The Design of IPO Lockups*

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Abstract

A model explaining the length of IPO lockups is developed and tested. The model demonstrates that, depending upon a firm’s characteristics, the length of the lockup period may be chosen to address either a moral hazard or an asymmetric information problem. The major empirical implication of the model concerns the relation between the optimal lockup length and the underpricing in the IPO. The length of the lockup and the underpricing in the IPO should be positively correlated in the cross section when the lockup solves an asymmetric information problem but should be uncorrelated for the moral hazard firms. Using proxies to identify firms for which the moral hazard problem or the asymmetric information problem is predicted to be the dominant consideration, we present empirical evidence consistent with this prediction.
1 Introduction

Recently, considerable attention has been paid to understanding the implications of the lockup provision, a period of time following an offer for which “firm insiders” are prevented from selling secondary shares in the market, that is a standard part of the contract between an underwriter and a firm engaged in its initial public offering of equity. Much of this literature mentions asymmetric information or moral hazard as potential motivations for the existence of the lockup provision.¹ Brav and Gompers (2003) take a deeper look at this question and are the first to offer an empirical analysis of the possible reasons for the inclusion of the lockup period in the IPO agreement.

This study extends the work of Brav and Gompers (2003). In a mechanism design framework we develop and test a model that explains the length of IPO lockups. Our model demonstrates that, depending on firm specific characteristics, the length of the lockup period may be chosen to solve either an asymmetric information problem or a moral hazard problem. The comparative static properties of the model yield its main testable implication; the correlation between the length of the lockup period and the underpricing in the IPO will be positively correlated for those firms for which the lockup period is chosen to address an asymmetric information problem while their correlation will be zero if the lockup period is chosen to address a moral hazard problem. Our empirical analysis presents results consistent with this prediction.

Brav and Gompers (2003) test three competing explanations for the cross-sectional difference in the length of lock-ups: (i) lockup length as a signal of firm quality, (ii) lockup length as a commitment device to alleviate moral hazard problems,

and (iii) lockup length as a means for investment banks to extract added compensation from the IPO firm. Brav and Gompers interpret their findings as supporting the hypothesis that lockup lengths are set to alleviate moral hazard problems and as not supporting the signaling or the rent extraction hypotheses. However, in their empirical analysis both the moral hazard and signaling hypotheses rely heavily on arguments based on asymmetric information making it difficult to distinguish between the two hypotheses.

Brau, Lambson, and McQueen (2004) (BLM) examine this issue further in a paper that is closely related to Brav and Gompers. BLM argue that the variables used in the Brav and Gompers (2003) study to indicate the severity of moral hazard problems may be more naturally interpreted as indicating the severity of an asymmetric information problem concerning firm value. BLM develop a signaling model of lockup length in which the insiders of good firms not only retain a greater exposure to the firm’s risk but also willingly commit to keep that exposure for a longer period than would the insiders of a “bad” firm (i.e. they consider a separating equilibrium). BLM argue that the empirical findings in Brav and Gompers (2003) are consistent with their model and also present empirical support for other predictions of their model (concerning firm transparency or the possible level of informational asymmetry and the level of firm specific risk). Their analysis, however, does not address the moral hazard question.

A common feature, of the Brav and Gompers (2003) and BLM (2004) papers, is that the analysis treats the signaling hypothesis and the commitment (or moral hazard) hypothesis as mutually exclusive. In other words, these papers take the stance that for the

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2 For example, Brav and Gompers (2003, pg. 9) note in reference to their primary indicators of the severity of any managerial moral hazard problem (firm size, underwriter quality, and whether the IPO firm is backed by venture capital financing) that: “Each of these variables is likely associated with less informational asymmetry about firm value in the aftermarket.”
cross section of firms that go public either the signaling hypothesis or the commitment hypothesis will explain the cross sectional differences in lockup length. As noted above, BLM highlight the similarities between the empirical predictions of these hypotheses. The two studies are also similar in that they use the intuition of a separating signaling equilibrium to generate their empirical predictions. This is a somewhat unfortunate choice since if the signaling equilibrium is a separating equilibrium, asymmetric information cannot be the explanation for the observed underpricing of the IPO.

Our analysis differs from the Brav and Gompers (2003) and BLM (2004) studies on both of the above mentioned dimensions. Our model demonstrates that the lockup of secondary shares can be beneficial both for firms suffering primarily from an asymmetric information problem and for firms suffering from a moral hazard problem. Thus, for a given firm, either asymmetric information or a moral hazard problem may be the friction that drives the choice of lockup period. The results of the model provide very different empirical predictions dependent upon which motivation for the choice of lockup period is dominant. The model uses a pooling equilibrium in the solution to the asymmetric information problem so the empirical predictions are able to relate the length of the lockup period to the underpricing at the IPO in an internally consistent model.

The model considers an economy with two types of managers differentiated by their cost of effort. Managers seek external equity financing for their firms. The firms require two inputs in order to have positive value, unobservable managerial effort and investment capital. If any firm receives both effort and funding they become “good” (high value) firms, if a firm receives funding and no effort it becomes a “bad” (low value) firm, and if it receives neither funding nor effort the firm has zero value. It is assumed,
given the personal cost of effort for the managers, that it is efficient to induce only low
cost managers to exert effort. Consequently, low cost managers select their most
preferred IPO strategy, choosing the offer price, the portion of the primary shares sold,
whether or not to exert effort, and the length of the lockup period. High cost managers
mimic the observable choices of low cost managers rather than be identified as seeking to
finance a low value firm. They, however, never exert effort. In equilibrium, low cost
managers bring good firms to the IPO market and high cost managers bring bad firms.

Our equilibrium, however, is pooling so the immediate post-IPO share prices of
the two types of firms are equal. Stochastic information arrival in the aftermarket
provides an incentive for good firms to delay selling shares, because delay increases the
chance that secondary shares will be sold at true value rather than average value; this is
the information asymmetry-based motivation for lockups.

Moral hazard provides a second motivation to lock up shares. Shares sold into an
uninformed secondary market will be sold at pooled price rather than true value. Because
the cost of effort is borne privately, without quality revelation there is no incentive for
low-cost managers to put forth effort. A lock up of the manager’s shares therefore also
serves to alleviate the moral hazard problem.

At the optimum of the design problem the incentive compatibility constraint
governing managerial effort may be binding or not depending on firm characteristics. If
the constraint is binding, the length of the lockup is selected to solve this moral hazard
problem. If the constraint is not binding at the optimum, the length of the lockup is
selected to address the asymmetric information problem derived from the fact that both
good and bad firms receive funding in equilibrium.
The difference in the comparative static analysis of the optima of the two types of problems, moral hazard and asymmetric information, provides us with our empirical predictions. The major prediction of the model relates to the relation between the length of the lockup period and the extent of underpricing measured as the difference between the aftermarket price and the offer price. For a set of firms with characteristics such that the length of the lockup is chosen to address the asymmetric information problem described above, the length of the lockup and the underpricing in the IPO is predicted to be positively correlated in the cross-section. When instead the lockup is chosen to address the moral hazard problem, there is predicted to be no correlation between lockup length and underpricing. This difference is due to the fact that underpricing is driven by information asymmetry rather than by managerial moral hazard. Our empirical analysis provides results that are consistent with this prediction.

The model also makes a prediction regarding the level of insider holdings. High capital needs necessitate that more equity is sold. These equity sales drive a wedge between management and ownership, thereby worsening the moral hazard problem. The length of the lockup increases in response to heightened moral hazard. We therefore predict a negative relationship between insider holdings and lockup length when the moral problem is the relevant imperfection. No such relationship is predicted for firm in which the lockup is set to address an information asymmetry problem.

The paper is organized as follows. Section 2 presents the model and its solution. Section 3 summarizes the main empirical predictions of the model. Section 4 describes the data and section 5 presents our empirical findings. Section 6 concludes.
2 The Model

The model considers an economy in which two types of managers/entrepreneurs, who run otherwise equivalent firms, compete for capital in the public equity market. Each firm seeking financing can become a “good” type firm (i.e. high value firm) worth \( X \) if it receives the required outside capital, \( I < X \), and unobservable managerial effort (simultaneous with the financing) as inputs. If the firm receives the necessary capital but no managerial effort it becomes a “bad” firm worth 0. If any firm receives less than the required external capital, a publicly observable event, it becomes worthless; regardless of whether there is a contribution of managerial effort. These binary payoffs imply that, without loss of generality, securities in this model may be described as equity.

Managers:

Firm managers can supply effort or not. The required effort is personally costly to the managers and they differ in the level of this unobservable cost. The managerial types are defined by the level of effort cost. There are low cost managers who face an effort cost of \( C_L \) and high cost managers who face an effort cost of \( C_H \), where \( C_L < C_H \). The \textit{ex ante} probability a given manager is a low cost manager is denoted \( \theta \). We assume that \( C_H \) is such that it is prohibitively costly to induce high cost managers to exert effort. Managers are risk neutral and face a personal discount factor \( \delta \) on date \( t \) income, where \( \delta < 1 \). We also assume the managers have no alternative to an IPO. There are no alternate sources of capital for the firm and the manager has no outside employment opportunities.

The Capital Market:

The capital market includes two types of investors, informed and uninformed. The informed investors can distinguish between good and bad firms that seek to issue public equity while the uninformed know only the \textit{ex ante} probability, \( \theta \), that firms are
good. The wealth of the uninformed, expressed as a percentage of the required capital ($I$) is $u \in (0, 1)$. We assume that investment by both the informed and the uninformed is required to carry the issue (this assumption, while shown to be unnecessary in Maksimovic and Pichler (1999), is made for simplicity).

The aftermarket for public equity is assumed to be partially revealing in the sense that after a length of time $T$ following the IPO with probability $Q(T)$ the true value of the firm is publicly revealed, otherwise equity is assumed to trade at its ex ante expected value, $EV = 0X + (1 - \theta)0 = 0X$. We assume that the function $Q(T)$ is such that $Q(0) = 0$, $Q'(\cdot) > 0$, and $Q''(\cdot) \leq 0$. These assumptions on $Q(\cdot)$ are sufficient for our purposes but are made stronger than necessary for simplicity.

*The Initial Public Offering:*

The choice of IPO design in this model includes the choice over the offer price ($p$), whether to structure the deal to induce managerial effort or not, the percent of equity to be sold in the IPO ($\alpha$), and the length of the lockup period ($T$) on the retained shares. The IPO price $p$ is established in equilibrium in the IPO market and is, therefore, not an unconstrained choice of the low cost manager. The percent of the equity sold in the IPO is constrained to be (at least) large enough to raise the required capital $I$. The length of the lockup is chosen in recognition of the partially revealing nature of the aftermarket. The longer the lockup the greater chance the manager sells into an informed aftermarket, however, the discount factor, $\delta$, implies that it is not optimal for any manager to choose an unlimited lockup. In equilibrium it must be incentive compatible for the low cost manager to elect to exert effort and the length of the lockup will influence their incentive

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3 The assumption that the aftermarket price may become perfectly revealing is made for simplicity. All that is required is that with probability $Q(T)$ there will be less adverse selection in the aftermarket than in the IPO market. This can be derived from a model of trade in the aftermarket with liquidity motives for the informed but such a model would be beyond the scope of the present paper.
to do so. A major theme of the paper is that for different firms the lockup of the retained shares plays different roles and so in the cross section of firms the length of the lockup will be set based on different tradeoffs.

In equilibrium, low cost managers design their preferred IPO strategy and ultimately bring good firms to the public equity market while high cost managers mimic these choices except that they exert no effort and so always run bad firms. The result is an IPO market which mirrors the market examined in Rock (1984). It is well known that this type of asymmetric information in the market leads to underpricing relative to the expected value of the population of firms going public.

**Proposition 1:** There exists a unique Bayesian-Nash Equilibrium for the IPO market with the per share price \( p \in (0, EV) \) in which the informed investors invest in an issue if and only if it is by a good firm and the uninformed investors always invest. The equilibrium price is

\[
p = \frac{\theta u X}{\theta u + (1 - \theta)}.
\]

**Proof:** See the appendix.

Defining underpricing as in the empirical IPO literature, \( UP = \frac{EV - p}{p} \), the equilibrium underpricing in Proposition 1 is given as

\[
UP = \frac{(1 - u)(1 - \theta)}{u}.
\]

**Corollary 1:** The comparative static properties for underpricing in the model are given by \( \frac{\partial UP}{\partial \theta} < 0 \), \( \frac{\partial UP}{\partial u} < 0 \), and \( \frac{\partial UP}{\partial X} = 0 \).

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4 We restrict attention to parameter values such that it is efficient to motivate the low cost managers to exert managerial effort. If this were not the case, all firms would be bad. The IPO market would have no private information, lockups or underpricing.
The comparative static properties for underpricing are intuitive. Underpricing is decreasing in $\theta$. The parameter $\theta$ indicates the level of overall risk in the IPO market for the uninformed. As the overall risk is reduced there is less underpricing needed to compensate the uninformed for their informational disadvantage. Underpricing is also decreasing in the relative wealth of the uninformed. As relatively less of the good offerings go to the informed, the uninformed face less of a “lemons problem” reducing the need for underpricing. Finally, the equilibrium level of underpricing is independent of $X$ so that the percent underpricing is independent of the difference in the value of a good versus a bad firm. The structure of the model implies that both $EV$ and $p$ are proportional to $X$ so $UP$ is independent of $X$.

In designing the IPO the low cost manager must ensure that at least $I$ in outside capital will be raised by the firm. This imposes the “capital constraint” (CC) on the design problem: $\alpha p \geq I$. In order for managerial effort to be exerted it must be incentive compatible for the low cost manager to do so. For a given lockup length $T$, the expected value of a low cost manager’s retained shares, given that he exerts effort, is $(1 - \alpha)[Q(T)X + (1 - Q(T))EV]\delta^T$. Instead of exerting effort, the low cost manager may shirk, in that case the expected value of his retained shares is $(1 - \alpha)[(1 - Q(T))EV]\delta^T$. For effort to be optimal it must be that the difference in these values is greater than the cost of effort $C_L$. The incentive compatibility constraint (IC) on effort can therefore be written $(1 - \alpha)Q(T)X\delta^T \geq C_L$.

Because the IPO price will be set in equilibrium (as a function of the parameters $\alpha$, $\theta$, $u$, and $X$; see Proposition 1) the problem faced by the low cost manager in designing
the optimal IPO strategy is to maximize the expected value of his shares subject to the capital constraint and the incentive compatibility constraint.

\[
\begin{align*}
\text{Max} & \quad \alpha p(u, \theta, X) - I + (1 - \alpha)[Q(T)X + (1 - Q(T))EV]T \\
\text{s.t.} & \quad (1 - \alpha)Q(T)X \delta T \geq C_L \quad \text{(IC)} \\
& \quad \alpha p(u, \theta, X) \geq I \quad \text{(CC)}
\end{align*}
\]

Finally, we note that the optimal \( \alpha \) makes the capital constraint hold with equality (define \( \alpha^* \) by \( \alpha^* p = I \)). This is proven formally in the appendix. Intuitively, information arrival over time implies that, quite generally, aftermarket sales suffer a less severe adverse selection problem than do IPO sales. Hence, the low cost manager’s problem becomes:

\[
\begin{align*}
\text{Max} & \quad (1 - \alpha^*)[Q(T)X + (1 - Q(T))EV]T \\
\text{s.t.} & \quad (1 - \alpha^*)Q(T)X \delta T \geq C_L \quad \text{(IC)}.
\end{align*}
\]

The design problem can therefore be reduced to the choice of the optimal length of lockup period on the manager’s retained shares. The characteristics of the optimal lockup differ depending upon whether the incentive compatibility constraint is binding at the optimum or not. In other words, the choice of lockup period differs depending upon whether the length of the lockup is chosen to solve a moral hazard problem with respect to managerial effort or to solve a problem of asymmetric information in the aftermarket.

Let \( T_{AI} = \arg \max \{[Q(T)X + (1 - Q(T))EV]T \} \). By definition, \( T_{AI} \) solves the unconstrained version of problem (2). The incentive compatibility constraint is slack at the optimum so, at the margin, the lockup is chosen to solve the asymmetric information problem. The mild regularity assumptions for \( Q(\cdot) \) imply that \( T_{AI} \) is uniquely defined.
Define $T_{MH}$ as the shortest lockup for which the low-cost manager would choose to exert effort. $T_{MH}$ is defined implicitly by $(1-\alpha)Q(T_{MH})X\delta^{T_{MH}} = C_L$. This is the solution to problem (2) when the incentive compatibility constraint is binding at the optimum (the solution to the moral hazard version of the problem). It may be the case that $T_{AI}$ exceeds $T_{MH}$, and the incentive compatibility constraint does not bind. Alternatively, it may be the case that $T_{MH}$ exceeds $T_{AI}$. In that case the optimal lockup length is driven by the moral hazard problem. The following proposition summarizes these two cases.

**Proposition 2:** Using the definitions above,

(a) (Asymmetric Information Case) If $Q(T_{AI})\delta^{T_{AI}} \geq \frac{C_L}{(1-\alpha^*)X}$ then the incentive compatibility constraint (IC) does not bind at the optimum and the optimal lockup length is $T_{AI}$.

(b) (Moral Hazard Case) If $Q(T_{AI})\delta^{T_{AI}} < \frac{C_L}{(1-\alpha^*)X}$ then the incentive compatibility constraint (IC) binds at the optimum and the optimal lockup length is $T_{MH}$.

**Proof:** See the Appendix.

Note that the incentive compatibility constraint is slack when $C_L$ is low; it is not difficult to motivate effort when it is not costly. In addition, when $I$ is small relative to the other parameters in the model, the amount of equity sold ($\alpha^*$) will be small. Again by inspection of proposition 2 this leads to a slack incentive compatibility constraint; all else equal, a manager holding a large equity stake has a strong incentive to exert effort.
Proposition 2 distinguishes the cases in which lockups are driven by moral hazard from those driven by asymmetric information. We now characterize the comparative static properties of these two cases.

**Corollary 2:** \( \frac{\partial T_{AI}}{\partial \theta} < 0 \) and \( \frac{\partial T_{AI}}{\partial X} = 0 \), i.e., \( T_{AI} \) is decreasing in \( \theta \) and independent of \( X \). \( \frac{\partial T_{MH}}{\partial \theta} = 0 \) and \( \frac{\partial T_{MH}}{\partial X} < 0 \), i.e., \( T_{MH} \) is independent of \( \theta \) and decreasing in \( X \).

**Proof:** See the Appendix.

Thus as the proportion of good firms (\( \theta \)) in the market rises the length of the lockup will fall if the lockup is chosen to solve the asymmetric information problem. Intuitively, the low cost manager faces the cost of delay when using the lockup period to distinguish the value of his retained shares from the *ex ante* expected value \( EV \). The cost and benefit are of course balanced at the margin. As \( \theta \) rises, the difference between \( X \) and \( EV \) falls reducing the advantage to delay. To the contrary, the length of a lockup period chosen to solve the asymmetric information problem is independent of the difference in value between a good and bad, \( X \). This occurs because the advantage to delay is unchanged with changes in \( X \) (\( X \) and \( EV = \theta X \) change proportionally).

The comparative static properties of \( T_{MH} \), the solution to the moral hazard problem, while very different from those of \( T_{AI} \) are also very intuitive. First, when the length of the lockup period is set to solve the moral hazard problem the optimal length is independent of the proportion of good and bad firms in the economy. The incentive to exert effort depends upon the expected value of the low cost manager’s retained shares when he exerts effort as compared to their value when he shirks. Clearly, this is independent of \( \theta \). Secondly, as the difference in value between good and bad firms, \( X \), is increased \( T_{MH} \) is reduced. The difference in firm value is a primary driver of the
incentive for managerial effort. By increasing this difference the incentive for effort is increased and consequently the need for a lengthy lockup period is reduced.

3 Empirical Predictions

The development in section 2 and Propositions 1 and 2 provide us with the main empirical implications of the model. The comparative static characteristics of the different lockups ($T_{MH}$ and $T_{AI}$) provide the main testable implication of the theory. To study the cross-section of firms we consider the response to changes in the fundamental parameters that govern the firm characteristics in the model; $X$ and $\theta$. Assuming that in the cross section of firms there is variation in both of these parameters the results reported in Corollaries 1 and 2 provide the following testable hypothesis.

$H1$: Shocks to the parameter $\theta$ cause underpricing and $T_{AI}$ to positively covary. Shocks to $X$ cause $T_{MH}$ but not $T_{AI}$ or $UP$, to vary. Thus, in general, lockups driven by asymmetric information positively covary with underpricing. Lockups driven by moral hazard do not covary with underpricing.

A decrease in the parameter $\theta$ increases the amount of risk in the capital market. The consequences of this are twofold. First, and most obviously, the uninformed investors will require more of a compensation for participating (this simply restates the result that $UP$ is decreasing in $\theta$). Secondly, $T_{AI}$ is increased while $T_{MH}$ remains constant as $\theta$ decreases. This implies that for a given firm, a decrease in $\theta$ tends to make it more likely that the asymmetric information problem is the motivating factor in establishing the length of the lockup.

A second empirical implication of the model can be developed by considering variation in the required investment, $I$. Holding the other parameters of the model fixed this is equivalent to examining variation in the value added of the investment. Recalling that the capital constraint (CC) will always bind we see that the impact of a change in the
required investment, $I$, is a change in the amount of external equity, $\alpha^*$, that must be sold by the manager. We generate the following hypothesis.

$H2$: In a sample of firms for which the lockup lengths are driven by asymmetric information there should be no correlation between lockup length and the fraction of the equity retained by the manager. For those firms for which the lockup length is driven by a moral hazard problem there should be a negative correlation between the length of the lockup and the fraction of the equity retained by the manager.

Hypothesis 2 suggests that for the moral hazard firms managerial ownership and lockup length are substitute mechanisms in the solution of the managerial effort problem. It is important to note that this result is derived in a model where the amount invested by the firm is given exogenously. In a richer model with an endogenous investment decision it is possible that these mechanisms may be compliments rather than substitutes.

Testing these two hypotheses requires a means by which we can separate the moral hazard firms and the asymmetric information firms. We use three characteristics of the IPO firms to separate the moral hazard from the asymmetric information firms; whether the IPO firm had venture capital financing prior to the IPO, whether the IPO firm is taken public by a high reputation underwriter, and whether the IPO occurred during the 1999-2000 bubble period.

It has been argued that venture capitalist (VC) backing of an IPO firm (see Megginson and Weiss (1991)) or the presence of a high reputation lead underwriter (see Beatty and Ritter (1986) and Carter and Manaster (1990)) serve a certification role by reducing the uncertainty concerning the value of the firm going public. If VC backing or a high reputation lead underwriter serves as a screening device for IPO firms then the asymmetric information problem should be less severe for such offerings. VC backing or the presence of a high reputation lead underwriter can then serve as an indication of firms
that are, all else equal, expected to suffer relatively more from the moral hazard problem while non-VC backed firms or those taken public by a low reputation lead underwriter are expected to suffer relatively more from the asymmetric information problem.

Ljungqvist and Wilhelm (2003) argue that there was a change in the incentives of investment banks caused by a change in the governance characteristics of the firms going public, and that the “IPO bubble period” of 1999 – 2000 can be characterized as having had a greater level of uncertainty concerning the value of the firms going public. The spike in underpricing that occurred during this period is consistent with this view. To the extent that investors responded to this period as if uncertainty and asymmetric information were heightened, we expect a subperiod analysis of this “event” to show a greater tendency by all types of firms to exhibit the characteristics of asymmetric information firms.

4 Data

The data for this study was drawn from the SDC data base. We use all initial public offerings of equity for the period January 1988 through December 2004, a total of 5,564 firms. Information was collected for each IPO concerning the proceeds of the offer, the offer price, the market value of the equity after the IPO, the underpricing on the first trading day, the identity of the lead underwriter(s), the length of the lockup period, insider ownership, and whether the offering was a unit offering, a carveout, VC backed, or from the high-tech industry. Alon Brav and Paul Gompers kindly shared their corrections to the lockup lengths reported on SDC. Information on underwriter rankings (based on Carter-Manaster (1990)) was gathered from Jay Ritter’s website. We
eliminated firms from the sample if observations for the main variables of interest, underpricing, lockup length, or insider ownership, were missing. This reduced the sample to 2,548 firms.

Table 1 provides summary statistics for the sample. As can be seen in panel B there are clear differences in average lockup length across subsamples. IPOs that are backed by venture capitalists or high-quality investment banks have much shorter lockups on average, which is consistent with a reduction in the severity of market imperfections.

As reported by others, the lockup length is highly standardized. Approximately three-quarters of IPOs in the sample have lockups of exactly 180 days. However, the degree of standardization varies across the subsamples. IPOs which are either backed by venture capitalists or underwritten by high-quality investment banks (≥ 8.0 ranking based on the Carter-Manaster (1990) technique) are much more likely to use the standard 180 day lockup than are other IPOs. Although this finding neither supports nor refutes the model, we find it surprising. Ex-ante, it might have been expected that firms more likely to choose off-the-shelf “boilerplate” contract terms would be less sophisticated issuers, that is, small issuers that are not VC backed nor underwritten by high quality banks.

Panel B of Table 1 shows a second curious stylized fact: there is a clear intertemporal trend in lockup lengths. Usage of the 180 day lockup length increased during the bubble period. This increasing standardization was, however, apparently not caused by market heat as the post bubble period did not exhibit a reversion towards the longer lockups that were more common before the bubble. Although we do not have an immediate explanation for this increasing standardization, the result parallels other results

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5 The standardization seen during the bubble may be at least partly explained by a substitution toward types of IPOs that are more likely to employ the 180 day standard, since the proportion of both IPOs underwritten by high-quality banks and venture capital backed IPOs increased during the bubble.
in the literature. Specifically, Hsuan-Chi and Ritter (2000) show that the proportion of IPOs for which the gross spread was exactly seven percent rose from fewer than half in the 1980s to more than 90% by the late 1990s. They attribute this standardization to lack of competition between banks. To the extent that underwriters compete for issuers, and issuers dislike long lockups, a similar explanation may drive our results.

5 Empirical Tests and Results

The empirical predictions generated by the model concern the relation between IPO underpricing, the length of the lockup period, and the amount of equity retained by firm insiders after the offering. Specifically, in a subsample of firms for which the lockup length is chosen to solve a moral hazard problem the lockup length should be uncorrelated with the underpricing in the IPO and negatively correlated with the value of the insider’s equity in the cross section of that subsample. In a subsample of firms for which the lockup length is chosen to solve an asymmetric information problem lockup length should be positively correlated with underpricing and uncorrelated with the value of insider holdings in the cross section.6

We test hypotheses 1 and 2 using a simple linear regression model. In a cross sectional regression of lockup length on underpricing, post IPO insider ownership in percentage terms, and other control variables we are able to measure the correlations of interest. The model is specified as:

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6 As we argue above, these predictions are most naturally generated in a pooling equilibrium for the asymmetric information problem because this type of equilibrium can lead to underpricing in the IPO market. These relations are less easily generated in a model in which the length of the lockup is used to separate good firms from bad because asymmetric information in the IPO market can no longer be used to explain the underpricing of the IPO.
The parameters of interest are of course $\beta_1$ and $\beta_2$ which provide the signs of the relevant correlations considered in the hypotheses. The other variables are included in an attempt to control for possible spurious correlation between the variables of interest. The model predicts that for the asymmetric information sample, the regression coefficients will satisfy $\beta_1 > 0$ and $\beta_2 = 0$ while for the moral hazard sample $\beta_1 = 0$ and $\beta_2 < 0$.

Panel A of table 2 shows that the main empirical prediction of the model, hypothesis 1, holds at the 5% level in four of the five subsamples. In particular, for IPOs that are not VC backed the coefficient $\beta_1$ is positive and significant, whereas for VC backed IPOs the coefficient is insignificantly different from zero. For IPOs with low-quality underwriters the coefficient $\beta_1$ is positive and significant, whereas for IPOs with high-quality underwriters the coefficient is insignificantly different from zero. The coefficient $\beta_1$ is insignificant in the bubble period regression, however, which is inconsistent with our model if the bubble period can serve as an indication of increased asymmetric information. However, note that none of the variables in that regression is significant except the coefficient on offering proceeds. As mentioned before, there was an exogenous trend towards standardization of the lockup period during the IPO bubble implying little cross sectional variation in lockup periods.

The data is broadly consistent with the main hypothesis of the model; that the length of the lockup period in the cross section of firms is chosen to address both moral

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7 In this regression, the coefficient $\beta_1$ has borderline significance (p-value = .0773). However, the economic significance is small; note that the coefficient is an order of magnitude smaller than the same coefficient in the non-VC and low reputation underwriter regressions.
hazard and asymmetric information problems. As a robustness test we also estimate the regressions over the period 1988 – 1998, to eliminate the bubble period, the (unreported) results are qualitatively the same.

Hypothesis 2 can also be examined with reference to table 2. The coefficients reported in the third line of this table provide the signs of the correlations between lockup length and the dollar value of insider holdings after the IPO. The model predicts that this correlation should be zero for asymmetric information firms and negative for moral hazard firms. This prediction holds in our sample when we split the sample into VC backed and non-VC backed firms. For the high reputation underwriter subsample, however, we do not find a significant coefficient. However, as in the bubble subsample, there is a high degree of standardization of the independent variable which results in generally insignificant coefficients. Note that all coefficients other than the constant are insignificant, and that the regression has an $R^2$ that is an order of magnitude lower than that in the other regressions. Again, this move to standardization has no obvious explanation save perhaps the decrease in competitiveness of the market, particularly among bulge bracket underwriters, that Hsuan-Chi and Ritter (2000) cite.

Table 3 shows the same regressions as Table 2, but employs the natural log of the dollar value of insiders’ post-IPO holdings as a control variable, rather than percentage holdings. Not surprisingly, the results are quite similar. However, most of the regressions show an improvement in fit and the negative coefficient on $\beta_2$ for the venture capital sample has much stronger economic and statistical significance. This specification therefore provides slightly stronger support for $H2$. This may be due to
chance, or it may be that dollar values of equity are a better proxy for the incentive effects of insider holdings than are percentage holdings.

6 Conclusions

A basic assumption of the model presented in this paper is that, depending upon firm characteristics, the length of the lockup period in an IPO may be chosen to solve either a moral hazard or an asymmetric information problem. In a mechanism design framework we derive the optimal lockup length, assuming a fixed price offering for a firm’s IPO. In the model the lockup length may indeed be chosen to address either a moral hazard or an asymmetric information problem. The comparative static properties of the model differ depending upon which of these frictions drives the choice of lockup period. The main empirical implication of the model is that there should be a positive correlation between the lockup length and the underpricing in the IPO in the cross section of a sample of firms for which the asymmetric information problem determines the length of the lockup. For a complementary sample of firms for which the lockup is chosen to address a managerial moral hazard problem there should be no correlation between the lockup length and the underpricing in the IPO. The intuition for this prediction can be explained by simply noting that underpricing is driven by asymmetric information, not by moral hazard. Thus increasing the severity of the asymmetric information problem should impact both of the variables of interest. Except for our use of the bubble period to indicate greater asymmetric information, the data strongly supports this prediction.

We also examine the comparative static properties of the model with respect to the amount of investment and find it predicts a negative correlation between lockup length and the proportion of equity owned by firm insiders after the IPO for moral hazard
firms and a zero correlation between these variables for asymmetric information firms. There is mixed support for this hypothesis in the data. Splitting the sample between venture capital backed IPOs backed and non-VC backed IPOs supports the prediction, whereas splitting the sample between the high-quality underwriter and low-quality underwriter does not. The latter failure is due to an extraordinary amount of standardization in the lockups chosen by high-quality underwriters. We are not aware of any paper in the literature which notes this differential (across investment bank types) tendency to standardize the IPO contract, nor are we able to fully explain this finding.

A final note on the model is in order. There is nothing special about the restriction to a fixed-price mechanism for the IPO made by the current model. All we require from the mechanism is that underpricing is positively correlated with the severity of any informational asymmetry in the market. This will hold for bookbuilding models as well as the fixed price model used here and for just about any auction model, etc. A fixed-price mechanism is used here only because of its simple characterization.
References:


Table 1
Summary Statistics

Panel A: Summary statistics, Full Sample ($ Millions)

<table>
<thead>
<tr>
<th>Variable</th>
<th>10th Percentile</th>
<th>Median</th>
<th>Mean</th>
<th>90th Percentile</th>
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<tbody>
<tr>
<td>Proceeds ($ Millions)</td>
<td>7.2</td>
<td>30.1</td>
<td>41.62</td>
<td>79.9</td>
</tr>
<tr>
<td>Offer Price ($)</td>
<td>5.5</td>
<td>11.125</td>
<td>11.5</td>
<td>17</td>
</tr>
<tr>
<td>Market Value of Equity</td>
<td>8.4</td>
<td>38.58</td>
<td>77.89</td>
<td>151.87</td>
</tr>
<tr>
<td>Underpricing (%)</td>
<td>-2.78</td>
<td>12.36</td>
<td>22.77</td>
<td>55.63</td>
</tr>
<tr>
<td>Days of Lockup</td>
<td>180</td>
<td>180</td>
<td>224.8</td>
<td>365</td>
</tr>
<tr>
<td>Underwriter Rank</td>
<td>3.1</td>
<td>8.1</td>
<td>6.77</td>
<td>9.1</td>
</tr>
<tr>
<td>VC Backed (%)</td>
<td>0</td>
<td>0</td>
<td>42.18</td>
<td>100</td>
</tr>
<tr>
<td>High Tech (%)</td>
<td>0</td>
<td>100</td>
<td>50.36</td>
<td>100</td>
</tr>
<tr>
<td>Carve Out (%)</td>
<td>0</td>
<td>0</td>
<td>5.98</td>
<td>0</td>
</tr>
<tr>
<td>Post-IPO insider holding (%)</td>
<td>14.6</td>
<td>46.25</td>
<td>45.16</td>
<td>71.3</td>
</tr>
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</table>

Panel B: Distribution of Lockup Length by Subsample

<table>
<thead>
<tr>
<th>Period</th>
<th>Average Lockup</th>
<th>Proportion of Lockups</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>&lt; 180 Days</td>
<td>180 Days</td>
<td>&gt; 180 Days</td>
<td></td>
</tr>
<tr>
<td>1988-1993</td>
<td>229.01</td>
<td>0.076</td>
<td>0.714</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>1994-1998</td>
<td>231.93</td>
<td>0.044</td>
<td>0.753</td>
<td>0.203</td>
<td></td>
</tr>
<tr>
<td>1999-2000</td>
<td>183.77</td>
<td>0.071</td>
<td>0.88</td>
<td>0.049</td>
<td></td>
</tr>
<tr>
<td>2001-2004</td>
<td>180.19</td>
<td>0.076</td>
<td>0.879</td>
<td>0.045</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subsample</th>
<th>Average Lockup</th>
<th>Proportion of Lockups</th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>&lt; 180 Days</td>
<td>180 Days</td>
<td>&gt; 180 Days</td>
<td></td>
</tr>
<tr>
<td>VC-Backed</td>
<td>191.48</td>
<td>0.057</td>
<td>0.865</td>
<td>0.078</td>
<td></td>
</tr>
<tr>
<td>Not VC-Backed</td>
<td>244.39</td>
<td>0.058</td>
<td>0.694</td>
<td>0.248</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subsample</th>
<th>Average Lockup</th>
<th>Proportion of Lockups</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>&lt; 180 Days</td>
<td>180 Days</td>
<td>&gt; 180 Days</td>
<td></td>
</tr>
<tr>
<td>Hi-Ranked Underwriter</td>
<td>183.29</td>
<td>0.044</td>
<td>0.914</td>
<td>0.043</td>
<td></td>
</tr>
<tr>
<td>Low-Ranked Underwriter</td>
<td>268.03</td>
<td>0.075</td>
<td>0.591</td>
<td>0.334</td>
<td></td>
</tr>
</tbody>
</table>
This table presents the coefficients and associated t-statistics, in parentheses, from the OLS regression of the equation:

\[ \text{Lockup} = \alpha + \beta_1 \text{underpricing} + \beta_2 \text{inside} + \beta_3 \text{proceeds} + \beta_4 \text{price} + \beta_5 \text{tech} + \beta_6 \text{momentum} + \beta_7 \text{Rank} + \beta_8 \text{VC} + \varepsilon \]

where the dependent variable is the number of days in the lockup period and the independent variables of interest are the percent of underpricing at the end of the first trading day and the holdings of firm insiders measured as a percent of outstanding equity. We use as control variables the log of the proceeds of the issue, the offer price, a dummy variable set to 1 if the issuing firm is a high tech firm, a measure of the momentum on the NASDAQ exchange, a dummy variable set to 1 if the underwriter ranking is greater than or equal to 8 and a dummy indicating whether the IPO was backed by venture capitalists. Statistical significance at the 1%, 5% and 10% level is denoted by the symbols ***, ** and * respectively.

<table>
<thead>
<tr>
<th></th>
<th>Asymmetric Information Sample</th>
<th>Moral Hazard Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Firms</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low Rep Underwriter</td>
<td>Non-VC</td>
</tr>
<tr>
<td>Intercept</td>
<td>497.11***</td>
<td>550.80***</td>
</tr>
<tr>
<td></td>
<td>(9.09)</td>
<td>(6.54)</td>
</tr>
<tr>
<td>Underpricing</td>
<td>0.44***</td>
<td>0.97***</td>
</tr>
<tr>
<td></td>
<td>(7.16)</td>
<td>(6.91)</td>
</tr>
<tr>
<td>Inside</td>
<td>-0.04</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(-.37)</td>
<td>(.08)</td>
</tr>
<tr>
<td>Proceeds</td>
<td>-51.73***</td>
<td>-61.82***</td>
</tr>
<tr>
<td></td>
<td>(-11.13)</td>
<td>(-6.51)</td>
</tr>
<tr>
<td>Price</td>
<td>-2.91***</td>
<td>-10.99***</td>
</tr>
<tr>
<td></td>
<td>(-3.69)</td>
<td>(-5.62)</td>
</tr>
<tr>
<td></td>
<td>(-3.18)</td>
<td>(-1.62)</td>
</tr>
<tr>
<td>Momentum</td>
<td>-0.42</td>
<td>-0.46</td>
</tr>
<tr>
<td></td>
<td>(-.98)</td>
<td>(-.62)</td>
</tr>
<tr>
<td>Rank Dummy</td>
<td>-11.57***</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>(-2.09)</td>
<td>*</td>
</tr>
<tr>
<td>VC Dummy</td>
<td>-28.03***</td>
<td>-22.06**</td>
</tr>
<tr>
<td></td>
<td>(-5.69)</td>
<td>(-2.12)</td>
</tr>
<tr>
<td>Year Dummies</td>
<td>Included</td>
<td>Included</td>
</tr>
<tr>
<td>Adjusted R^2</td>
<td>0.2595</td>
<td>.2970</td>
</tr>
<tr>
<td>N</td>
<td>2548</td>
<td>1145</td>
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</table>
Table 3
Determinants of the Length of Lockup

This table presents the coefficients and associated t-statistics, in parentheses, of the same regression presented in Table 2, except that Inside is defined as the natural log of the dollar value of insiders’ post-IPO holdings. Statistical significance at the 1%, 5% and 10% level is denoted by the symbols ***, ** and * respectively.

<table>
<thead>
<tr>
<th>All Firms</th>
<th>Asymmetric Information Sample</th>
<th>Moral Hazard Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Rep Underwriter</td>
<td>Non-VC</td>
</tr>
<tr>
<td>Intercept</td>
<td>531.50***</td>
<td>607.14***</td>
</tr>
<tr>
<td>(9.27)</td>
<td>(6.85)</td>
<td>(5.84)</td>
</tr>
<tr>
<td>Underpricing</td>
<td>.53***</td>
<td>1.02***</td>
</tr>
<tr>
<td>(.719)</td>
<td>(6.51)</td>
<td>(6.16)</td>
</tr>
<tr>
<td>Inside</td>
<td>-6.72**</td>
<td>-10.56*</td>
</tr>
<tr>
<td>(-2.44)</td>
<td>(-2.00)</td>
<td>(-7.2)</td>
</tr>
<tr>
<td>Proceeds</td>
<td>-49.41</td>
<td>-66.32***</td>
</tr>
<tr>
<td>(-8.13)</td>
<td>(-5.89)</td>
<td>(-6.32)</td>
</tr>
<tr>
<td>Price</td>
<td>-3.68***</td>
<td>-9.37***</td>
</tr>
<tr>
<td>(-3.67)</td>
<td>(-4.19)</td>
<td>(-4.12)</td>
</tr>
<tr>
<td>Tech</td>
<td>-20.36***</td>
<td>-18.21*</td>
</tr>
<tr>
<td>(-3.57)</td>
<td>(-1.7)</td>
<td>(-2.22)</td>
</tr>
<tr>
<td>Momentum</td>
<td>-0.67</td>
<td>-0.83</td>
</tr>
<tr>
<td>(-1.37)</td>
<td>(-.98)</td>
<td>(-1.18)</td>
</tr>
<tr>
<td>Rank Dummy</td>
<td>-10.09</td>
<td>*</td>
</tr>
<tr>
<td>(-1.54)</td>
<td>*</td>
<td>(-.011)</td>
</tr>
<tr>
<td>VC Dummy</td>
<td>-26.26***</td>
<td>-19.29</td>
</tr>
<tr>
<td>(-4.57)</td>
<td>(-1.6)</td>
<td>*</td>
</tr>
<tr>
<td>Year Dummies</td>
<td>Included</td>
<td>Included</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>.2841</td>
<td>.3143</td>
</tr>
<tr>
<td>N</td>
<td>1956</td>
<td>914</td>
</tr>
</tbody>
</table>
Appendix: Proofs of the Propositions

Proof of Proposition 1:
The uninformed investors in the IPO market face an informational disadvantage and will lose money if the price is set at the *ex ante* expected value of the shares. The uninformed investors must at least breakeven on their investments in the IPO market for it to be rational for them to participate in the market. Set the uninformed investors expected profit equal to zero $0 = \theta u X + (1 - \theta) 0 - p(\theta u + (1 - \theta))$ and rearrange to find the price function $p(u, \theta, X)$ in proposition 1. Note that there is underpricing relative to the *ex ante* expected value of the shares. The informed investors invest only in the good offerings and derive rents from their superior information. Uninformed investors invest in all issues, receiving all of the bad offerings and the portion $u$ of the good offerings, and make zero profits at the offer price. Good firms have no way of separating themselves from bad firms in the model so sell underpriced equity rather than accept the value zero alternative. Bad firms must mimic good firms in the publicly observable choices (the unobservable effort decision is the exception) or they will not get funded.

Proof of Proposition 2
The proof has three steps: we show that $T_{AI}$ and $T_{MH}$ are well-defined, we show that the capital constraint is binding, and finally we derive the inequalities in the proposition.

*Step 1:* Define $T_{\max} = \arg\max\{Q(T)X\delta^T\}$. The first-order condition implied by this definition is $Q'\delta^T + Q\delta^T\ln(\delta) = 0$, which has a unique solution if $Q' = -Q\ln(\delta)$ does. The left hand side of this last equation is a decreasing function of $T$ by the assumed concavity of the function $Q$, while the right hand side is an increasing function of $T$. Note that $T_{AI} \leq T_{\max}$. The two expressions are equal when $\theta = 0$, but $T_{AI}$ is a decreasing function of $\theta$ while $T_{\max}$ is independent of $\theta$. The first-order condition that defines $T_{AI} = \arg\max\{[Q(T)X + (1 - Q(T))EV]\delta^T\}$ is

$$Q'X(1 - \theta)\delta^T = -(\ln \delta)[X\theta + QX(1 - \theta)\delta^T]$$

The left-hand side of this equation is a decreasing function of $T$. The right-hand side of this equation is an increasing function of $T$ over the relevant range since $T_{AI} \leq T_{\max}$. Thus $T_{AI}$ is uniquely defined. The proof for $T_{MH}$ is similar.

*Step 2:* We establish that, in each version of the solution, the optimal $\alpha$, $\alpha^*$, satisfies the equation $\alpha^* p = I$. For the Asymmetric Information Case (when the IC constraint does not bind) the manager’s objective function is $\alpha p - I + (1 - \alpha)[Q(T)X + (1 - Q(T))EV]\delta^T$. The choice of $\alpha$ therefore is based on a comparison of the equilibrium IPO price $p$ and the quantity $[Q(T)X + (1 - Q(T))EV]\delta^T$ which we label $\pi(T)$. Evaluating $\pi(T)$ at $T = 0$ we can see that $\pi(0) = EV = \theta X + (1 - \theta)0 > p$ due to the underpricing of IPO shares. Because the optimal lockup length in the Asymmetric Information Case is chosen to maximize $\pi(T)$ it is clear that $\pi(T_{AI}) > \pi(0) > p$ which implies that the minimal $\alpha$ is optimal. Because in the solution to the Moral Hazard Case the optimal lockup length $T_{MH} > T_{AI}$ the argument is not as simple as in the
Asymmetric Information Case. In particular, it may be beneficial to select an \( \alpha > \alpha^* \) and a larger \( T \) in order to increase the incentives for managerial effort. This can easily be shown not to be an optimal strategy. The problem in the Moral Hazard Case, by assumption, can be written with the IC constraint binding:

\[
\begin{align*}
\text{Max}_{\alpha, T} & \quad \alpha p(u, \theta, X) - I + (1 - \alpha)[Q(T)X + (1 - Q(T))EV]\delta^T \\
\text{s.t.} & \quad (1 - \alpha)Q(T)X\delta^T = C_L \quad \text{(IC)} \\
& \quad \alpha p(u, \theta, X) \geq I \quad \text{(CC)}
\end{align*}
\]

Solving the IC constraint for \( \alpha \) and substituting this into the maximand provides:

\[
(p - I) + \left[ \frac{C_L}{Q(T)X\delta^T} \right] \left( [Q(T)X + (1 - Q(T))EV]\delta^T - p \right)
\]

Noting that both terms involving \( T \) are decreasing in the relevant range implies the optimal choice will always involve the minimum \( T \) that satisfies the IC constraint. The optimal \( \alpha \) therefore will be \( \alpha^* \) (defined by \( \alpha^* p = I \)) in the Moral Hazard Case.

**Step 3:** Note that the objective function above takes its unconstrained maximum at \( T_{AI} \). If \( Q(T_{AI})\delta^{T_{AI}} \geq \frac{C_L}{(1 - \alpha)X} \) then the incentive compatibility constraint is slack at the optimum and \( T_{AI} \) is the solution to the design problem. If \( Q(T_{AI})\delta^{T_{AI}} < \frac{C_L}{(1 - \alpha)X} \) then the incentive compatibility constraint binds. The lockup is of insufficient length to resolve the moral hazard problem and the solution must have \( T > T_{AI} \). Because the objective function is decreasing over this range (by definition of \( T_{AI} \)) the firm prefers the shortest lockup satisfying the incentive compatibility constraint. Thus \( T_{MH} \) defined by \( Q(T_{MH})\delta^{T_{MH}} = \frac{C_L}{(1 - \alpha)X} \) solves the design problem.

**Proof of Corollary 2:**

First we derive the comparative static properties of \( T_{MH} \). Recall that \( Q(T)\delta^T \) takes a unique maximum at some value \( T_{Max} \); see step 1 of the proof of Proposition 2. Since this lockup length maximizes the incentive to put forth effort, it follows that \( T_{MH} \leq T_{Max} \). Hence the function \( Q(T)\delta^T \) is increasing over the relevant range, and the comparative static results \( \frac{\partial T_{MH}}{\partial X} < 0 \) and \( \frac{\partial T_{MH}}{\partial \theta} = 0 \) follow immediately from the definition of \( T_{MH} \).

Next we define \( M(T) = [Q(T)X + (1 - Q(T))\theta X]\delta^T \) and let \( N(T) = M'(T) \) be the derivative with respect to \( T \). \( N(T) = Q'(T)(X - \theta X)\delta^T + [\theta X + Q(T)(X - \theta X)]\delta^T \ln(\delta) \), and is zero by definition. Using the implicit function theorem on \( N(T_{AI}) = 0 \) allows us to derive the comparative static characteristics for \( T_{AI} \).
By the implicit function theorem, $\frac{\partial T_{AI}}{\partial \theta} = \frac{-\frac{\partial N(T_{AI})}{\partial \theta}}{\frac{\partial N(T_{AI})}{\partial T}}$. Note that $\frac{\partial N(T_{AI})}{\partial T} < 0$ given the regularity assumptions on $Q(T)$ as this is the second order condition for the maximization problem. The derivative of $N$ with respect to $\theta$ is

$$\frac{\partial N(T_{AI})}{\partial \theta} = Q'(T_{AI})\delta^T_{AI}(-X) + [X - Q(T)X']\delta^T_{AI} \ln(\delta).$$

Both terms in the sum are negative which implies that $\frac{\partial (T_{AI})}{\partial \theta} < 0$. Finally, inspection of $M(T)$ shows that the solution to the asymmetric information problem $T_{AI}$ is independent of $X$. 