4. Options Markets

4.7. Hedging Strategies
How Does the OTC Market Hedge?

(Hull) A F.I. wrote for $300,000 a European call option on 100,000 shares of a non dividend-paying stock, where

\[ S = 49 \]
\[ X = 50 \]
\[ r = 0.05 \]
\[ \sigma = 0.2 \]
\[ T = \frac{20}{52} \]
Stay Naked or Run for Cover?

• Naked position:
  write the call
  \[300,000 - \max\{S_T - 50, 0\} \times 100,000\]

• Covered position:
  write the call and buy the stock
  \[300,000 - \max\{S_T - 50, 0\} \times 100,000 + (S_T - 49) \times 100,000\]
Stop-Loss?

- Covered position when \( S > 50 \)
  “buy high”

- Naked position when \( S < 50 \)
  “sell low”
Delta-Hedging?

- Binomial trees
  \[ \Delta = \frac{f_u - f_d}{S_u - S_d} \]

- Black-Scholes formulas
  \[ \Delta = \frac{\partial c}{\partial S} = N(d_1) \]
  \[ \Delta = \frac{\partial p}{\partial S} = N(d_1) - 1 \]
The $\Delta$ of Different Options

<table>
<thead>
<tr>
<th></th>
<th>call</th>
<th>put</th>
</tr>
</thead>
<tbody>
<tr>
<td>stock</td>
<td>$N(d_1)$</td>
<td>$N(d_1) - 1$</td>
</tr>
<tr>
<td>stock index</td>
<td>$N(d_1)e^{-qT}$</td>
<td>$[N(d_1) - 1]e^{-qT}$</td>
</tr>
<tr>
<td>currency</td>
<td>$N(d_1)e^{-r_f T}$</td>
<td>$[N(d_1) - 1]e^{-r_f T}$</td>
</tr>
<tr>
<td>futures</td>
<td>$N(d_1)e^{-r T}$</td>
<td>$[N(d_1) - 1]e^{-r T}$</td>
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</tbody>
</table>

$\Delta$ is positive for calls, negative for puts.
Example

(Hull) A F.I. wrote a European put option on £1 million, where

\[ S = \frac{\text{USD}1.62}{\text{£1}} \]
\[ X = \frac{\text{USD}1.6}{\text{£1}} \]
\[ r = 0.1 \]
\[ r_f = 0.13 \]
\[ \sigma = 0.15 \]
\[ T = \frac{6}{12} \]
The “Greeks”

- Delta
- Gamma for curvature
- Theta for time
- Vega for volatility
- Rho for risk-free interest rate
Gamma for Curvature

\[ \Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 f}{\partial S^2} \]

The \( \Delta \)-hedging of a higher \( \Gamma \) portfolio needs to be readjusted more frequently than that of a lower \( \Gamma \) portfolio.

\( \Gamma \) is positive.
Theta for Time

$$\Theta = \frac{\partial f}{-\partial T}$$

$\Theta$ is typically negative.
Δ, Γ, and Θ

\[ \Theta + r S \Delta + \frac{1}{2} \sigma^2 S^2 \Gamma = r f \]

Θ proxies for Γ when the portfolio is Δ-neutral.
Vega for Volatility

\[ \text{Vega} = \frac{\partial f}{\partial \sigma} \]

Vega is positive.
Rho for Risk-Free Interest Rate

\[ \rho = \frac{\partial f}{\partial r} \]

\( \rho \) is positive for calls, negative for puts.
Hedging Versus Creating an Option Synthetically

- riskless = writing $f+$ holding $\Delta S$
  buying $f = $ holding $\Delta S$—riskless

- Hedging is the reverse of creating an option synthetically
Hedging a Portfolio

• Static
  Buy put options

• Dynamic
  Create synthetic put options
  Sell shares ($\Delta < 0$)
  $S$ increases: $\Delta$ increases towards 0
  $S$ decreases: $\Delta$ decreases towards $-1$
Momentum Versus Contrarian

- Momentum strategy:
  buy high & sell low
  for hedging

- Contrarian strategy:
  buy low & sell high
  for profits
With price jumps, there is no way to synthetically replicate a put option pay-off.
Homework

1. (Baby Hull 15.11, Papa Hull 15.10) What is the delta of a short position in 1000 European call options on silver futures? The options mature in eight months, and the futures contract underlying the option matures in nine months. The current nine-month futures price is $8 per ounce, the exercise price of the options is $8, the risk-free interest rate is 12 percent per annum, and the volatility of silver is 18 percent per annum.

2. (Baby Hull 15.12, Papa Hull 15.11) In the previous problem, what initial position in nine-month silver futures is necessary for delta hedging? If silver itself is used, what is the initial position? If one-year silver futures are used, what is the initial position? Assume no storage costs for silver.

3. (Baby Hull 15.17, Papa Hull 15.16) A fund manager has a well-diversified portfolio that mirrors the performance of the S&P 500 and is worth $360 million. The value of the S&P 500 is at 1200, and the portfolio manager would like to buy insurance against a reduction of more than 5 percent in the value of the portfolio over the next six months. The risk-free interest rate is 6 percent per annum. The dividend yield on both the portfolio and the S&P 500 is 3 percent, and the volatility of the index is 30 percent per annum.

   (a) If the fund manager buys traded European put options, how much would the insurance cost?
   (b) Explain carefully alternative strategies open to the fund manager involving traded European call options, and show that they lead to the same result.
   (c) If the fund manager decides to provide insurance by keeping part of the portfolio in risk-free securities, what should the initial position be?
   (d) If the fund manager decides to provide insurance by using nine-month index futures, what should the initial position be?