4. Options Markets

4.6. Applications
4.6.1. Options on Stock Indices

• e.g.:
  Dow Jones Industrial (European)
  S&P 500 (European)
  S&P 100 (American)
  LEAPS

• $S$ becomes to $Se^{-QT}$
Basic Properties with $Se^{-qT}$

$$c \geq Se^{-qT} - Xe^{-rT}$$

$$p \geq Xe^{-rT} - Se^{-qT}$$

$$c + Xe^{-rT} = p + Se^{-qT}$$

$$Se^{-qT} - X < C - P < S - Xe^{-rT}$$
Binomial Trees with $Se^{-qT}$

$$\Delta = \frac{f_u - f_d}{S_u - S_d}$$

$$p = \frac{e^{(r-q)T} - d}{u - d}$$

$$f = e^{-rT}[pf_u + (1 - p)f_d]$$
Black-Scholes with $Se^{-qT}$

$$c = Se^{-qT}N(d_1) - Xe^{-rT}N(d_2)$$

$$p = Xe^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$$

$$d_1 = \frac{\ln \frac{S}{X} + (r - q + \frac{\sigma^2}{2})T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$
Hedging a Stock Portfolio

\[ E[r_p] - r = \beta(E[r_m] - r) \]

\[ N = \frac{V_p}{100 \times V_m} \]
4.6.2. Options on Currencies

- Currency options on PHLX
- Contract size varies across currencies
- Active OTC market

$S$ becomes $Se^{-rfT}$
Basic Properties with $Se^{-r_f T}$

\[ c \geq Se^{-r_f T} - X e^{-r T} \]
\[ p \geq X e^{-r T} - Se^{-r_f T} \]
\[ c + X e^{-r T} = p + Se^{-r_f T} \]
\[ Se^{-r_f T} - X < C - P < S - X e^{-r T} \]
Binomial Trees with $Se^{-rfT}$

$$\Delta = \frac{f_u - f_d}{S_u - S_d}$$

$$p = \frac{e^{(r-r_f)T} - d}{u - d}$$

$$f = e^{-rT}[pf_u + (1 - p)f_d]$$
Black-Scholes with $S e^{-r_f T}$

\[ c = S e^{-r_f T} N(d_1) - X e^{-r T} N(d_2) \]

\[ p = X e^{-r T} N(-d_2) - S e^{-r_f T} N(-d_1) \]

\[ d_1 = \frac{\ln \frac{S}{X} + (r - r_f + \frac{\sigma^2}{2})T}{\sigma \sqrt{T}} \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]
4.6.3. Options on Futures

A derivative on a derivative, why?

• Futures may entail lower transaction costs than the underlying asset.

• Exercising the option does not lead to delivery of the underlying asset.

• Futures options and futures typically trade in adjacent pits.
What Happens at Exercise?

• When a call (put) spot option is exercised, the holder buys (sells) the underlying asset worth $S$ at $X$.

• When a call (put) futures option is exercised, the holder gets a long (short) position in the futures.

Because of marking-to-market, the holder also gets a cash amount $F - X (X - F)$. 
Basic Properties, Binomial Trees, and Black-Scholes

• $F = Se^{rT}$
  thus $S$ becomes $Fe^{-rT}$
Basic Properties with $Fe^{-rT}$

\[ c \geq (F - X)e^{-rT} \]

\[ p \geq (X - F)e^{-rT} \]

\[ c + Xe^{-rT} = p + Fe^{-rT} \]

\[ Fe^{-rT} - X < C - P < F - Xe^{-rT} \]
Binomial Trees with $Fe^{-rT}$

\[ \Delta = \frac{f_u - f_d}{F_u - F_d} \]

\[ p = \frac{1 - d}{u - d} \]

\[ f = e^{-rT}[pf_u + (1 - p)f_d] \]
Black-Scholes with $Fe^{-rT}$

\[ c = e^{-rT} \left[ FN(d_1) - XN(d_2) \right] \]

\[ p = e^{-rT} \left[ XN(-d_2) - FN(-d_1) \right] \]

\[ d_1 = \frac{\ln \frac{F}{X} + \frac{\sigma^2 T}{2}}{\sigma \sqrt{T}} \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]
Homework

1. (Baby Hull 13.11, Papa Hull 14.15) The S&P 100 index currently stands at 696 and has a volatility of 30 percent per annum. The risk-free rate of interest is 7 percent per annum and the index provides a dividend yield of 4 percent per annum. Calculate the value of a three-month European put with an exercise price of 700.

2. (Baby Hull 13.10, Papa Hull 14.18) Consider a stock index currently standing at 250. The dividend yield on the index is 4 percent per annum and the risk-free rate is 6 percent per annum. A three-month European call option on the index with a strike price of 245 is currently worth $10. What is the value of a three-month put option on the index with a strike price of 245?

3. (Baby Hull 13.16, Papa Hull 14.21) Suppose that a portfolio is worth $60 million and the S&P 500 is at 1200. If the value of the portfolio mirrors the value of the index, what options should be purchased to provide protection against the value of the portfolio falling below $54 million in one year’s time?

4. (Baby Hull 13.17, Papa Hull 14.22) Consider again the situation in the previous problem. Suppose that the portfolio has a beta of 2.0, the risk-free interest rate is 5 percent per annum, and the dividend yield on both the portfolio and the index is 3 percent per annum. What options should be purchased to provide protection against the value of the portfolio falling below $54 million?

5. (Baby Hull 13.19, Papa Hull 14.41) A stock index currently stands at 300. It is expected to increase or decrease by 10 percent over each of the next two time periods of three months. The risk-free interest rate is 8 percent and the dividend yield on the index is 3 percent. What is the value of a six-month put option on the index with a strike price of 300 if it is
   (a) European?
   (b) American?

6. (Baby Hull 13.20, Papa Hull 14.42) Suppose that the spot price of the Canadian dollar is U.S.$0.75 and that the Canadian dollar/U.S. dollar exchange rate has a volatility of 4 percent per annum. The risk-free rates of interest in Canada and the United States are 9 percent and 7 percent per annum.
annum, respectively. Calculate the value of a European call option with an exercise price of $0.75 and an exercise date in 9 months.

7. (Baby Hull 14.10, Papa Hull 14.25) Consider a two-month call futures option with a strike price of 40 when the risk-free interest rate is 10 percent per annum. The current futures price is 47. What is a lower bound for the value of the futures option if it is
   (a) European?
   (b) American?

8. (Baby Hull 14.11, Papa Hull 14.26) Consider a four-month put futures option with a strike price of 50 when the risk-free interest rate is 10 percent per annum. The current futures price is 47. What is a lower bound for the value of the futures option if it is
   (a) European?
   (b) American?

9. (Baby Hull 14.12, Papa Hull 14.27) A futures price is currently 60. It is known that over each of the next two three-month periods it will either rise by 10 percent or fall by 10 percent. The risk-free interest rate is 8 percent per annum. What is the value of a six-month European call option on the futures with a strike price of 60? If the call were American, would it ever be worth exercising early?

10. (Baby Hull 14.13, Papa Hull 14.28) In the previous problem what is the value of a six-month European put option on futures with a strike price of 60? If the put were American, would it ever be worth exercising it early? Verify that the call prices calculated in the previous problem and the put prices calculated here satisfy put-call parity relationships.

11. (Baby Hull 14.14, Papa Hull 14.29) A futures price is currently 25, its volatility is 30 percent per annum, and the risk-free interest rate is 10 percent per annum. What is the value of a nine-month European call on the futures with a strike price of 26?

12. (Baby Hull 14.15, Papa Hull 14.30) A futures price is currently 70, its volatility is 20 percent per annum, and the risk-free interest rate is 6 percent per annum. What is the value of a five-month European put on the futures with a strike price of 65?