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Optimal Financial Instruments

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ABSTRACT

Debt and equity are developed as optimal financial instruments in a model where cash flows and control rights are allocated to investors endogenously. When investment decisions must be made by a single party, the debtholder's cash flows are fixed in order to provide the equityholder with efficient incentives for investment. Ownership of control may be transferred to the debtholder to attenuate the impact of asymmetric information, concerning the investment opportunity, on the efficiency of the decision making.

SINCE THE CLASSIC ARTICLE by Modigliani and Miller (1958), a great deal of attention has been focused on the problem of determining the optimal capital structure of the firm. Spanning, taxes and/or transaction cost, agency cost, and signalling arguments have been offered as explanations for why a firm's cash flows might be split between debt and equity claimants in a particular way. Here we address the more basic question of the optimal design of the securities that firms issue.

The majority of the existing literature takes the form of the allowable financial instruments as given. In defense of this approach, it may be noted that to a first approximation firms have used two standard financial instruments, debt and equity. We argue here, however, that the characterization of the standard instruments used in much of the literature has been incomplete and that the incentives of decision makers within the firm cannot be completely understood until the problem is addressed from a more basic level than is typically done. By concentrating on the cash flow characteristics of these instruments, the equally important issue of the way the standard instruments distribute the control rights of the firm has been ignored. As a consequence, the objectives in corporate decision making have been assumed rather than developed endogenously.

We consider the allocation of cash flows and the distribution of control rights to optimal financial instruments in a security design problem. By considering the design of securities in this way we are able to consider the

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incentives of the decision maker from a more primitive level than has traditionally been done. Our model involves two potentially conflicting interest groups. Each is represented as a single agent. An investor who is initially assigned control of the corporation enjoys the right to make decisions concerning investment and the distribution of intermediate cash flows. The resolution of disagreements between the active investor and the second investor, who is assumed to be passive, are governed by optimal contracts that we interpret as financial instruments.

We focus on disagreements concerning investment and dividend policy. These arise because investment expenditures are unobservable to the passive investor. Our main result is that standard debt and equity contracts are optimal financial instruments in this setting; they maximize the value of the firm subject to the constraints implied by private information.

Debt and equity contracts distribute both the cash flow generated by the firm and define state-contingent ownership of control rights. The analysis provided here suggests that the debtholder's cash flows are fixed because the controlling party realizes the marginal product of investment if, and only if, this is the case. The tendency of the fixed payment schedule to induce under-investment or risk-shifting when there is private information about investment is offset by the potential transfer of control. The timing of this transfer explains the disparity between our characterization of the incentive properties of debt and the characterization that arises in much of the literature. In other models, suboptimal decision making occurs because the controlling party has an incentive to invest inefficiently, and is permitted to do so. In our model, the contracting parties realize ex ante that this incentive may be present ex post. A transfer of control occurs whenever a publicly observed signal indicates that the incentive to invest inefficiently is present. Thus, the contracting parties act to mitigate the incentive problem before it arises. Although the model developed here is restrictive, these insights seem to be quite general.

The interpretation that securities are designed to implement optimal investment decisions, providing a possible explanation for the state-contingent transfer of control, allowing an endogenous examination of decision-making incentives within the firm, and providing a model of the intuition that the existing instruments are designed so that in each state the owner of the residual control rights owns the residual cash flow are the major contributions of this study.[1]

Allowing the contracts to distribute both control rights and cash flows results in a significant generalization of previous research. Diamond (1984) and Townsend (1979) show that a "debt" or fixed payment contract is the

[1] Easterbrook and Fischel (1983) (see pp. 403–404) advance the argument that shareholders, due to their status as residual claimants, "are the group with the appropriate incentives to make discretionary decisions," and that "when the firm is insolvent, the bondholders and other creditors eventually acquire control." The model presented here formalizes this claim and provides a framework within which this argument is valid.

only incentive-compatible contract when there is limited information available concerning the cash flow generated by the firm. In these models, it is the incentive compatibility of the truthful reporting of final cash flows that determines how the cash flow is distributed by the contracts. In contrast, our model specifies that the joint distribution of the cash flows and the control rights is driven by incentive compatibility for the investment decision (value creation).

Hart and Moore (1989) develop a dynamic model of debt considering the control rights associated with the debt contract rather than the incentive properties of debt. In their model, decision-making power remains with the "active" investor and is only transferred to the outside claimant via the "passive" claimant's ability to sell off portions of the firm's assets.

Williams (1987) and Chang (1986) also provide arguments for the cash flows associated with debt and equity instruments. They do not, however, consider the way in which control over decision making is distributed by the standard contracts.

In an incomplete markets environment, Allen and Gale (1988) provide the negative result that if there are costs to issuing securities then the standard debt and equity contracts will not be issued. Instead, instruments are designed so that each state's cash flows may be sold to those investors who value them most. Their analysis also ignores the way in which control is distributed by the different securities.

Harris and Raviv (1989) have independently considered optimal securities that distribute both cash flows and control rights (which they define as voting rights). The focus of Harris and Raviv is on the market for corporate control. Neither the incentives for decision making within the firm nor the possibility of bankruptcy (the state-contingent assignment of control across securities) are considered in their model.

A more general version of bankruptcy is developed in this model than is considered in much of the literature. In traditional models focusing on inefficient investment behavior, bankruptcy is represented as terminal cash flows being less than the required payment on the debt. Debt and equity holders are viewed as adversaries. In contrast, we view bankruptcy as a tool that broadens the investment opportunity set and facilitates cooperation between the contracting parties. The state-contingent transfer of control relaxes an incentive constraint that would bind if bankruptcy were not allowed, enhancing the value of the firm as a going concern.

Hart and Moore (1988) consider the optimal ex ante distribution of the ownership of assets between coalitions of agents. In our model, bankruptcy is an extension of this idea which allows state-contingent changes in the ex ante ownership scheme as new information arrives. The possibility of bankruptcy vastly expands the set of potential arrangements.

In a model similar in spirit to that given here, Aghion and Bolton (1988) also provide a positive role for bankruptcy. In their model, disputes between security holders are a consequence of state-contingent differences in private nonpecuniary costs associated with the investment decision. Aghion and

Bolton focus on the differences in the control rights associated with debt and equity as the distinguishing features of the contracts, leaving their cash flow characteristics in the background. The main focus of Aghion and Bolton is on the possible renegotiation of the contracts based on information revealed concerning the prospects of the firm. Bankruptcy is a state-contingent transfer of control that facilitates the renegotiation of the contracts. The main difference between the two characterizations is that, in our model, the possibility of bankruptcy induces efficient investment decisions, rather than providing a framework for renegotiation.

The paper is organized as follows. Section I presents the model and describes the class of allowable financial instruments. Section II presents the optimal instruments for the model. In Section III, the results are discussed, and conclusions are presented in Section IV.

I. The Model

In this model of corporate control, we consider three agents. An entrepreneur owns the rights to a firm and designs the financial instruments used to finance the firm. The entrepreneur is assumed to have no personal capital. Two identical investors, neither of which has sufficient resources to capitalize the firm, have combined wealth sufficient to establish the firm. Securities must therefore be created to overcome the investors' wealth constraints.

The entrepreneur is a perspective from which we judge the optimality of the financial instruments. After selling the securities the entrepreneur completely divorces himself from the firm. The securities are therefore designed to maximize the entrepreneur's initial wealth, i.e., the market value of the securities. We allow that securities distribute both the cash flows and control rights between the two investors. Optimal financial instruments will therefore align, as closely as possible, the interests of the controlling investor with firm-value maximization.

The role of the entrepreneur in this model does not influence the design of the optimal securities in any significant way. The entrepreneur is simply an artifice used to induce efficiency. One standard story of an entrepreneur with insufficient funds to finance a firm is that the shortfall is made up with outside debt, and that ownership of control remains with the entrepreneur. In this model, the entrepreneur may also be the first investor. The contract sold to the second investor would then be the optimal contract for use by the entrepreneur/manager to raise the needed investment capital.

The two identical investors, labeled investor *1* and investor *2*, are assumed to have utility functions that are risk neutral in wealth. In order to highlight wealth constraints as the motivation for the creation of the financial instruments, we assume that each investor has a limited amount of investment capital, denoted W. Each investor's wealth, W, is assumed to be less than I^0, the amount of capital required to begin the firm, while $2W$ is more than enough (to be made precise later). The capital market is assumed to price assets in a competitive, risk-neutral manner. For simplicity, the alternative investment opportunity available to the investors is a risk-free asset where the riskless rate of return is taken to be zero.

The investor assigned control at a given time receives information concerning the value of investment opportunities facing the firm and makes the investment decision for the firm. We use a simple model of the firm's decision making (in which investment decisions influence the distribution of the cash flow generated by the firm) to examine the noncooperative game played between the firm's different claimants.

The action of the model is as follows. There are four points in time. At time *0* the contracts are designed and sold, an initial investment of I^0 is made, and the firm is established. At time *1* valuable information concerning time *3* cash flow potential is publicly observed and an intermediate cash flow is received by the firm. Based on this news the financial instruments assign control over the time *2* decision making. At time *2*, an investment opportunity will become available to the firm. The investor assigned control over the time *2* decision making owns control of the intermediate cash flow and makes the investment decision for the firm based upon his knowledge of the investment opportunity facing the firm. Publicly all that is known is that the investment opportunity will be drawn from a set of potential projects; the available project is privately revealed to the controlling investor at time *2*. We assume that the level of time *2* investment expenditure, I, is also unobservable to outside parties. The controlling investor divides the realized cash flow between investment expenditure and dividends paid to himself. The investment level and the available project jointly determine the probability distribution of Y, the time *3* cash flow generated by the firm. At time *3* the realization of Y is publicly observed, the firm is liquidated, and payments are made to the claimants as specified by the financial instruments.[2]

Uncertainty in the model is over intermediate cash flow, denoted X, the investment project available at time *2*, and final cash flow, Y. The distribution of intermediate cash flow is common knowledge at time *0*. The random variable X is assumed to be discrete with outcomes X_i, $i = 1, \cdots, N$, which are ordered to be increasing in i. The realization of X is publicly observable

[2] In the analysis of the model we do not allow for renegotiation of the financial instruments by the claimants. Initially we assume that there are no transactions costs, and so there must be a version of the "Coase Theorem" that provides that the initial contracts are indeterminate, i.e., renegotiation based on the public information should provide for optimal decision making regardless of the form of the initial contracts. However, because of the structure of the model, the renegotiated contracts must provide that the controlling investor is the residual claimant. The analysis of the model simply provides these contracts initially. We could alternatively assume that there is some positive cost to renegotiation. In this case, the initial contracting arrangement becomes important, and there is no value to renegotiation in the model. If there are costs associated with the transfer of control and complete contingent contracts cannot be written at time *0*, then there will be room for profitable renegotiation of the financial instruments (see Aghion and Bolton (1988) or Kalay and Zender (1990) for models which include this feature). Kalay and Zender (1990) also consider circumstances under which information can lead to a restriction on the exercise of control by the controlling investor rather than an outright transfer of the ownership of control.

at time *1*. The random variable Y is also assumed to be discrete with realizations Y_j, $j = 1, \cdots, M$, where the Y_j are ordered so that they are increasing in j. We interpret Y as a liquidating cash flow, and it is assumed to be publicly observable.

We assume that an informative signal concerning Y is publicly observed at time *1*. The public signal, denoted $\phi^{j'}$, has two realizations H or L. In the security design problem, the level, j', of the signal $\phi^{j'}$ is a choice variable for the entrepreneur.[3] The signal $\phi^{j'}$ is valuable because of the following relation between $\phi^{j'}$ and Y. When the signal $\phi^{j'} = H$, the conditional support of Y is $\{Y_j \mid j = j', \cdots, M\}$ and when $\phi^{j'} = L$, the conditional support of Y is $\{Y_j \mid j = 1, \cdots, j' - 1\}$.[4] The information received from $\phi^{j'}$ therefore indicates future cash flow potential in a plausible way. The signal indicates that time *3* cash flow will be drawn from one of two disjoint sets,[5] i.e., the signal provides information that time *3* cash flows will be good or bad relative to a given benchmark. The realization $\phi^{j'} = H$ indicates good prospects for the future (Y will not fall below some level). If $\phi^{j'} = L$, the signal indicates poor future prospects, relative to the chosen benchmark.[6]

There is a known set of potential new investment projects, one of which will be available to the firm at time *2*. The production function associated with each of the projects in the set of potential projects and the probability with which each of these projects becomes available are common knowledge at time *0*. We assume that there are $K > 1$ projects, indexed by $k = 1, \cdots, K$, that may be available at time *2*, and label the associated probabilities π_k, where $\sum_k \pi_k = 1$.

We denote the conditional distribution of Y by the probability density functions $p_{k,j}(I)$ (where $p_{k,j}(I)$ is the probability of Y_j occurring given that project k was available and investment level I was chosen) for $j = 1, \cdots, j' - 1$ if $\phi^{j'} = L$ and the functions $q_{k,j}(I)$ for $j = j', \cdots, M$ otherwise. We make the following assumptions concerning these functions.

Assumption 1: *$p_{k,j}(I) > 0$ and $q_{k,j}(I) > 0$ for all k, j, and I.*

[3] We assume that $\phi^{j'}$ is not subject to moral hazard and is publicly observable. In this model, the signal could be thought of as being a function of any observable variable. Practical examples of such a signal (as a function of intermediate cash flows for example) include coupon payments or sinking fund contributions. Other examples, such as interest coverage ratios or minimum net worth constraints, might be based on accounting data for which an acceptable audit is available.

[4] It is not important for our model that the signal itself be binary, only that the information described above, concerning the conditional support of the final cash flows, may be derived from some function of the publicly available information. We may think of the realization of ϕ as a "reduced form" of this signal extraction problem.

[5] The technology can easily be changed to accommodate a less precise signal which indicates that Y will be drawn from two overlapping sets.

[6] We assume that it is prohibitively costly for the controlling investor to communicate his/her private information concerning the investment environment to outside parties beyond that contained in the public signal. In a working paper, Kalay and Zender (1990) consider the possibility that an imprecise signal is communicated at low cost *by the firm* to the market while more precise information concerning the investment decision may only be transmitted at high cost.

Assumption 2: *The $p_{k,j}(I)$ and $q_{k,j}(I)$ are twice continuously differentiable for all k and j.*

Assumption 3: *The sets of functions $\{p_{k,j}(I)\}_1^{j'-1}$, and $\{q_{k,j}(I)\}_j^M$, satisfy the stochastic dominance condition (SDC); the derivative with respect to I of the associated distribution function is nonpositive for all j, k, and all I.*

Assumption 4: *The probability functions $\{q_{k,j}(I)\}_{j'}^M$ and $\{p_{k,j}(I)\}_1^{j'-1}$, satisfy the convexity of the distribution function condition (CDFC); the second derivative of the distribution function is non-negative for each j, k, and all I.*

Under these assumptions investment expenditure shifts the conditional probability distribution over final cash flow to the right, in the sense of stochastic dominance. The SDC implies that the distribution function is decreasing in I for each j so that expected cash flows increase with investment. The CDFC implies the distribution function is decreasing in I at a decreasing rate, providing a concave investment decision problem for the controlling investor. Note that these conditions extend easily to the ex ante density functions $p_j(I) = \sum_k \pi_k p_{k,j}(I)$ and $q_j(I)$ (defined analogously).

At time *2* the controlling investor privately observes the realized k, the available project, and based on the realization of the public signal and k, makes an investment choice. Because the level of investment is assumed to be unobservable to the outside investor, the amount of the intermediate cash flow that remains after investment is also unobservable. For simplicity we assume that the residual is paid out to the controlling investor and that this dispersement cannot be contracted upon.[7] The investment level, I, is chosen to maximize the expected value of the controlling investor's contract plus the dividend, given the production function implied by the realization of the public signal and the available investment project.

To summarize, the signal $\phi^{j'}$ provides the investors with information concerning the support of Y. The investor assigned control over time *2* decision making selects a level of investment that, given project k is available, shifts the probability distribution over the conditional support according to the functions $\{p_{k,j}(I)\}$ or $\{q_{k,j}(I)\}$. This production function is observed only by the controlling investor. The realized intermediate cash flow that is not invested is assumed to be paid out to the controlling investor at time *2*. Financial instruments are designed so that the controlling investor

[7] Because the chosen level I is private information to the controlling investor, the "free cash flow" $X_i - I$ is also unknown to the public. We label this cash flow as dividends and assume that they are dispersed to the controlling investor for simplicity. What is required is only that the controlling investor perceive a tradeoff between investment and the benefits of this "free cash flow," not that the cash be distributed to the controlling investor. The benefits may be taken by the controlling investor as excess on-the-job perquisite taking, or in various other forms. The assumption that dividend payments are unobservable need not therefore be taken literally. In an earlier version of the paper the investment versus dividend decision was characterized as a personally costly effort choice made by the controlling investor. In that version of the model, shirking by the controlling investor reduced the value of the firm unless the controlling investor was faced with the marginal product of his effort decision as in the usual principal-agent model. In this model, the potential problem is an under-investment problem that is formally the same as that given by Myers (1977). The same results obtain in either version.

makes efficient investment decisions in the presence of this informational asymmetry. The action and resolution of uncertainty in the model are illustrated in the time line provided in Figure 1.

We now describe the allowable securities and the security design problem solved by the entrepreneur at time *0*. General financial instruments in our model can be represented as

$$C_1 \equiv \{j', d(\phi^{j'}), S(\phi^{j'}, Y)\}$$

and

$$C_2 \equiv \{j', d(\phi^{j'}), B(\phi^{j'}, Y)\},$$

where C_h is the contract designed to be sold to investor h. The elements of these vectors represent, respectively, the threshold level of the signal, the state-contingent distribution of control rights, and the state-contingent allocation of cash flows.

The choice of j' indicates which signal $\phi^{j'}$ ($j' = 1, \cdots, M$) is to be used. We restrict the entrepreneur to the choice of one such signal by assumption. We will see below that this is not a restrictive assumption and is justified by assuming that it is costly (for a variety of reasons) to release more information concerning the firm to the public. The function $d(\phi^{j'})$ is a state-dependent indicator function defined by

$$d(\phi^{j'}) = \begin{cases} 1 & \text{if investor } \mathit{1} \text{ owns the time } \mathit{2} \text{ decision rights} \\ 0 & \text{if investor } \mathit{2} \text{ owns the time } \mathit{2} \text{ decision rights.} \end{cases}$$

The assignment of control over the time *2* decision making may only depend upon information received before time *2*. Since the signal $\phi^{j'}$ is assumed to be publicly observable it is possible for the securities to depend upon this signal. The cash flows that accrue to each investor at time *3* are specified by the "sharing rules" $S()$ and $B()$. These functions may be dependent upon all publicly available information at time *3*.[8]

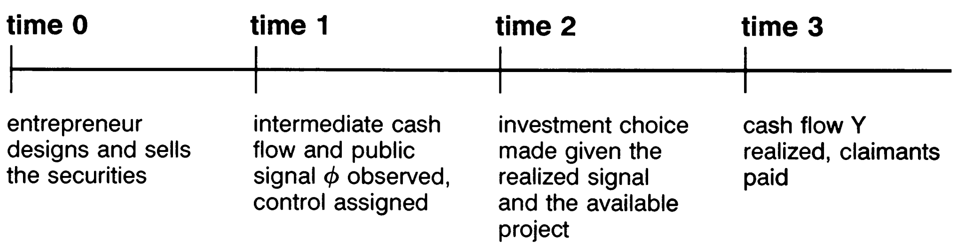

Figure 1. Time line. A summary of the timing of the action and the resolution of uncertainty in the model.

[8] The realization of X is public knowledge and therefore the functions $d()$, $S()$, and $B()$ may, in general, depend upon this realization. The realization of X is, however, not valuable to the investment decision or informative concerning the level of the investment that is made. There is, therefore, no value in making the contracts depend upon the realization of X as doing so simply adds noise to the contracts.

Definition: The permissible financial instruments satisfy the following conditions:

1) $d(\phi^{j'}) = 0$ or $1\ \forall \phi^{j'}$,
2) $S(\phi^{j'}, Y)$ and $B(\phi^{j'}, Y)$ are non-negative functions, and
3) $S(\phi^{j'}, Y) + B(\phi^{j'}, Y) = Y\ \forall\ \phi^{j'}$.

Condition 1 simply requires that one and only one of the investors be assigned the control rights at time *2*. This requirement is provided by the assumption that it is prohibitively costly to inform outside agents concerning the decision problem. Suppose that coordination of decision making is costly, or that one party enjoys a comparative advantage in running the firm, or that the cost of administering the investment is a fixed positive amount, per manager. Then contracts that award control to a single party provide economic benefit, and it is unnecessary to assume that contracts have this feature as is done here.

Condition 2 constrains the possible instruments to provide limited liability to the investors. This is a characteristic common to many corporate securities. Given the risk neutrality of our model this assumption is necessary for the problem to be interesting. We can justify this assumption in part by noting that it is the limited liability feature of the standard financial instruments that has allowed them to be freely traded among individuals. We therefore take this condition to be a basic requirement in the security design problem. Finally, condition 3 requires that the cash flows distributed to the investors at time *3* exactly equal the firm's realized cash flow.

Given this definition of permissible contracts, the entrepreneur's time *0* security design problem is written as follows. The entrepreneur chooses C_1, C_2, $I(\phi^{j'}, k)$, P_S, and P_B to maximize

$$P_S + P_B - I^0 \tag{1}$$

subject to the constraints

$$B\big(d(\phi^{j'}), Y\big) \geq 0 \quad \forall d\big(\phi^{j'}\big), \quad \text{and} \quad \forall Y \tag{2}$$

$$S\big(d(\phi^{j'}), Y\big) \geq 0 \quad \forall d\big(\phi^{j'}\big), \quad \text{and} \quad \forall Y \tag{3}$$

$$S\big(d(\phi^{j'}), Y\big) + B\big(d(\phi^{j'}), Y\big) = Y \quad \forall d\big(\phi^{j'}\big) \quad \text{and} \quad \forall Y \tag{4}$$

$$P_S + P_B \geq I^0 \tag{5}$$

$$I\big(\phi^{j'}, k\big) \leq X_i \tag{6}$$

$$P_S = E\big(S\big(d(\phi^{j'}), Y\big) + \big(X - I(\phi^{j'}, k)\big) d(\phi^{j'})\big) \tag{7}$$

$$P_B = E\big(B\big(d(\phi^{j'}), Y\big) + \big(X - I(\phi^{j'}, k)\big)\big(1 - d(\phi^{j'})\big)\big) \tag{8}$$

$$W \geq P_S \tag{9}$$

$$W \geq P_B \tag{10}$$

and I solves the controlling investor's maximization problem

$$\operatorname*{Max}_{I}\ \left[E\left(S\left(d(\phi^{j'}), Y\right) + \left(X - I(\phi^{j'}, k)\right) \mid k, X = X_{i'}\right] d(\phi^{j'})\ \forall \phi^{j'} \quad (11)$$

$$\operatorname*{Max}_{I}\ \left[E\left(B\left(d(\phi^{j'}), Y\right) + \left(X - I(\phi^{j'}, k)\right) \mid k, X = X_{i'}\right] \left(1 - d(\phi^{j'})\right)\ \forall \phi^{j'} \quad (12)$$

The entrepreneur designs the securities to maximize the rents accruing to ownership of the time *0* rights to the firm. Constraints (2), (3), and (4) restrict the chosen instruments to belong to the set of allowable instruments. Constraint (5) ensures that only firms with a positive net present value are established. We assume that this constraint holds throughout the analysis and so can solve it out and henceforth ignore it. Constraint (6) requires that the amount invested at time *2* is not more than the realized level of intermediate cash flow. For simplicity it is assumed that X_i (for all i) is at least as large as the first best level of investment (for all $\phi^{j'}$), and so this constraint is also satisfied at the optimum.[9] Constraints (7) and (8) are the rational risk neutral pricing functions for the securities. Inequalities (9) and (10) ensure that neither investor violates his/her own budget constraint. Because investment at time *2* is unobservable, constraints (11) and (12) are necessary to ensure that the chosen securities implement the desired investment policy in all states.

II. The Optimal Securities

The optimal financial instruments are given in the following proposition.

Proposition 1: *Optimal financial instruments for the problem given in equations* (1) *through* (12) *are given by*

$$C_1 = \left\{j', d(H) = 1, d(L) = 0, S(H, Y) = Y - F, S(L, Y) = 0\right\}$$

and

$$C_2 = \left\{j', d(H) = 1, d(L) = 0, B(H, Y) = F, B(L, Y) = Y\right\}$$

where the first best investment level, $I^*(\phi^{j'}, k)$, is chosen at time 2 for all j', and feasible pairs (j', F) are chosen so the investors' budget constraints are not violated. Feasible pairs (j', F) are defined as pairs for which $F < Y_{j'}$.

[9] The assumption is without loss of generality as added capital could instead be raised at time *0* to cover any possible shortfall. If we alter the model to allow additional capital to be raised in the capital market at time *2* and include an additional "security design" problem at that point, the analysis below makes clear that a contract resembling junior debt will be optimal.

Proof: Use the first-order condition approach to derive the Kuhn-Tucker conditions for the agency problem contained in equations (1) through (12). Then note that because the public signal ϕ divides the conditional support of Y into two intervals, feasible pairs of j' and F can be chosen so that the wealth constraints are not binding and the contracts given in the proposition satisfy the Kuhn-Tucker constraints. Because F is chosen so that $F < Y_{j'}$ the incentive compatibility constraint is never binding *at the optimum* and the Lagrange multiplier on this constraint will be equal to zero. The first-order condition approach therefore provides a valid solution to the problem.

We can describe these contracts as follows. The financial instruments specify that investor *1* owns the decision making rights if the realization of the public signal is high, and in these states owns all of the cash flows generated at time *3* less a constant payment made to investor *2*, denoted F. When the realization of the public signal is low, control is owned by investor *2* along with all the cash flows generated by the firm at time *3*. Feasibility simply requires that the level of the public signal be set to ensure that if its realization is high, then realized time *3* cash flows will be greater than the constant amount owned by investor *2* with probability 1.

These securities possess features characteristic of standard debt and equity instruments. First note that a state-contingent change in the ownership of control (bankruptcy), as is observed in practice, is derived endogenously in the model. This feature of the optimal financial instruments derives directly from the limited liability assumption and the investors' wealth constraints, the motivation for selling the financial instruments. Removing the wealth constraint in our model allows the entrepreneur to sell the entire firm to a single investor, removing the need for financial instruments and trivially allowing the first-best solution to the model. If the limited liability constraint is relaxed, it is possible to provide the controlling party with a payment schedule that represents the marginal product of investment, without resorting to the transfer of control.

A second notable feature is that the optimal securities provide that the investor in possession of the control rights is also the residual claimant. When $\phi^{j'} = L$, ownership of control is transferred to investor *2* ($d(L) = 0$) and all cash flows generated by the firm accrue to this investor.[10] If $\phi^{j'} = H$, investor *1* retains ownership of the control rights ($d(H) = 1$) and all cash flows generated by the firm, less the "face value" of the debt instrument, accrue to investor *1*. Note that if there is only one project available at time *0* the linear contracts developed here will not be uniquely optimal. Other contracts that depend explicitly on the conditional density functions of the investment project will also achieve the first best. Only in special cases will such contracts provide an optimal solution when the exact "production

[10] The value that is transferred to the "bondholder" at time *2* is represented by the time *2* market price of the firm given that the realization of $\phi^{j'} = L$, or $\sum p_j(I^*)Y_j$, where the summation is over $j = 1, \cdots, j'$. Note that in this version of the model this value may exceed F. This problem is addressed below.

function" is unknown at time *0*.[11] Because at time *0* the exact characteristics of the new investment project are, in practice, likely to be unknown, the robustness cash flow schemes given in Proposition 1 are an important feature of these contracts.

The level, j', of the signal $\phi^{j'}$ and the face value, F, of the "debt instrument" are chosen by the entrepreneur so that the budget constraints of the investors are not violated, and the investment decision rule implemented is I^*. The set of feasible pairs (j', F) is potentially large. We therefore establish a version of the traditional *MM* irrelevance proposition. This result stems from the fact that for all feasible pairs (j', F) the residual claimant makes firm value-maximizing choices.

The results of the security-design problem can be summarized as follows. The optimal contracts in this model initially assign the control rights and the residual cash flows to the "equityholder" as this arrangement implements optimal investment decisions. Because of the wealth constraints, the incentive compatibility constraint becomes binding when expected cash flows are low. A state-contingent change in ownership of the control rights (bankruptcy) is used to relax this constraint when it becomes binding. Bankruptcy provides the second investor with the ownership of control and all generated cash flows as this investor then has incentives for optimal decision making.

A. Uniqueness of the Instruments

The financial instruments provided in Proposition 1 are not uniquely optimal in the model. The indeterminacy lies in the state-contingent assignment of the ownership of control. Because we have assumed the two investors are identical and that there are no costs associated with the transfer of the ownership of control, optimal ownership structures need not be dependent upon the public signal ϕ as specified in Proposition 1. At the extreme, it is possible for the entrepreneur to conduct a lottery (with appropriately chosen probabilities) at time *0*, the winner of which is given the entire firm.[12] Because no decisions are made by the investors until time *2*, the ownership of control before that period is indeterminate in our framework. We speak of

[11] The fact that the contracts provide that the controlling investor is the residual claimant implies that the decision problem can be substantially enriched without affecting the results. For example, if we enrich the decision maker's action space to include the *choice* of a project $k \in \{1, \cdots, K\}$ (where one of the "projects" might represent liquidation of the firm), or if the controlling investor possesses other private information valuable in the investment decision, the derived contracts will remain optimal. One theme of this paper is the robustness of these contracts to such changes in the decision maker's action space.

[12] It is also true that for large values of W, relative to I^0, it is possible for investor *1* to retain ownership of control in all states (let $d = 1$, $S(H, Y) = Y - F$, and $S(L, Y) = Y$). Since the motivation for designing and selling financial instruments in this model is the wealth constraints faced by the investors, it seems counterproductive to consider contracts that are optimal only when these constraints do not, in some sense, bind. It is also clear that such a contract would not be incentive compatible if the public signal ϕ were announced by the firm and subject to moral hazard.

investor *1* owning the control rights from time *0* to time *2* (and so a potential transfer of control) and have not examined the alternative schemes in order to preserve the simplicity of the model. We demonstrate below how it is possible to alter the model's structure and provide that the contracts in Proposition 1 are uniquely optimal.[13]

B. A Comparative Advantage in Control

The introduction of exogenous risk into corporate financial instruments is not commonly observed. The features of this model that allow for the possibility of conducting a lottery for control of the firm are that the investors have been assumed to be identical and that the transfer of control between investors is costless. Natural assumptions to use to extend the current model are that investor *1* has a comparative advantage in running the firm, or that there is a cost to the transfer of control. (For example, we might think of the entrepreneur as being investor *1* and being endowed with limited wealth and a comparative advantage over outside investors in running the firm.) This advantage may be thought of as a superior knowledge of the project that is costly to transfer to other agents, or as private or less costly access to information that is important in the decision-making process for the firm, or, in a bounded rationality sense, as being derivative of a superior skill at evaluating the necessary information.

We model this comparative advantage as a cost ($c(\,)$) of transferring control of the firm to investor *2*. Consider how this "bankruptcy cost" might vary across the different states of nature. If this cost does represent a comparative advantage in making decisions for the firm (based on nontransferrable access to more precise information, for example), it is reasonable to think of this cost as increasing in the time *2* value of the firm given the controlling investor's information set. In other words, the advantage in running the firm with which investor *1* is endowed is more valuable the greater is the value that can be created by the decision maker. For example, when a large amount of

[13] In an earlier version of this paper, a model was presented (with $M = 4$, $N = 2$) in which two investment decisions were required. In terms of the current presentation, an investment decision determining the probability distribution over the X's was made at time *0*. This extension provides optimal contracts that delegate control over the time *0* decision to investor *1* and, to provide correct incentives, it is important that control may be transferred to investor *2* at time *2* (regardless of the relative size of W, see footnote 11). The contracts given in Proposition 1 are uniquely optimal in the extended model.

It is possible to extend the current model to allow for this "second" investment decision. The extended model provides the interesting intuition that incentives for the initial investment decision are provided not only by final cash flows but also by the possibility of the loss of control (and so further payouts) at time *2*. We do not present this extension for two reasons. First, this extension seems to provide only this added intuition while sacrificing the simplicity of the model. A great deal of tedious algebra is required as well as much more cumbersome notation. Second, the extended model includes results, such as an optimal capital structure, that are specific to the model and clearly will not hold for more general technologies. The optimal capital structure derives from the above-mentioned intuition. It is necessary to "punish" poor initial decisions to "just the right extent." In the model this is accomplished by a choice of "debt" level.

value may be created for the firm via the investment decision, the advantage to more precise information concerning the productive environment is relatively large, while if the most valuable "project" is liquidation of the firm, it matters little which investor controls the liquidation. In the context of our model, this structure translates into an expected cost of transfer or bankruptcy that is increasing in the public signal ϕ.

Define j^* as the smallest j' such that given the contracts given in Proposition 1 (with $F^* \leq Y_{j*}$) solve the entrepreneur's maximization problem given above without violating the wealth constraint of investor *1*. Label the resulting contracts as C_1^* and C_2^*. In other words, investor *2*'s capital contribution given C_2^* is at a minimum. The contracts C_1^* and C_2^* clearly provide for optimal investment decisions in all states. There is, however, an expected bankruptcy cost of $\text{Prob}(\phi^{j^*} = L)E(c \mid \phi^{j^*} = L)$. Given the investors' wealth constraints, this cost can be reduced, by choosing the benchmark ϕ^{j^*-1}, only at the expense of a suboptimal investment policy when investor *1* owns control over the time *2* decision. This provides the following proposition.

Proposition 2: *If the production technology is such that, when the benchmark level of the public signal is lowered (when $j' = j^* - 1$ rather than j^* is chosen), the increase in the expected cost of inefficient investment decisions exceeds the decrease in the expected cost of the transfer of control, then the contracts from Proposition* 1 *which minimize investor 2's capital contribution (C_1^* and C_2^* from above) are the uniquely optimal financial instruments.*

Note that when we include a cost of the transfer of control in the model, we derive an "optimal capital structure" for the firm which includes a minimal amount of "debt" as expected. In this version of the model, it is also true that the value transferred to investor *2* under the conditions of the contracts is always less than or equal to the face value of investor *2*'s contract, F^*. It is also true that the use of a single benchmark is uniquely optimal when there is a cost to the transfer of control. When a cost of control (which in expectation is increasing in the public signal) is included in the model, the states in which control is transferred at the optimum are precisely identified.

III. Discussion

Formally, the analysis provided here is based on a model of two agents. The results, therefore, apply most directly to proprietorships, partnerships, or closely held corporations. The securities in this model have been designed to address the "stockholder/bondholder" conflict. Allowing for multiple agents within the classes of active and passive investor would not have a substantial effect on our results if the stockholder/bondholder conflict remains the focus of the analysis. In the absence of heterogeneity within these groups, the objectives of group members are identical, and there is no loss of generality in treating each group as a single agent.

A different situation occurs when the environment is enriched to include factors that generate disputes among the members of a group. Then securities must contain provisions for resolving disputes within groups, as well as between groups. For large corporations we may define the control rights as the right to provide the incentive contract to the firm's manager. The securities, following the framework used here, would be designed to provide the proper incentives for an efficient choice of managerial incentive contract (see Dybvig and Zender (1991)). The question then becomes how best to distribute cash flows and the ownership of control to groups of investors.

One approach considered in the literature (see Grossman and Hart (1988) and Harris and Raviv (1988)) is to consider how votes should be distributed across equity claimants. The analysis given here suggests that if the ownership of control must be given, in aggregate, to one group of shareholders, then the votes attached to the different securities should be state-contingent. The optimal securities might specify that different groups of claimants have the right to vote dependent upon the firm's future prospects. This of course begs two questions. The first is, is a voting scheme the optimal rule for the allocation of the ownership of control across diverse claimants? And the second is, what plausible restrictions on contracting imply that only one group of claimants must own control in any state? An interesting research question is, can we provide conditions under which a voting rule and a particular distribution of votes to the claimants can implement (constrained) efficient decision making? Another interesting research objective would be a separation result showing that provisions for resolving disputes between groups are independent of resolving disputes among groups. This, however, is beyond the scope of our analysis.

It is important to understand which assumptions drive our results and how. There are two critical assumptions that determine the shape of the cash flows associated with the two contracts: risk neutrality and the particular information structure chosen. Risk neutrality provides that the optimal sharing rules $S()$ and $B()$ may be linear in final cash flow. The financial instruments in this model act as incentive contracts for the investors. If risk aversion were included in the model, the optimal schemes would in general include the usual tradeoff between risk sharing and decision-making incentives.

The chosen information structure is also a vital assumption in deriving contracts that appear to be debt and equity. We assume that the information revealed at time *2* provides a lower bound for the support of Y. While the signal $\phi^{j'}$ is a noisy signal of future cash flows, for feasible pairs (j', F) it is a perfect signal of the absence of default (defining default as $Y < F$ at time *3* when investor *1* retains control). This result is similar to one by Brennen, Detemple, and Kalay (1988), who find that the existence of such a perfect signal of default results in optimal managerial decisions, despite the fact that managers act to maximize shareholder welfare. The Brennen, Detemple, and Kalay result, however, is derived in a model that takes the financial instruments as given and *assumes* that managerial incentives are to maximize shareholder wealth, while here the instruments and the managerial incentives are developed endogenously.

While this information structure is plausible, it is restrictive. If we alternatively assume that a "good" signal implies that the bulk of the probability mass is in the right tail of the conditional distribution of Y and a poor signal implies that it is in the left tail (where the conditional support is the same in each case), then our "debt" and "equity" contracts are no longer optimal. Instead of these robust contracts the optimal contracts would depend directly on the functions $\{p_j(I)\}$ and $\{q_j(I)\}$. In words the "incentive scheme" in this case would exhibit the traditional result that extreme (high or low) payments are made in those states whose realization are most informative about the agent's decision.

It is not clear that the events which trigger bankruptcy in actual debt contracts provide the required type of information. In practice the type of signal used in this model will be dependent upon the distribution of cash flows generated by the firm. Alterations in the distribution of future cash flows or changes in the holdings of fixed assets will clearly destroy the validity of a given benchmark. It is possible to interpret many of the standard bond covenants (see Smith and Warner (1979)) as providing and/or maintaining the validity of a given signal. For example, requirements to maintain minimum working capital or net tangible asset levels, restrictions on investments in financial assets on the disposition of assets or on merger activity conditional on maintaining certain minimum levels of net tangible assets (as a dollar amount or a percentage of funded debt) may exist to provide and maintain a signal of possible default on the debt contract.

There is one case where this type of information is clearly available. When debt is collateralized with assets whose market value is readily known, the required information is clearly available. The model presented here suggests that firms for which this is possible will have a "debt capacity" at least as great as that of a firm without such assets. This observation is consistent with the stylized facts.

One should note that the public signal $\phi^{j'}$ may be a function of any variable that provides the necessary information concerning final cash flows Y. A natural specification might be that the public signal is a function of intermediate cash flows. Then if time *1* and time *3* cash flows are positively correlated, payout requirements may provide the necessary information. In practice bankruptcy may be triggered in a variety of ways, a minimum level of intermediate cash flow (measured by coupon payments, sinking fund contributions, etc.) is only one example. Interest coverage ratios or minimum net worth constraints provide others. It may even be that large intermediate cash flows signal poor future prospects. That good intermediate cash flows may be associated with bankruptcy may seem counter-intuitive, but it is important to note that bankruptcy is contingent upon current outcomes only to the extent to which they provide information concerning future prospects.

The information structure of our model also allows us to compare our results to those provided by the existing literature. Asymmetric information between the firm and the market is often used to derive inefficient decision making and to "break" the capital structure irrelevance propositions. In this model, despite the presence of asymmetric information concerning the investment environment and risky "debt" in the capital structure, efficient decision making is implemented. In the traditional literature, bankruptcy, which is typically modeled only as the result of a limited liability constraint, introduces inefficient decision making in the presence of asymmetric information. The traditional version of bankruptcy introduces a kink into the "incentive contract" of the decision maker. Here it is bankruptcy, which in our optimal contracts takes the form of a state-contingent change in the ownership of the decision-making rights, that allows first-best "incentive contracts" to be derived despite the required limited liability and the wealth constraints of the investors.

IV. Conclusion

In the analysis presented here, the optimal financial instruments completely resolve incentive problems induced by asymmetric information, and allow agents to maximize the surplus extracted from production when limited wealth makes cooperation a necessity. Securities which closely resemble standard debt and equity contracts implement efficient investment decision rules through the joint distribution of cash flows and control rights. The endogeneity of the ownership of control accounts for the disparity between our results and the results of studies which consider only the distribution of cash flows provided by the standard instruments, taking control as fixed.

We may interpret the securities in this model as sharing rules in standard agency models. The importance difference is that the ownership of control is also assigned by the "sharing rules" developed here. Two identifying features of debt contracts, a fixed payment and the state-contingent transfer of control, provide critical incentives in this model. The agent who owns control will make efficient decisions if, and only if, he realizes the marginal product of investment; this requires that the return on investment to the passive investor be insensitive to performance. A central contribution of our analysis is the demonstration that the state-contingent transfer of control can mitigate opportunistic behavior by the controlling investor and in some situations eliminate that behavior entirely. The optimal financial instruments provide that control remains in the hands of the "shareholder" until the incentive compatibility constraint on the investment decision becomes binding. At this point, control over decision making is assigned to the "debtholder" and all generated cash flows accrue to this claimant. This form of "bankruptcy" allows, in this model, that the decision makers are provided with first-best decision-making incentives.

A question often raised in the analysis of the separation of ownership and control is: Why are residual claimants assigned control rights while a claimant with rights to a fixed cash flow stream is denied direct control over decision making? In our model we find that because one claimant is denied control over decision making, the payments made to this claimant (absent bankruptcy) are fixed to ensure optimal decision making by the owner of the control rights.

Extensions to this analysis currently underway consider whether the crucial assumptions of risk neutrality and a "perfect" signal of future default may be relaxed while preserving the form of the optimal contracts. The intuitions developed in this model will likely carry over to less restrictive models; it is, however, important to consider what circumstances provide debt and equity contracts as "second-best" instruments. Along these lines, if bankruptcy is costly and only an imperfect signal of default is available, we may be able to endogenously develop "bond covenants." Such covenants may transfer control over specific decisions to the debtholder when shareholder's incentives are only slightly distorted. The robustness of the contracts considered here suggest that interesting results may be derived by recognizing that there exists an important difference between the standard principle agent environment and the workings of the modern corporation. In a firm, the manager has a very rich action set and, clearly, in his decision making is able to do more than "shift the distribution over payoffs to the right." Other important extensions are to allow for trading in the derived contracts and to consider that the ownership of control does not rest in a single agent's hands.

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