.25) Consider again the situation in the previous problem. Suppose that the portfolio has a beta of 2.0, the risk-free interest rate is 5 percent per annum, and the dividend yield on both the portfolio and the index is 3 percent per annum. What options should be purchased to provide protection against the value of the portfolio falling below \$54 million?

portfolio value: 60,000,000
s&P500: 300
$\beta = 2$
$r = 0.05$
$q = 0.03$
portfolio value floor: 54,000,000
$T = 1$

- *From the Portfolio...*
  capital gains: $\frac{54M-60M}{60} = -0.1$
  dividends: 0.03
  rate of return: $E[r_p] = -0.07$
- *To the Index...*
  rate of return: $E[r_m]$*?*

$$\begin{aligned} E[r_p] - r = &\quad \beta(E[r_m] - r) \\ -0.07 - 0.05 = &\quad 2(E[r_m] - 0.05) \\ E[r_m] = &\quad -0.01 \end{aligned}$$

  dividends: 0.03
  capital gains: -0.04
- *You should purchase 4000 put contracts with a 288 strike price.*
  $X = 300(1 + (-0.04)) = 288$
  $N = 2\frac{60,000,000}{100 \times 300} = 4000$

4. Options Markets
4.6. Applications

6. (Continued)

To check that the answer is correct, consider what happens when the value of the portfolio declines to \$48M.

- *From the Portfolio...*

  capital gains: $\frac{48M-60M}{60} = -0.2$

  dividends: 0.03

  rate of return: $E[r_p] = -0.17$

- *To the Index...*

  rate of return: $E[r_m]$?

$$\begin{aligned} E[r_p] - r = &\quad \beta(E[r_m] - r) \\ -0.17 - 0.05 = &\quad 2(E[r_m] - 0.05) \\ E[r_m] = &\quad -0.06 \end{aligned}$$

  dividends: 0.03

  capital gains: -0.09

- *The total value is insured at \$54,000,000.*

  value of the S&500: $S = 300(1 + (-0.09)) = 273$

  value of the portfolio: 48,000,000

  value of put options: $4000 \times 100 \times \max\{288 - 273, 0\} = 6,000,000$

  total value: 48,000,000+6,000,000=54,000,000

4. Options Markets
4.6. Applications

7. (Baby Hull 13.3, Papa Hull 12.29) Consider a two-month call futures option with a strike price of 40 when the risk-free interest rate is 10 percent per annum. The current futures price is 47.

$T = \frac{2}{12}$
$X = 40$
$r = 0.1$
$F = 47$

What is a lower bound for the value of the futures option if it is
(a) European?

$$\begin{aligned} c \geq & \quad (F - X)e^{-rT} \\ \geq & \quad (47 - 40)e^{-0.1\frac{2}{12}} \\ \geq & \quad 6.88 \end{aligned}$$

(b) American?

$$\begin{aligned} C \geq & \quad (F - X) \\ \geq & \quad (47 - 40) \\ \geq & \quad 7 \end{aligned}$$

4. Options Markets
4.6. Applications

8. (Baby Hull 13.4, Papa Hull 12.30) Consider a four-month put futures option with a strike price of 50 when the risk-free interest rate is 10 percent per annum. The current futures price is 47.

$T = \frac{4}{12}$
$X = 50$
$r = 0.1$
$F = 47$

What is a lower bound for the value of the futures option if it is
(a) European?

$$\begin{aligned} p \geq & \quad (X - F)e^{-rT} \\ \geq & \quad (50 - 47)e^{-0.1\frac{4}{12}} \\ \geq & \quad 2.90 \end{aligned}$$

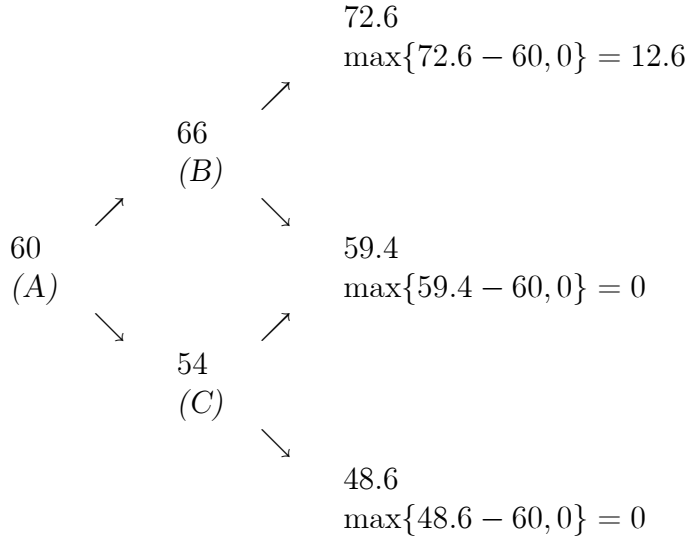
(b) American?

$$\begin{aligned} C \geq & \quad (X - F) \\ \geq & \quad (50 - 47) \\ \geq & \quad 3 \end{aligned}$$

4. Options Markets
4.6. Applications

9. (Baby Hull 13.6, Papa Hull 12.31) A futures price is currently 60. It is known that over each of the next two three-month periods it will either rise by 10 percent or fall by 10 percent. The risk-free interest rate is 8 percent per annum. What is the value of a six-month European call option on the futures with a strike price of 60? If the call were American, would it ever be worth exercising early?

$step = \frac{3}{12}$
$r = 0.08$
$X = 60$



$$
\begin{aligned}
p = & \quad \frac{1-d}{u-d} = \frac{1-0.9}{1.1-0.9} = 0.5 \\
1-p = & \quad 0.5 \\
f_{(B)} = & \quad (0.5 \times 12.6 + 0.5 \times 0)e^{-0.08\frac{3}{12}} = 6.18 \\
f_{(C)} = & \quad (0.5 \times 0 + 0.5 \times 0)e^{-0.08\frac{3}{12}} = 0 \\
f_{(A)} = & \quad (0.5 \times 6.18 + 0.5 \times 0)e^{-0.08\frac{3}{12}} = 3.03
\end{aligned}
$$

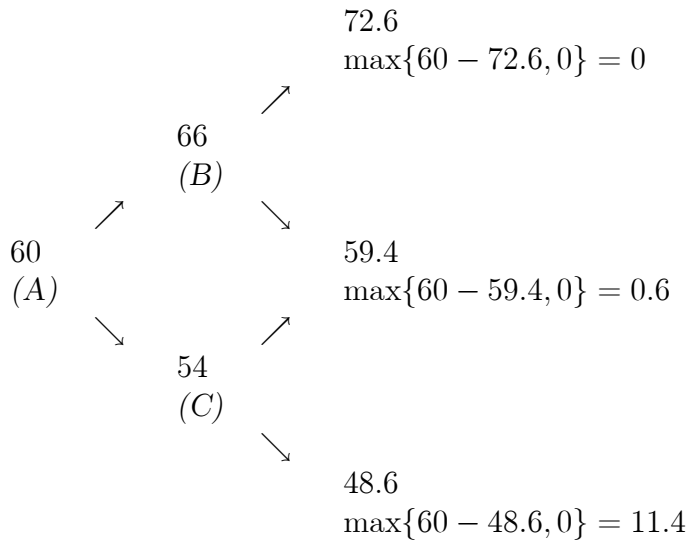
9. (Continued)

- *Exercise at (B)? No*
  $\max\{66 - 60, 0\} = 6 < 6.18$
- *Exercise at (C)? No*
  $\max\{54 - 60, 0\} = 0 = 0$
- *Exercise at (A)? NO*
  $\max\{60 - 60, 0\} = 0 < 3.03$

4. Options Markets
4.6. Applications

10. (Baby Hull 13.7, Papa Hull 12.32) In the previous problem what is the value of a six-month European put option on futures with a strike price of 60? If the put were American, would it ever be worth exercising it early? Verify that the call prices calculated in the previous problem and the put prices calculated here satisfy put-call parity relationships.

$step = \frac{3}{12}$
$r = 0.08$
$X = 60$



$$
\begin{aligned}
p = &\quad \frac{1-d}{u-d} = \frac{1-0.9}{1.1-0.9} = 0.5 \\
1-p = &\quad 0.5 \\
f_{(B)} = &\quad (0.5 \times 0 + 0.5 \times 0.6)e^{-0.08\frac{3}{12}} = 0.29 \\
f_{(C)} = &\quad (0.5 \times 0.6 + 0.5 \times 11.4)e^{-0.08\frac{3}{12}} = 5.88 \\
f_{(A)} = &\quad (0.5 \times 0.29 + 0.5 \times 5.88)e^{-0.08\frac{3}{12}} = 3.02
\end{aligned}
$$

4. Options Markets
4.6. Applications

10. (Continued)

- *Exercise at (B)? No*
  $\max\{60-66,0\} = 0 < 0.29$
- *Exercise at (C)? Yes*
  $\max\{60-54,0\} = 6 > 5.88$
  $f_{(A)} = (0.5 \times 0.29 + 0.5 \times 6)e^{-0.08\frac{3}{12}} = 3.08$
- *Exercise at (A)? NO*
  $\max\{60-60,0\} = 0 < 3.08$

European put-call parity:

$$\begin{aligned} p + Fe^{-rT} &= c + Xe^{-rT} \\ 3.02 + 60e^{-0.08\frac{6}{12}} &= 3.03 + 60e^{-0.08\frac{6}{12}} \\ 60.67 &\approx 60.68 \end{aligned}$$

American put-call relationship:

$$\begin{aligned} Fe^{-rT} - X &< C - P < F - Xe^{-rT} \\ 60e^{-0.08\frac{6}{12}} - 60 &< 3.03 - 3.08 < 60 - 60e^{-0.08\frac{6}{12}} \\ -2.35 &< -0.05 < 2.35 \end{aligned}$$

4. Options Markets
4.6. Applications

11. (Baby Hull 13.8, Papa Hull 12.33) A futures price is currently 25, its volatility is 30 percent per annum, and the risk-free interest rate is 10 percent per annum. What is the value of a nine-month European call on the futures with a strike price of 26?

$F = 25$
$\sigma = 0.3$
$r = 0.1$
$T = \frac{9}{12}$
$X = 26$

$$\begin{aligned} d_1 = & \frac{\log\left(\frac{F}{X}\right)+\frac{\sigma^2}{2}T}{\sigma\sqrt{T}} \\ = & \frac{\log\left(\frac{25}{26}\right)+\frac{0.3^2}{2}\frac{9}{12}}{0.3\sqrt{\frac{9}{12}}} = -0.0211 \approx -0.02 \end{aligned}$$

$$\begin{aligned} d_2 = & d_1 - \sigma\sqrt{T} \\ = & -0.0211 - 0.3\sqrt{\frac{9}{12}} = -0.2808 \approx -0.28 \end{aligned}$$

$$\begin{aligned} N(d_1) = & N(-0.02) \\ = & 0.4920 \end{aligned}$$

$$\begin{aligned} N(d_2) = & N(-0.28) \\ = & 0.3897 \end{aligned}$$

$$\begin{aligned} c = & e^{-rT}[SN(d_1) - XN(d_2)] \\ = & e^{-0.1\frac{9}{12}}[25 \times 0.4920 - 26 \times 0.3897] \\ = & 2.01 \end{aligned}$$

4. Options Markets
4.6. Applications

12. (Baby Hull 13.9, Papa Hull 12.34) A futures price is currently 70, its volatility is 20 percent per annum, and the risk-free interest rate is 6 percent per annum. What is the value of a five-month European put on the futures with a strike price of 65?

$F = 70$
$\sigma = 0.2$
$r = 0.06$
$T = \frac{5}{12}$
$X = 65$

$$\begin{aligned} d_1 &= \frac{\log\left(\frac{S}{X}\right) + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}} \\ &= \frac{\log\left(\frac{70}{65}\right) + \frac{0.2^2}{2}\frac{5}{12}}{0.2\sqrt{\frac{5}{12}}} = 0.6386 \approx 0.64 \end{aligned}$$

$$\begin{aligned} d_2 &= d_1 - \sigma\sqrt{T} \\ &= 0.6386 - 0.2\sqrt{\frac{5}{12}} = 0.5095 \approx 0.51 \end{aligned}$$

$$\begin{aligned} N(-d_1) &= N(-0.64) \\ &= 0.2611 \end{aligned}$$

$$\begin{aligned} N(-d_2) &= N(-0.51) \\ &= 0.3050 \end{aligned}$$

$$\begin{aligned} p &= e^{-rT}[XN(-d_2) - SN(-d_1)] \\ &= e^{-0.06\frac{5}{12}}[65 \times 0.3050 - 70 \times 0.2611] \\ &= 1.51 \end{aligned}$$

4. Options Markets
4.7. Hedging Strategies

1. (Baby Hull 14.4, Papa Hull 13.10) What is the delta of a short position in 1000 European call options on silver futures? The options mature in eight months, and the futures contract underlying the option matures in nine months. The current nine-month futures price is \$8 per ounce, the exercise price of the options is \$8, the risk-free interest rate is 12 percent per annum, and the volatility of silver is 18 percent per annum.

$F = 8$
$X = 8$
$r = 0.12$
$\sigma = 0.18$
$T = \frac{8}{12}$
$T^* = \frac{9}{12}$

$$\begin{aligned} d_1 = & \frac{\log\left(\frac{F}{X}\right) + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}} \\ = & \frac{\log\left(\frac{8}{8}\right) + \frac{0.18^2}{2}\frac{8}{12}}{0.18\sqrt{\frac{8}{12}}} = 0.0735 \approx 0.07 \end{aligned}$$

$$\begin{aligned} N(d_1) = & N(0.07) \\ = & 0.5279 \end{aligned}$$

$$\begin{aligned} \Delta = & N(d_1)e^{-rT} \\ = & 0.5279 \times e^{-0.12\frac{8}{12}} \\ = & 0.4873 \end{aligned}$$

The Δ of a short position in 1000 futures options is $-1000 \times 0.4873 = -487.3$.

4. Options Markets
4.7. Hedging Strategies

2. (Baby Hull 14.5, Papa Hull 13.11) Assume no storage costs for silver. In the previous problem, what initial position in nine-month silver futures is necessary for delta hedging?

$$\Delta = \frac{\partial f}{\partial F} = 0.4873$$

A long position in $0.4873 \times 1000oz = 487.3oz$ *of nine-month silver futures.*

If silver itself is used, what is the initial position?

$$\Delta = \frac{\partial f}{\partial S} = \frac{\partial f}{\partial F}\frac{\partial F}{\partial S}$$

$$\begin{aligned} F &= Se^{rT^*} \\ \frac{\partial F}{\partial S} &= e^{rT^*} \end{aligned}$$

$$\begin{aligned} \Delta = \frac{\partial f}{\partial S} &= \frac{\partial f}{\partial F}\frac{\partial F}{\partial S} \\ &= 0.4873e^{0.12\frac{9}{12}} = 0.5332 \end{aligned}$$

A long position in $0.5332 \times 1000oz = 533.2oz$ *of silver.*

If one-year silver futures are used, what is the initial position?

$$\Delta = \frac{\partial f}{\partial F_{1yr}} = \frac{\partial f}{\partial S}\frac{\partial S}{\partial F_{1yr}}$$

$$\frac{\partial S}{\partial F} = e^{-rT_{1yr}}$$

$$\begin{aligned} \Delta = \frac{\partial f}{\partial F_{1yr}} &= \frac{\partial f}{\partial S}\frac{\partial S}{\partial F_{1yr}} \\ &= 0.5332e^{-0.12\frac{12}{12}} = 0.4729 \end{aligned}$$

A long position in $0.4729 \times 1000oz = 472.9oz$ *of one-year silver futures.*

4. Options Markets
4.8. Synthetic Payoff Replication

1. (Baby Hull 14.12, Papa Hull 13.16) A fund manager has a well-diversified portfolio that mirrors the performance of the S&P 500 and is worth $90 million. The value of the S&P 500 is at 300, and the portfolio manager would like to buy insurance against a reduction of more than 5 percent in the value of the portfolio over the next six months. The risk-free interest rate is 6 percent per annum. The dividend yield on both the portfolio and the S&P 500 is 3 percent, and the volatility of the index is 30 percent per annum.
(a) If the fund manager buys traded European put options, how much would the insurance cost?

3000 put contracts should be bought with a strike of 285.

$$\begin{aligned} N &= \beta\frac{N_S S}{Q_{S\&P} S\&P} \\ &= 1\frac{90{,}000{,}000}{100\times 300} \\ &= 3000 \end{aligned}$$

$$\begin{aligned} N &= \beta\frac{N_S S}{Q_{S\&P} X} \\ 3000 &= 1\frac{85{,}500{,}000}{100\times X} \\ X &= 285 \end{aligned}$$

$S = 300$
$r = 0.06$
$\sigma = 0.3$
$T = \frac{6}{12}$
$q = 0.03$

4. Options Markets
4.8. Synthetic Payoff Replication

1. (a) (Continued)

$$\begin{aligned} d_1 &= \frac{\log\left(\frac{S}{X}\right)+\left(r-q+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ &= \frac{\log\left(\frac{300}{285}\right)+\left(0.06-0.03+\frac{0.3^2}{2}\right)\frac{6}{12}}{0.3\sqrt{\frac{6}{12}}} = 0.4186 \approx 0.42 \end{aligned}$$

$$\begin{aligned} d_2 &= d_1 - \sigma\sqrt{T} \\ &= 0.4186 - 0.3\sqrt{\frac{6}{12}} = 0.2064 \approx 0.21 \end{aligned}$$

$$\begin{aligned} N(d_1) &= N(0.42) \\ &= 0.6628 \end{aligned}$$

$$\begin{aligned} N(d_2) &= N(0.21) \\ &= 0.5832 \end{aligned}$$

$$\begin{aligned} N(-d_1) &= 1 - N(d_1) \\ &= 1 - 0.6628 \\ &= 0.3372 \end{aligned}$$

$$\begin{aligned} N(-d_2) &= 1 - N(d_2) \\ &= 1 - 0.5832 \\ &= 0.4168 \end{aligned}$$

$$\begin{aligned} p &= Xe^{-rT}N(-d_2) - Se^{-qT}N(-d_1) \\ &= 285e^{-0.06\frac{6}{12}} \times 0.4168 - 300e^{-0.03\frac{6}{12}} \times 0.3372 \\ &= 15.62 \end{aligned}$$

The insurance costs $\$15.62 \times 3000 \times 100 = \$4,686,000.$

4. Options Markets
4.8. Synthetic Payoff Replication

1. (Continued)
(b) Explain carefully alternative strategies open to the fund manager involving traded European call options, and show that they lead to the same result.

$$\begin{aligned} c = & \quad Se^{-qT}N(d_1) - Xe^{-rT}N(d_2) \\ = & \quad 300e^{-0.03\frac{6}{12}} \times 0.6628 - 285e^{-0.06\frac{6}{12}} \times 0.5832 \\ = & \quad 34.58 \end{aligned}$$

From put-call parity $p = c + Xe^{-rT} - Se^{-qT}$.

	0	$\frac{6}{12}$
sell index	$+300e^{-0.03\frac{6}{12}}$	$-S_T$
buy call	-34.58	$+\max\{S_T - 285, 0\}$
invest	$-285e^{-0.06\frac{6}{12}}$	$+285$
buy put	-15.62	$+\max\{0, 285 - S_T\}$

(c) If the fund manager decides to provide insurance by keeping part of the portfolio in risk-free securities, what should the initial position be?

$$\begin{aligned} \Delta = & \quad [N(d_1) - 1]e^{-qT} \\ = & \quad [0.6628 - 1]e^{-0.03\frac{6}{12}} \\ = & \quad -0.3322 \end{aligned}$$

33.22% of the portfolio $(90,000,000 \times 0.3322 = 29,898,000)$ *should be sold and invested in risk-free securities.*

4. Options Markets
4.8. Synthetic Payoff Replication

1. (Continued)
(d) If the fund manager decides to provide insurance by using nine-month index futures, what should the initial position be?

$$\Delta = \frac{\partial f}{\partial F} = \frac{\partial f}{\partial S}\frac{\partial S}{\partial F}$$

$$\begin{aligned} F &= Se^{(r-q)T^*} \\ S &= Fe^{-(r-q)T^*} \\ \frac{\partial S}{\partial F} &= e^{-(r-q)T^*} \end{aligned}$$

$$\begin{aligned} \Delta = \frac{\partial f}{\partial F} &= \frac{\partial f}{\partial S}\frac{\partial S}{\partial F} \\ &= -0.3322e^{-(0.06-0.03)\frac{9}{12}} \\ &= -0.3248 \end{aligned}$$

A short position of 32.48% of the portfolio $(90,000,000 \times 0.3248 = 29,232,000)$ *should be taken in index futures.*