Asymmetric Information, Debt Capacity, And Capital Structure[*]

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Current Draft: June 2, 2016

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We are especially indebted to Phil Dybvig and Jeff Zwiebel for insightful comments on a previous draft. We also thank Matthias Kahl, Chris Leach, Michael Roberts, Iulian Obreja, Paul Pfleiderer, Roberto Pinheiro, Chris Yung, and seminar participants at Georgetown, Stanford, the University of Nebraska, the University of Texas at Austin, the University of Texas at Dallas, and Washington University in St Louis for helpful comments.

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Abstract:

Capital structure choice based on costs associated with asymmetric information is examined and used to present a new perspective on the standard pecking order and tradeoff theories. In the model, both the amount of debt and the restrictiveness of the associated debt covenants are chosen as part of the financial structure, allowing a more complete characterization of this choice. Leverage choice balances *ex ante* adverse selection against *ex post* moral hazard allowing a number of interesting empirical implications to be developed.

The standard tradeoff theory of corporate capital structure considers the allocation of cash flow across debt and equity securities to be the primary impact of capital structure choice. An optimum is found by considering the implications of this allocation for taxes, bankruptcy costs, decision making incentives, information transmission, or other frictions. This paper seeks to develop a parsimonious model of capital structure choice that allows consideration of both the allocation of cash flow and the allocation of control rights via debt covenants in determining the firm's ideal debt structure. The model develops a tradeoff theory of capital structure in which costs associated with asymmetric information between the firm and external investors are the sole friction.

The model can be thought of as an extension of Myers and Majluf (1984). A firm with uncertain cash flow facing asymmetric information between the firm and the market is considered. To establish the firm, a capital constrained entrepreneur seeks financing from an inferiorly informed capital market. Myers and Majluf examine asymmetric information at the time of financing and show that existing equity holders benefit from the sale of securities with low information sensitivity (debt). Their model does not, however, consider decision-making subsequent to the financing event nor the impact of the initial financing scheme on these decisions.[1]

Extending this traditional framework, we assume that subsequent to the initial financing, the information asymmetry remains, however, a signal concerning the strength of the economy is publicly observed. Based on this signal, the firm may either continue, generating an uncertain future cash flow, or liquidate, for an immediate and certain value.

[1] The examination of the implications of the initial financing choice for subsequent decision-making is the fundamental departure of our model from traditional pecking order models (Myers and Majluf (1984) or Vishwanath (1993) in a dynamic context). In a recent paper, Davis (2015) makes a related point.

The liquidation decision is a simple representation of investment decision-making after the financing is in place. The presence of debt financing generates the standard bias toward continuation. The cost of this *ex post* moral hazard balances the entrepreneur's initial motivation to issue debt at the financing stage in response to the adverse selection faced at that time. The incentive problem implies that debt covenants, which allow the lender to demand immediate repayment of the debt on a state contingent basis, may be valuable features of debt contracts. In the model, the debt covenant effectively allocates control of the liquidation decision to the firm or the lender based on the realization of the public signal. The lender's inferior information implies this allocation may be costly. This cost may be mitigated by renegotiation of the covenant. The balance of this net expected cost and the benefit of initially issuing debt represents the tradeoff in the initial capital structure choice. The greater is the *ex ante* asymmetric information and the lower the expected cost of renegotiation, the more likely it is that firms will elect to use high leverage and restrictive debt covenants. A measure of debt capacity arises endogenously within the model; however, equilibrium debt levels may lie significantly below this benchmark.

Covenants are standard features of debt contracts. They take a variety of forms and may restrict firms from taking certain actions (engaging in mergers, the payment of dividends, issuing additional debt) that transfer wealth from lenders to shareholders or may proscribe conditions that must be maintained (minimum levels of net worth or interest coverage) to avoid technical default. The covenants modeled here are "proscriptive" covenants which act as an "early warning system" for deterioration in the financial health of the firm and/or future default.

Covenants have been shown to have a significant *ex post* impact on firm behavior. For example, Asquith, Gertner, and Scharfstein (1994) found technical violation of covenants to be the leading reason for default (slightly ahead of failure to make scheduled payments) in their sample of Junk-Bond issuers. Chava and Roberts (2008) demonstrate that capital investment declines following the violation of a covenant, as creditors use the threat of default to intervene in managerial decision making. Nini, Smith, and Sufi (2012) further show that violations of covenants are followed by reductions in acquisitions, capital expenditures, leverage, and payouts to shareholders, and an increase in CEO turnover. They attribute these policy changes to creditor's influence on the firm's decision making. The absence of control right allocations in the form of debt covenants from the *ex ante* capital structure discussion may therefore be an important omission. By presenting a model in which the amount of debt as well as the structure of its control rights are jointly considered in the leverage decision we hope to further understand the role of debt covenants, provide more structure regarding their role in the capital structure decision, and to shed light on the broader capital structure question.

Garleanu and Zwiebel (2009) present a model that is closely related to our work. They point out that debt covenants provide an interesting and important example of the broader property rights literature developed in Grossman and Hart (1986) and Hart and Moore (1990). Garleanu and Zwiebel develop a model that illustrates one way (the motivation of costly information acquisition) in which restrictive covenants, that allocate control of decisions to a party with *inferior* information, may enhance value. Our model differs from theirs primarily in the nature and role of the covenants. Their focus is on the nature of the covenants themselves and they take the leverage decision and the managerial

incentive problem as exogenous. The leverage decision is our focus, and leverage and managerial incentives are endogenous in our model.

Our model provides a complementary explanation to that in Garleanu and Zwiebel (2009) for the use of restrictive rather than unrestrictive covenants. Furthermore, we show that an important consideration in determining the restrictiveness of debt covenants is the firm's ability to renegotiate the covenants when they prevent efficient actions from being pursued. Thus, the use of restrictive covenants and the fact that they are frequently renegotiated are closely tied.[2]

Dynamic models of optimal financial contracting (*e.g.* DeMarzo and Sannikov (2006), and DeMarzo and Fishman (2007a and 2007b) have been developed in which the history dependent dynamic contracts can be interpreted as the result of the renegotiation of violated covenants. For example, in the papers cited above, the fundamental contracting friction is the agent's ability to appropriate a portion of the realized cash flows. The optimal dynamic contract can be implemented using equity, long-term debt, and a line of credit. The agent holds a portion of the equity, while the long-term debt, the line of credit and the ability of the external investors to terminate the project provide the agent with incentives to truthfully reveal the amount of cash flow generated each period (or instant in time). Interpreting the dynamically changing overall debt level as the result of covenants that are violated and renegotiated, these models would suggest debt levels that are renegotiated upwards and downwards presumably in exchange for covenants that are made less and more restrictive respectively (features that are not consistent with the data).

[2] Leland (1994) is another important paper considering the impact of debt covenants on the capital structure decision. It is a model of debt value and capital structure choice whose focus is very different from ours. In his model, debt covenants primarily determine the value of the firm at the bankruptcy point. Our focus is on how covenants affect investment/liquidation decision making and so leverage choice.

The main theoretical results of the model can be summarized as follows. Adverse selection at the financing stage introduces a standard pecking order in the model. Firms prefer to issue first riskless debt, then risky debt, and finally external equity. The model identifies liquidation value as a measure of the firm's debt capacity, the point at which there is no longer any motivation for the firm to issue debt in response to the adverse selection problem. However, the model also indicates that the optimal level of debt may be significantly below the firm's debt capacity. If the renegotiation of covenants is not possible, it is optimal for firms to use low amounts of debt combined with unrestrictive covenants and never optimal to use a restrictive covenant. When renegotiation is possible and costless, the use of high levels of debt combined with restrictive covenants represents equilibrium initial financial structures; these results establishes the connection between the use of restrictive covenants and the ability to renegotiate them. In the most natural case, when renegotiation is possible but costly, depending on parameter values, firms may find either a "high" level of debt combined with a restrictive covenant or a "low" level of debt and an unrestrictive covenant to represent an equilibrium initial financial structure. The equilibrium balances the benefit of initially selling debt rather than equity under asymmetric information against the costs associated with distorted decision making and the costs of renegotiating the covenants associated with higher debt levels.

Empirically, the main capital structure implications in the model may be summarized as follows. The firm is more likely to choose higher leverage and more restrictive covenants, the greater is the value added of the investment opportunity, the greater is the *ex ante* adverse selection problem, the greater is the firm's debt capacity (the firm's ability to use debt), the stronger is the economic outlook, and the lower is the cost

of renegotiation. Finally, when firms attempt to renegotiate violated covenants, the concession the firm must make to waive the violation will be greater the stronger is the then prevailing economic environment.

Consideration of the standard tradeoff theory costs and benefits of debt would be accomplished in this model very differently than in the pecking order. The pecking order theory assumes that tradeoff theory frictions (taxes and bankruptcy costs) are second order effects except at extreme debt levels at which point they dictate financing choices in the usual way. Here, tradeoff theory frictions, were they to be included, would affect leverage choice within an intermediate range of leverage. Therefore, if the tensions examined in the model are important considerations in firms' capital structure choice, the model suggests a new approach to empirical examinations of capital structure.

1. The Basic Model

In the model, an entrepreneur/manager seeks funding in order to establish a firm. The entrepreneur's type or quality (t) is assumed to be known privately by the entrepreneur (we will equivalently refer to the entrepreneur's type or the firm's type; i.e., good entrepreneurs run good firms). Managers are assumed to be observationally equivalent to the external investors/market. *Ex ante*, external investors (the market) know only that type is drawn from the set $\{B, G\}$ where $0 < B < G$, the prior probability of a good type (G) is θ, and the mean type is denoted $\bar{t}_\theta$. Market participants do observe the financing structure chosen by the firm and update their beliefs based on this choice. The model is therefore, a signaling model in which managers choose a financial structure in order to establish the firm and, based on beliefs about the type of firm, market participants price the associated

securities. Asymmetric information between the manager and the market is the fundamental friction in the model. We denote the market's posterior probability that a manager is of type *G* by μ and the resulting expected type by $\bar{t}_{\mu}$.

The level of initial capital, *I*, required to initiate a project and establish a firm is assumed to be common knowledge. The investment's time 2 payoff is assumed to depend upon the entrepreneur's type and the value of a signal, *w*. This signal is publicly observable and verifiable at time *1*, where *w* is drawn from the set $\{w_2, w_1\}$, with $0 < w_2 < w_1$ and the probability that $w = w_1$ is equal to *p*. The random variable *w* is assumed to be independent of type (*t*). For expositional convenience, the signal will be discussed as an indication of the strength of the overall economy or the industry as it will affect the fortunes of all observationally equivalent firms. However, we could equivalently consider the signal to be an industry or firm specific release of information. As long as each possible realization of the signal may be reported by all firm types the implications of the model remain the same. We refer to the realization $w = w_1$ as a strong market and $w = w_2$ as a weak market.

If the project is initiated and is allowed to continue until time *2* it generates a cash flow of *H* or *L*, where $H > L > 0$. The cash flow *H* (success) is generated at time *2* with a probability equal to the product of the entrepreneur's type and the realization of the public signal, *tw*, and the cash flow *L* (failure) is realized with the complementary probability (1 – *tw*). For internal consistency we assume $1 > Gw_1$ and note that $Gw_1 > Bw_2 > 0$.

An alternative to continuation of the investment project, available at time 1, is that it may be liquidated (or "quit"). Liquidation of any firm generates a time *1* cash flow of *Q* with certainty. The timing of the model is such that the liquidation decision is made conditional on the realization of the public signal *w*. The liquidation decision introduces

the possibility of *ex post* moral hazard in the model and serves as a simple representation of *ex post* decision making that may be impacted by the incentive structure that is induced by the initial financing decision. As is common, we assume all agents are risk neutral and that the risk free rate is zero.

The entrepreneur/manager owns the rights to the project but has no capital. The required capital, *I*, must be raised by issuing a combination of equity and debt.[3] We consider that the entrepreneur chooses the face value of debt, *F*, and the restrictiveness of the debt's covenant. The debt covenant effectively allocates the right to make the liquidation decision on a state contingent basis. We indicate the restrictiveness of the debt covenant by a level of the signal w' below which control of the liquidation decision is allocated to the lender (the lender has the right to call the loan). The parties may attempt to renegotiate this allocation of control rights at time *1* at a cost *c*. The choice of debt structure (the face value and the level of the covenant) and the level of the required capital determine the proportion of the firm's equity to be sold externally, α. Entrepreneurs make decisions considering their informed valuation of their retained equity. Note that this is equivalent to assuming that the entrepreneur acts in the interest of shareholders (given his/her superior information). Intuitively, this structure generates an agency problem that is increasing in the amount of debt financing. Asymmetric information motivates the *ex ante* use of debt and the resulting agency problem provides an *ex post* cost to the use of debt financing.

[3] As in Garleannu and Zwiebel (2008) we assume the use of debt contracts is optimal based on standard arguments from the security design literature. Continuing work seeks to incorporate an optimal managerial incentive contract (see Dybvig and Zender (1991)). Note, however, that only the bad firm's manager need suffer from an agency problem for the development of our results so that modeling career concerns for managers generates the problems discussed here in the presence of incentive contracting.

First Best Liquidation/Continuation

We assume that in a strong market ($w = w_1$), it is efficient for both types of firms to continue operations. In contrast, in a weak market ($w = w_2$), it is efficient for a good firm to continue while a bad firm should liquidate. Thus we restrict the parameter values so that:

$$\begin{aligned} &Gw_1H + (1-Gw_1)L = Gw_1(H-L) + L > Q \\ &Bw_1(H-L) + L > Q \\ &Gw_2(H-L) + L > Q \\ &Bw_2(H-L) + L < Q \end{aligned} \qquad \textbf{(1)}$$

This will allow us to examine interesting aspects of the financing choice in a relatively simple environment.

The Agency Problem

The time *1* liquidation decision of an entrepreneur of type *t* (who is not constrained by a debt covenant), with debt outstanding, follows standard intuition. The entrepreneur acts to maximize the informed *ex post* value of his retained shares for a given face value of debt. Assuming $L \le F \le Q$ (restrictions we address below) the informed conditional expected value of levered equity is either $Q - F$ when firm is liquidated or $tw_2(H-F)$ when the firm is allowed to continue. The convexity of their claim in the final cash flow implies that equityholders have a bias towards continuation, a bias that increases with leverage. Similarly, debtholders have a bias towards liquidation.

Given this bias, managers of good firms will always make efficient liquidation decisions (it is always efficient for good firms to continue). Also, both types of firms will

make efficient decisions in a strong market. However, managers of bad firms will make an inefficient liquidation decision in a weak market if the face value of debt is sufficiently high. Our assumption that $Q-L>Bw_2(H-L)$ indicates that for face values of debt that are risk free $(F \le L)$ entrepreneurs in bad firms will liquidate efficiently. When the face value of debt increases sufficiently $Q-F<Bw_2(H-F)$. In particular there exists a level of debt, F^L defined implicitly by $Q-F^L=Bw_2(H-F^L)$, at which the manager of a bad firm is indifferent between continuation and liquidation in a weak market. The lender has the opposite bias and when $L<F \le Q$ will strictly prefer liquidation to continuation. Thus, the basic model presents a simple representation of standard intuition.

The nature of the managerial incentives and the structure of the model imply that it is sufficient to limit the analysis to two levels of the covenant, which we label a restrictive covenant and an unrestrictive covenant. We denote an unrestrictive covenant with $w'=w_2$ where the covenant conveys the right to call the loan to the lender if the $w<w'$. Therefore, with an unrestrictive covenant the lender will never be allocated the right to call the loan and consequently the manager will make the liquidation decision regardless of the realization of the signal *w*. Under a restrictive covenant $(w'=w_1)$ the lender is assigned the right to call the loan (and so to make the liquidation decision) when $w=w_2$. Note that it can never be optimal to assign control of the liquidation decision to the lender when $w=w_1$ because the lender strictly prefers liquidation and because the manager will always make the efficient decision in this case.

2. The Capital Structure Problem – No Renegotiation

We begin by examining the model in a restricted setting in order to illustrate its basic tensions. In this section we assume renegotiation is impossible (or infinitely costly), while in Section 3 we assume renegotiation is costless. Section 4 presents the main results, assuming renegotiation is possible but costly.

In the absence of renegotiation, equity, debt, and firm value, as well as the value of the manager's retained shares are defined as follows. A generic representation of time *0* equity value for a given firm type, *t*, can be written:

$$\begin{aligned} S^t(F,w') = p[(1-\ell(F,w_1,w'))tw_1 \max(H-F,0) + \ell(F,w_1,w')\max(Q-F,0)] \\ +(1-p)[(1-\ell(F,w_2,w'))tw_2 \max(H-F,0) + \ell(F,w_2,w')\max(Q-F,0)] \end{aligned} \quad \textbf{(2)}$$

where $\ell(F,w,w')$ is an indicator function that takes the value one in the event the firm will be liquidated given the signal *w*, a debt covenant *w'*, and a face value of debt *F*. Because of the cumbersome notation we will commonly suppress the indicator function and the max operator (in almost all instances we can also, without loss of generality, restrict the initial face value of debt to be in the interval $[L,Q]$). The value of the good firm's equity (assuming efficient liquidation) for example is then more simply written as:

$$S^G(F,w') = pGw_1(H-F) + (1-p)Gw_2(H-F) \quad \textbf{(3)}$$

and the bad firm's equity is:

$$S^B(F,w') = pBw_1(H-F) + (1-p)(Q-F). \quad \textbf{(4)}$$

The value of the good and bad firm's debt and firm (the sum of the equity plus the debt) values are defined analogously (see appendix). Uninformed firm value, given market beliefs about the probability a firm is good (μ), is written

$$V^U(F,w') = p\bar{t}_\mu w_1(H-L) + (1-p)[\mu Gw_2(H-L) + (1-\mu)(Q-L)] + L \quad \textbf{(5)}$$

Uninformed values of equity $(S^U(F,w'))$ and debt $(D^U(F,w'))$ are defined analogously.

The manager of a firm of type *t* will choose the initial financial structure (F, w', α) (where α represents the fraction of the firm's equity retained by the manager) of the firm in order to optimize his/her informed value of their retained shares subject to raising capital, *I*, by selling external claims.[4] The capital constraint can be used to solve for α and the objective function of a manager of type *t* is written as dependent upon the choice of the face value of debt and the restrictiveness of the covenant.

$$\max_{F,w'} \quad \frac{S^t(F,w')}{S^U(F,w')}\left(V^U(F,w') - I\right) \qquad \textbf{(6)}$$

The relation between our model and that of Myers and Majluf (1984) can be illustrated using equation (6). While this is not strictly true, we can think of the ratio $\left({}^{S^t(,)}\!/_{S^U(,)}\right)$ as capturing the impact of asymmetric information on the manger's choice while the firm value $(V^U(,))$ captures the impact of any inefficiencies in decision-making. Loosely speaking, if we ignore the dependence of these values on the restrictiveness of the covenant (hold *ex post* decision-making constant) uninformed firm value is independent of the amount of debt, and the Myers and Majluf notion that when good and bad firms pool, good firms pay a premium for financing based on their inability to separate from bad firms while bad firms benefit from the pooling is illustrated in the ratio. It is also true that, holding liquidation decision constant, this ratio is strictly increasing (decreasing) in the face value of debt for a good (bad) firm for F in the interval $[L, Q]$. This is the basis of the pecking order developed by Myers and Majluf (1984). For $F \geq Q$ the ratio is independent of *F* reflecting their notion of debt capacity.

[4] For simplicity we assume that firms raise an amount equal to *I* in the external markets. This can be derived from the fact that good firms pay a premium for external capital given the asymmetric information but the complications from doing so detract from the main focus of the paper.

Proposition 1: The Pecking Order and Debt Capacity: Asymmetric information at the time of financing (time *0*), *holding liquidation decision-making constant,* implies there is a pecking order for external financing in that the manager of a good firm prefers to issue riskless debt to the extent possible ($F = L$), then risky debt to the point of its informational equality with external equity ($F = Q$). At this level of debt financing, the entrepreneur is indifferent between issuing additional risky debt or external equity.

Proof: Immediate from the discussion above.

Proposition 1 illustrates that the model captures the standard pecking order preference of the good manager for issuing debt rather than external equity given the *ex ante* asymmetric information. The qualification is that this is a generic characterization of financing choice only when we ignore *ex post* costs associated with the induced incentives. Proposition 1 further demonstrates that by including a liquidation decision in the model a notion of "debt capacity" arises endogenously. When $F = Q$ there is no further motivation (derived from asymmetric information) to use debt rather than external equity.[5] This result and noting that $F = L$ represents risk free debt implies that there is no loss of generality to restricting attention to the range $L \leq F \leq Q$.

The innovation in this model is that we also consider the subsequent decision making and how this decision making is influenced by the firm's response to the *ex ante* informational asymmetry. There are two issues to consider. The first is that the use of risky debt in the initial financing of the firm may distort the incentives of the manager for

[5] Williamson (1988), Hart and Moore (1994), and Shleifer and Vishny (1992) have all, for varied reasons, identified liquidation value as a measure of debt capacity. This simple version of debt capacity is a "soft constraint" in that while there is no positive motivation for the further use of debt there is also no cost.

making the time *1* liquidation/continuation decision. The second is that debt covenants, which effectively allocate control of the liquidation decision on a state contingent basis, can help to limit the cost of the distorted incentives. The use of debt covenants will not perfectly control the incentive problem because the lender's incentives are also distorted if the debt is risky and because the lender makes decisions based upon inferior information. The cost of the *ex post* distortions in the incentives of the decision maker induced by the use of debt financing is balanced against the adverse selection benefits of the *ex ante* sale of debt in determining the firm's capital structure.

We examine Pure Strategy Perfect Bayesian Equilibria (PSPB) of the firms' choice over debt structure. An equilibrium entails (1) a debt structure choice (F and w') and (2) (when allowed) a renegotiation strategy for both types of manager, and (3) a set of beliefs held by market participants (posterior probabilities) regarding the type of firm issuing a given set of securities, such that neither type of manager can increase the *ex ante* informed value of their retained equity by choosing an alternative strategy and market pricing is an unbiased expectation of the value of the issued securities given the market's posterior probabilities concerning firm type where the market's posterior probabilities are formed following Bayes Law (when possible) given the equilibrium behavior of each type of firm.

In order to focus attention on the most interesting region, we restrict attention to the set of parameter values that admit only pooling equilibria.[6] The capital structure decision making in the model will therefore consist of identifying *ex ante* capital structures

[6] For some parameter values, there exist separating equilibria in the model based on the two dimensional signal provided by choice over face value and the restrictiveness of the covenant. Constantinides and Grundy (1989)) discuss this issue. Assumption 1 is sufficient to rule out separating equilibria.

which represent pooling equilibria in which good and bad firms choose the same initial financing structure.

Assumption 1: Parameter Restriction

In order to focus on pooling equilibria in the model we assume that the following inequality holds:

$$\left(1-\frac{B}{G}\right)(I-L) > (1-p)[(Q-L) - Bw_2(H-L)] \qquad \textbf{(7)}$$

The right hand side of equation (7) represents the expected change in value for a bad firm making efficient versus inefficient liquidation decisions in a weak market. In order to rule out separating equilibria, this value needs to be "small" because the most effective way for the good firm to separate from the bad is to induce inefficient decision making on the part of the bad firm if the bad firm mimics its financial structure. When this cost is small, the cost of mimicking the good firm is small. Similarly, the left hand side of the inequality is larger the greater the difference between the good and bad firm types (*G* and *B*). The larger is this difference, the greater is the benefit to the bad firm from mimicking the good.

Lemma 1: In the absence of renegotiation, with an unrestrictive covenant ($w' = w_2$), the range for the face value of debt $[L, Q]$ may be usefully separated into "low" debt $[L, F^L]$ and "high" debt $(F^L, Q]$ in the analysis. Using low debt and an unrestrictive covenant will induce efficient liquidation for both firm types in weak and strong markets. The cutoff point is given by $F^L = \frac{Q - Bw_2 H}{1 - Bw_2} > L$. For debt levels above F^L, the liquidation decision of bad firms is inefficient in the absence of renegotiation.

Proof: See Appendix A

The debt level F^L is the face value of debt for which the bad firm is indifferent to liquidation or continuation in a weak market. As will be shown, for most of the results it suffices to discuss low or high debt levels rather than specific choices within these intervals.

A final issue that must be considered before we can establish the PSPB equilibria of the signaling game are the off the equilibrium path beliefs of the market participants. In a pooling equilibrium for the initial financial structure, both types firms will choose the same financial structure to raise the initial capital, *I*. The market, believes there is a probability μ that a firm seeking the equilibrium financing is a good firm. In equilibrium this belief must equal the objective probability the firm is good (i.e., $\mu = \theta$). Further, for any financial structure chosen by a firm that is not the candidate equilibrium one must to proscribe reasonable beliefs for the market that support the equilibrium.

Given that in this model bad firms have an incentive to mimic choices of the good firms to improve the pricing of their financial claims, one possible set of beliefs is that observing *any* deviation from the equilibrium financial structure the market believes the firm is bad $(\mu = 0)$. It is straightforward to show that these beliefs support a large set of pooling equilibria. For some parameter values, all low levels of debt $(L \le F \le F^L)$ combined with an unrestrictive covenant are PSPB equilibria of the signaling game.

However, consider one such equilibrium; $F = L$ (risk free debt) and an unrestrictive covenant. For some parameter values this is an equilibrium financial structure.[7] From the perspective of a good firm it is the least desirable of all the possible equilibria and the most desirable for the bad firm. This equilibrium is supported only because the market is assumed to view any deviation as coming from a bad firm. However, from this equilibrium, the type that would benefit most from a deviation to higher debt is a good firm. Bad firms benefit from low debt levels and are not interested in such a deviation, unless they could convince the market that they were good firms by doing so. In this case, the market's off the equilibrium path belief that any deviation is from a bad firm is not reasonable.

For this reason, we will assume that for a given deviation from any candidate equilibrium, if one type of firm would strictly benefit from this deviation under the assumption that market beliefs are unchanged (i.e. were represented as $\mu = \theta$) then the market will assign a probability equal to one that the deviation is from that type of firm. In the event neither type of firm or both types of firms strictly benefit from a given deviation under the beliefs $\mu = \theta$ then the market will assign "passive" beliefs to the firm type that has deviated and assign beliefs $\mu = \theta$. For example, consider these off the equilibrium path beliefs for a candidate equilibrium with debt F such that $L < F < F^L$ and an unrestrictive covenant. Under the assumption that market beliefs are given as $\mu = \theta$ following any deviation, good firms have an interest in increasing the level of debt and bad firms have an interest in decreasing the debt level. The market will then assign beliefs so

[7] For all parameter values it is easily shown that for these off the equilibrium path beliefs there is some level of debt $F < F^L$ combined with an unrestrictive covenant that serves as a pooling equilibrium for which the same discussion applies. The choice here is simply for illustration.

that any deviation to higher debt sets $\mu = 1$ and any deviation to lower debt sets $\mu = 0$,[8] implying that only $F = F^L$ survives as an equilibrium strategy in the signaling game.

The initial financial structure decision in the absence of renegotiation may now be considered.

Proposition 2: When renegotiation of debt covenants is not allowed (or is infinitely costly) and off the equilibrium path beliefs are as described above, only the debt level F^L combined with an unrestrictive covenant represents a Pure Strategy Perfect Bayesian Equilibrium.

Proof: See Appendix A

Proposition 2 establishes that it is not an equilibrium to use high debt or restrictive covenants in the absence of renegotiation. When the initial financing choice considers both the *ex ante* adverse selection as well as the *ex post* moral hazard induced by this financial choice, neither high debt nor a restrictive covenant are selected in equilibrium. The market, anticipating the distorted incentives associated with high debt for a bad firm, in any pooling equilibrium will "charge" firms for the resulting inefficient decision-making. This implies that the use of high levels of debt $(F > F^L)$ cannot represent an equilibrium strategy. In the absence of renegotiation, there is a pecking order for financing choice but the point at which a firm turns to external equity (F^L) is well below the standard notion of debt capacity.

[8] While this set of beliefs has the "flavor" of the Intuitive Criterion introduced by Cho and Kreps (1987), their refinement does not generate this set of beliefs. This is because these beliefs do not satisfy the equilibrium dominance requirement specified by Cho and Kreps. In this setting, if a bad firm deviates from a pooling equilibrium by increasing the debt level and such a deviation convinces the market it is a good firm the bad firm may have an incentive to do so. Note, however, good firms have an even stronger incentive to make such a deviation. We note that these beliefs do not support any additional equilibrium rather they serve only to eliminate those that are not "reasonable" as argued above.

Similarly, the absence of renegotiation implies that the use of restrictive covenants, which place decision making in the hands of the uninformed lender, will not be part of an equilibrium financial structure.

3. Costless Renegotiation of Covenants

For simplicity we will assume that in the renegotiation process the firm makes a take-it-or-leave-it offer to the lender (and is responsible for any associated costs). This assumption gives all the bargaining power in the renegotiation to the firm and may be justified by the availability of alternative financing after the covenant is violated. This assumption does not affect our main conclusions; we highlight below the circumstances in which the assignment of bargaining power has a significant impact on outcomes.

An interesting aspect of the renegotiation offer made by a firm following the realization of a weak market is that the relation between the offered and the initial debt level differs depending on the initial debt level. Suppose that the initial financial structure involved a debt level F^R with a restrictive covenant (where $L \leq F^R < Q$ and the *R* denotes an initial level of debt associated with a restrictive covenant). A restrictive covenant allows the lender to make the liquidation decision in a weak market. The lender will always prefer to liquidate, introducing an inefficiency for a good firm. However, because it is efficient for a good firm to continue in a weak market and because a good firm's cash flow distribution in continuation stochastically dominates that of a bad firm, there is, in a weak market, a separating renegotiation offer the good firm is willing to make, $F^N > F^R$, that a bad firm will not mimic (F^N is high enough so that a bad prefers to liquidate under the initial debt contract) and that the lender will accept in exchange for waiving the covenant

violation, believing the offer has been made by a good firm. However, conditional on the realization of a weak market, firms not inherently interested in separation, rather they only seek to find the lowest acceptable renegotiation offer.

Renegotiation offers made by firms will be those that are just acceptable to the lender in exchange for waiving the covenant violation. What is acceptable to the lender will depend on the type or type or types of firms the lender believes have made the offer. For any set of parameter values there are two versions of the lender's participation constraint in a renegotiation; one if the offer separates good from bad firms and another if the offer does not. Offers to renegotiate a violated restrictive covenant must be such that good firms will be willing to bear the increase in the face value of debt in exchange for waiver of the covenant violation. A final consideration is a separation constraint that indicates whether bad firms will also renegotiate (if an acceptable offer entails an increase in face value that is small enough the manager of a bad firm will prefer the gamble to liquidation) or whether bad firms will prefer the lender liquidate the firm.

Figure 1 illustrates the possible combinations of an initial debt level with a restrictive covenant (F^R) and an acceptable renegotiation offer (F^N) given the realization of a weak market. All allowable combinations of F^R and F^N lie within the outlined area. Combinations on the lower edge of this area are those that extract the good firm's greatest benefit. The combinations on the upper edge of this area are those for which the good firm's participation constraint binds, representing renegotiation offers for which the good firm is indifferent between renegotiating the covenant and liquidating the firm in a weak market. If the initial debt level is low $(L \le F^R \le F^{R_L})$, the lender's participation constraint (assuming separation) will govern renegotiation offers. Because this offer will satisfy the

separation constraint, when the lender observes such an offer he will believe it was made by a good firm and the separating version of the lender's constraint will govern the renegotiation.

For higher initial debt levels, the set of acceptable renegotiation offers depends on whether it is efficient for an "average" type firm to continue in a weak market. If parameter values are such that it is inefficient for firm with an "average" type $(\bar{t})$ to continue, then for initial debt levels $F^{R_L} \le F^R \le Q$, the best acceptable renegotiation offer is governed by the separation constraint. Again, observing such offers will lead the lender to believe that it is negotiating with a good firm and it will accept the renegotiation.

If parameter values are such that it is efficient for an average type firm to continue the acceptable renegotiation strategies are a little more complex. For initial debt levels given by $F^{R_L} \le F^R \le F^{R_H}$ acceptable renegotiation offers are governed by the separation constraint as discussed above. However, for initial debt level in the range $F^{R_H} \le F^R \le Q$ the acceptable offers will be instead governed by the lender's pooling participation constraint, as this offers the best terms from the firm's perspective. Renegotiation of restrictive covenants is described more formally in *Lemma 3* and illustrated in Figure 1.

Lemma 3: When a restrictive covenant is chosen and there is costless renegotiation, the initial debt level can be separated into a "low" debt and a "high" debt region. When good firm is faced with the violation of a restrictive covenant in a weak market, if the initial debt level, F^R, is low

$$L \le F^R \le \frac{Gw_2((Q-L) - Bw_2(H-L))}{Gw_2 - Bw_2} + L = F^{R_L}$$

the renegotiation offer will be chosen to satisfy the lender's participation constraint

$$F^N(F^R) \geq \frac{F^R - L}{Gw_2} + L \ .$$

If parameter values are such that $\bar{t}w_2(H-L)+L \leq Q$, then when the initial debt level is high $F^{R_L} < F^R \leq Q$ then the renegotiation offer will be chosen to satisfy the separation constraint

$$F^N(F^R) \geq H - \frac{(Q - F^R)}{Bw_2}.$$

When $\bar{t}w_2(H-L)+L > Q$, for initial debt in the interval $F^{R_L} < F^R \leq F^{R_H}$ the renegotiation offer will satisfy the separation constraint and for initial debt in the interval $F^{R_H} < F^R \leq Q$ the renegotiation offer will be governed by the lender's pooling participation constraint

$$F^N(F^R) \geq \frac{F^R - L}{\bar{t}w_2} + L \ .$$

For cases in which the separation constraint is satisfied, bad firms will not mimic the renegotiation offer, preferring liquidation. The lender, believing a good firm has made the offer, will accept the offer and waive the covenant violation. In the case that the lender's pooling participation constraint binds, both types of firms make the same renegotiation offer in a weak market, and the lender accepts believing that the firm is of average type.

Proof: See Appendix A

An interesting conclusion of Lemma 3 is that the offer a good firm must make in order to separate from bad firms *ex post* is increasing in w_2. Given that it is efficient for good firms to continue in a weak market and for bad firms to liquidate, all else equal, the

"stronger" is the "weak" state the more difficult it is for the good firm to separate from bad firms. The model predicts that when restrictive debt covenants are renegotiated, the increase in the debt burden generated by that renegotiation should be increasing in the strength of the overall economy.

If the initial financial structure includes an unrestrictive covenant, there is an induced inefficiency for bad firms for debt levels greater than F^L. When renegotiation is possible, the inefficiency induced by the initial debt level can be mitigated. A renegotiation offers exist in which a bad firm offers to liquidate the firm in a weak market (despite the fact that the manager possesses the right to make the liquidation decision and under the initial debt level prefers continuation) in exchange for a reduction in the face value of the debt. As with a restrictive covenant, any such offer must satisfy the participation constrains for the bad firm and the lender. Figure 2 illustrate the set of possible offers from a bad firm with an unrestrictive covenant in a weak market.

Lemma 4: Assume renegotiation is costless and that a debt level F^U in combination with an unrestrictive covenant was chosen at *t = 0*. Then in a weak market, for very low levels of initial debt F^U such that $L \le F^U \le F^L$ bad firms already have an incentive to liquidate the firm in a weak market so there is no renegotiation the lender will accept.

For levels of debt such that[9]

$$F^L < F^U \le \frac{Gw_2H - Bw_2L - (Q-L)}{Gw_2 - Bw_2} = F^{U_L}$$

[9] Lemma 4 assumes that $F^L < F^{U_L}$ which is the most general case. It is possible that $F^{U_L} < F^L$ in which case F^L simply becomes a lower bound on the initial debt levels that permit renegotiation of an unrestrictive covenant. Because, in the analysis of the initial financial structure high debt associated with an unrestrictive covenant is never an equilibrium we do not discuss this issue any further.

the bad firm will make a take-it-or-leave-it offer to liquidate the firm in exchange for a reduction in the face value of debt which satisfies the lender's participation constraint

$$F^N(F^U) = Bw_2(F^U - L) + L.$$

In this case, the lender accepts the offer (believing a bad firm has made the offer) and good firms continue.

For debt levels $F^{U_L} < F^U < H$ the renegotiation offer that will be made by the bad firm is parameter dependent. If it is efficient for the average firm to continue in a weak market, $\bar{t}w_2(H-L)+L > Q$, then for values of $F^U > F^{U_L}$ a bad firm's offer will satisfy the separation constraint

$$F^N(F^U) = Q - Gw_2(H - F^U)\ ,$$

again, the lender accepts the offer (believing a bad firm has made the offer) and good firms continue.

If it is efficient for the average firm to liquidate in a weak market, then for debt levels, $F^{U_L} < F^U \le F^{U_H}$, where

$$F^{U_H} = \frac{Gw_2H - \bar{t}w_2L - (Q-L)}{Gw_2 - \bar{t}w_2}\ ,$$

then the offer made by the bad firm satisfies the separation constraint (see above). If the initial required capital, *I*, is large enough such that the initial debt level is $F^{U_H} < F^U < H$, then a renegotiation offer to liquidate in exchange for a reduction in the debt's face value will be made by both firm types, the offer is governed by the lender's participation constraint

$$F^N(F^U) = \bar{t}w_2(F^U - L) + L\ ,$$

the lender (believing that the offer may have come from either type of firm) will accept.

Proof: See Appendix A

We are now able to examine the *ex ante* financial structure choice assuming covenants may be renegotiated without cost.

Proposition 3: When renegotiation of covenants is costless, the set of possible PSPB equilibria of the signaling game include initial financial structures with any level of "high debt" in the range $[F^{R_L}, Q]$, where

$$F^L < F^{R_L} = \frac{Gw_2\left((Q-L)-Bw_2(H-L)\right)}{Gw_2 - Bw_2} + L < Q \text{ ,}$$

combined with a restrictive covenant and an initial financial structure with low debt, $F = F^L$, combined with an unrestrictive covenant. Following the realization of a weak market, in the high debt restrictive covenant equilibria depending on the initial level of debt and parameter values, separating or pooling renegotiation may occur.

There are no equilibria which include an unrestrictive covenant other than at the initial debt level $F = F^L$.

Proof: See Appendix A

Note that a good firm is indifferent between low debt and an unrestrictive covenant and high debt with a restrictive covenant when renegotiation is costless. Intuitively, for initial debt levels in the range identified in Proposition 3, increasing the level of debt *ex ante* (and so increasing the good firm's benefit from the initial sale of debt rather than equity), increases the cost (in the form of a higher offer) of renegotiating *ex post* to achieve efficient investment. With costless renegotiation, the increased cost of implementing

efficient decision making *ex post* just balances the increase in the *ex ante* benefit derived from the use of the high initial debt. Simple algebra shows that the range of high debt levels, F^R, exists as long as it is strictly efficient for a good firm to continue in a weak market.

Proposition 3 illustrates the usefulness of bond covenants in controlling the agency costs of debt but also the importance of the ability of firms to renegotiate these covenants. If renegotiation is impossible, restrictive covenants are never employed in equilibrium. When renegotiation is allowed, restrictive covenant may be used in combination with high levels of debt financing. An unrestrictive covenant with a positive probability of being renegotiated *ex post* is, however, never part of an equilibrium financial structure.

Proposition 3 again identifies liquidation value as the firm's debt capacity. There is never a positive incentive for the firm to choose an initial debt level $F^R > Q$. Further, when parameters are such that it is efficient for an average type firm to liquidate, if the initial debt level is greater than the liquidation value then there is no renegotiation offer a good firm is willing to make following violation of a restrictive covenant that the lender will accept, implying inefficient *ex post* decision making.

Figure 1 is useful for considering how a change in the allocation of the bargaining power between the firm and the lender affects the results. We have assumed that the firm has all the bargaining power so that in renegotiation the either the separation constraint or the lender's participation constraint will be binding. Separating renegotiation offers, F^N, share the efficiency gains between the lender and the good firm. If the lender were assumed to have some (or all) of the bargaining power in the renegotiation, accepted offers would, for each initial F^R, lie on the interior (or the upper edge) of the shaded area in Figure 1. *Ex*

ante, such outcomes would be inferior from the perspective of the good firm's manager, and the set of equilibria would reduce to the low debt/unrestrictive covenant equilibrium.

Proposition 3 illustrates that an important aspect of the decision to use restrictive or unrestrictive covenants is the identity of the party at the bargaining table in any renegotiation. When an unrestrictive covenant is renegotiated, the bad firm engages in bargaining with the lender. If the bad firm extracts any of the gains from trade in the renegotiation, the value of the good manager's *ex ante* value will be reduced given that these gains are anticipated in the pricing of the initial financial contracts. Because, with an unrestrictive covenant that is to be renegotiated, the bad firm must volunteer to liquidate absent default, it is likely that the bad firm will have most if not all of the bargaining power in that situation. This result offers an explanation, complementary to that in Garleanu and Zwiebel (2009), for the observed use of restrictive covenants and their renegotiation to be less restrictive rather than the use of initially less restrictive covenants that are renegotiated to be more restrictive (voluntary liquidation in our model).

4. Costly Renegotiation of Covenants

We now consider the model including a dissipative cost $c > 0$ of renegotiating debt; a cost paid by the firm at time *1*.[10] Given the equivalence of the low debt, unrestrictive covenant and the high debt, restrictive covenant capital structures with costless renegotiation, at first glance it may seem that with costly renegotiation high debt must be strictly inferior. However, the cost of renegotiation changes the separation constraint faced

[10] Costly renegotiation must be *ex post* efficient, therefore, we restrict parameters so that $Gw_2(H-L)+L-Q>c$ (for restrictive covenants) and $Q-Bw_2(H-L)-L>c$ (for unrestrictive covenants). These restrictions simply place a limit on the cost of renegotiation.

by the good firm in the renegotiation of a restrictive covenant (a bad firm must also bear this cost if it attempts a renegotiation). This change implies that the equilibrium is parameter dependent. For some parameter values, high debt and a restrictive covenant (that may require costly renegotiation) is the equilibrium in the signaling game and for others the equilibrium is the low debt, unrestrictive covenant financial structure.[11]

With a cost *c* for renegotiating a violated restrictive covenant the constraints on a separating renegotiation offer are as follows. The lender's constraint is written:

$$F^R \leq Gw_2(F^N - L) + L \,.$$

The offer required for separation from a bad firm is now written:

$$Q - F^R \geq Bw_2(H - F^N) - c \,.$$

Finally, any offer the good firm makes must be such that it will prefer renegotiating the covenant to liquidation

$$Gw_2(H - F^N) - c \geq Q - F^R$$

and, limited liability of the equity claims also implies that,

$$Gw_2(H - F^N) \geq c \,.$$

These constraints identify a set of possible combinations for F^R and F^N as shown in Figure 3. Notice that the separation constraint and the good firm's participation constraint are shifted downward relative to these same constraints in the case of a costless renegotiation. The final constraint restricts the level of the renegotiated face value which, via the good firm's participation constraint limits the initial debt level to the firm's debt

[11] For simplicity, in this section we ignore the pooling renegotiation strategies identified in Section 3. Their inclusion complicates the discussion while contributing no additional understanding. This can be done formally by assuming that $\bar{r}(H-L)+L<Q$ in which case there are no pooling renegotiation strategies for restrictive covenants and while the strategies exist for the case of unrestrictive covenants, they are never part of an equilibrium financial structure.

capacity, $F^R = Q$. As above, we first examine the *ex post* renegotiation game and then consider the *ex ante* financial structure choice.

Lemma 5: Assume that the cost of renegotiation is such that there are gains to a good firm renegotiating a restrictive covenant, $Gw_2(H-L)+L-Q>c$. Consider a good firm faced with the violation of a restrictive covenant in a weak market. For a low initial debt level, $L \le F^R \le F^{R_L}$ with

$$F^{R_L} = \frac{Gw_2\left((Q-L)-Bw_2(H-L)\right)}{Gw_2 - Bw_2} + L + \frac{Gw_2 c}{Gw_2 - Bw_2},$$

the renegotiation offer made by a good firm in a weak market is governed by the lender's participation constraint

$$F^N(F^R) = \frac{F^R - L}{Gw_2} + L.$$

Bad firms elect to liquidate and the lender (believing a good firm has made the offer will accept the increase in face value in exchange for waiving the covenant violation.

For a high initial debt level, $F^{R_L} \le F^R \le Q$, where the upper bound is established by the good firm's *ex post* participation constraint, renegotiation offers made by a good firm in a weak market satisfy the separation constraint:

$$F^N(F^R) = H - \frac{(Q-F^R)}{Bw_2} - \frac{c}{Bw_2}.$$

Bad firms will not mimic this renegotiation strategy and the lender, believing a good firm has made the offer, will accept it in exchange for a waiver of the violation of the covenant.

Proof: See Appendix A

Note that the renegotiation offer necessary to waive a restrictive covenant, that will allow a good firm to separate from bad firms, is again increasing in w_2. This indicates that renegotiation results in a larger increase to the original debt burden the stronger is the outlook in a weak market.

Renegotiation strategies following the realization of a weak market when the initial financial structure included an unrestrictive covenant are identified in a similar way.

Lemma 6: Assume that the cost of renegotiation is such that there are gains to a bad firm renegotiating an unrestrictive covenant, $Q-\left(Bw_2(H-L)+L\right)>c$. For an initial debt level, F^U, such that

$$F^L < F^U \leq F^{U_L} = \frac{Gw_2H - Bw_2L - (Q-L) + c}{Gw_2 - Bw_2}$$

the renegotiation offer a bad firm makes is governed by the lender's participation constraint

$$F^N(F^U) = Bw_2(F^U - L) + L \ .$$

Because the separation constraint is also satisfied, good firm's will continue under the initial financial structure and the lender, believing the renegotiation offer comes from a bad firm will accept the reduction in the initial face value of debt in exchange for liquidation of the firm (making the covenant more restrictive).

For an initial debt level, F^U, such that $F^{U_L} < F^U < H$, the renegotiation offer a bad firm will make in a weak market is governed by the separation constraint

$$F^N(F^U) = Q - Gw_2(H - F^U) - c \ .$$

Good firms will continue in a weak market and the lender believing the offer has been made by a bad firm will accept the reduction in the initial face value of debt in exchange for liquidation of the firm.

Proof: See Appendix A

The equilibrium initial financial structure with costly renegotiation may now be considered.

Proposition 4: Under costly renegotiation, two initial financial structures may be PSPB equilibria of the signaling game. If

$$\left(V^U(F^L, w_2) - I\right) - \left(\frac{\theta \bar{t}}{(1-\theta)(G-B)}\right)\left(pGw_1(H-Q) + (1-p)\left(\frac{G-B}{B}\right)c\right) > 0, \quad \textbf{(8)}$$

$F^R = Q$ (debt equal to debt capacity) combined with a restrictive covenant is the equilibrium financial structure. If the reverse is true, the equilibrium financial structure is debt with an initial face value of F^L and an unrestrictive covenant. In the razor's edge case of an equality, both are equilibria of the signaling game.

Proof: See Appendix A

As Proposition 4 shows there are only two possible equilibria, the first is one in which firms choose a relatively low level of debt in combination with an unrestrictive covenant; with no *ex post* moral hazard, a covenant which leaves the informed party in control of the decision making is efficient. This equilibrium is almost diametrically opposed to the result derived by Myers and Majluf (1984) and is driven by the consideration of the impact of the initial financial structure on subsequent decision

making. The second possible equilibrium, is very similar to that introduced by Myers and Majluf; debt equal to the firm's debt capacity combined with a restrictive covenant (which will be renegotiated by good firms in a weak market). The difference is that our definition of debt capacity and the use of the restrictive covenant both explicitly recognize the *ex post* moral hazard problem introduced by the firm's response to the *ex ante* asymmetric information.

Interestingly, the simple structure of the model is not responsible for this stark result. Enriching the model to include a richer set of markets (*w*) and signals, the set of equilibrium financial structures does not change. Consider a richer model that includes a third "very weak" and fourth "disastrous" states, $w_4 < w_3 < w_2$, with the associated signals. Furthermore, include a "medium" quality firm, with $G > M > B$. Assume that bad firms should liquidate in all but the best state, medium firms should liquidate in states w_4 and w_3, and that good firms should continue in all but the worst state, w_4. First note that any positive amount of debt will include a covenant that transfers control of the liquidation decision to the lender in state w_4. It is efficient for all firms to liquidate in this state, therefore, there is no value in the managers' superior information concerning firm type in this state. There will also be no renegotiation if this minimally restrictive covenant is violated.

As the face value of initial debt is increased above a marginal amount, depending on parameter values, the first point of induced inefficiency may be either the point at which the medium type firm would prefer to continue in state w_3 rather than liquidate as is efficient or the level (previously identified as $F = F^L$) for which the bad firm will prefer to continue in state w_2 rather than liquidate. Assuming that for some debt level

$F < F^L$ the medium type firm will begin to make inefficient liquidation decisions, then in the case of costly renegotiation, this will allow for a third possible equilibrium initial financial structure that includes an even lower level of initial debt associated with a covenant that transfers control to the lender in the state w_3 rather than w_2 (*i.e.*, is *less* restrictive than considered in Proposition 4).[12]

More interestingly, as soon as the initial debt level is such $F > F^L$, the point at which the "worst" type firm begins to make inefficient decisions, the equilibrium outcome incorporating a "high" level of initial debt sets debt equal to debt capacity and the associated covenant will be "very restrictive." By very restrictive we mean that control over the liquidation decision will be transferred to the lender in all states except for those in which it is efficient for all types of firms to continue.[13] The equilibrium outcomes in the richer model *may* then include multiple low debt/unrestrictive covenant equilibria and a high debt/restrictive covenant equilibria. Therefore, for firms with significant amounts of debt, the associated covenants should be very restrictive, allowing the manager to control the liquidation decision only in states for which all types of firms should continue. The existing model captures exactly this aspect of the set of equilibria. Enriching the model in this way, however, quickly complicates the statement of the model and the analysis with no other identified advantages.

A comparative static analysis of equation (8) identifies when it is likely that the equilibrium will include a low or high debt level. The greater is the initial capital

[12] It is also possible that the medium type firm does not begin to make inefficient decisions until initial debt is such that $F > F^L$ in which case there will only be the two equilibria considered in Proposition 4.
[13] This result stems from the fact that the "worst" type of firm in the pool of observationally equivalent firms makes inefficient decisions in "better" states than will the other firm types.

required, *I*, or equivalently, the lower is the expected value added (NPV) of the investment project, the less likely it is that the equilibrium will be a high debt initial financial structure. The required capital, I (or value added, $V^U(F^L, w_2) - I$), represents the market's valuation of the securities sold externally (retained) and all else equal, the greater is the value of the securities sold (retained) the lower (higher) is the incentive to use a large amount of initial debt. The reason equation (8) is written as shown is that other comparative statics, due to the simple nature of the model, are confounded by the fact that many of the basic parameters of the model also impact the value added of the investment opportunity. For consistency with the empirical results which control for firm value or size, we will consider the effect of a comparative statics change in a parameter, holding the value added of the investment opportunity fixed. We also discuss the total impact of such a change.[14]

All else equal, the larger is the firm's debt capacity (Q), and so the greater is the *ex ante* benefit from the use of debt, the more likely it is that the equilibrium will include high debt and a restrictive covenant. This can be seen by taking a partial derivative of equation (8) with respect to Q. There are two effects of a change in debt capacity, the first is due to the increase in the NPV of the investment project that results from an increase in the liquidation value and the second is due to the impact of a change in debt capacity, holding the value added of the investment constant. In the case of an increase in debt capacity, both effects make it more likely the equilibrium will include high debt.

[14] Alternatively, comparative statics, holding the value added of the investment opportunity constant could be developed by making the structure of the model more complex.

As expected, as the cost (c) of renegotiation rises (representing the net cost of moral hazard in the model) it is less likely that firms will use high debt and a restrictive covenant. Raising the cost of renegotiation affects the cost but not the benefit of initial debt use. The results concerning debt capacity and the renegotiation cost reflect the basic tradeoff in the model, *ex ante* adverse selection balanced against *ex post* moral hazard.

Holding the value added of the investment project constant, as the pool of observationally equivalent firms is stronger (i.e., all else equal, the larger is the probability of a good firm, θ, alternatively the larger is B) the less likely it is that the equilibrium in the signaling game is an initial financial structure featuring high debt and a restrictive covenant. The stronger is the pool of observationally equivalent firms, the lower is the adverse selection discount faced by the good firm in the pricing of its securities and so the lower is the benefit from initially issuing a large amount of debt. Increasing B (θ) also increases the (expected) concession in renegotiation, further reducing the incentive for the use of a high debt initial financial structure. Increasing θ or B increases the value added of the investment opportunity, which generates an opposite effect, the net of which may be positive or negative.

Finally, holding the value added of the investment opportunity constant, an increase in the probability of a strong market, p (an improvement in the *ex ante* economic outlook), makes it more likely the initial financial structure will include high debt and a restrictive covenant. In this sense, the simple model captures the usual notion that with costly renegotiation, an important consideration is to restrict the likelihood the initial contract will require renegotiation. Because an increase in the probability of a strong market, p, also increases the value added of the investment opportunity, the net effect of

an increase in the probability of a strong market is also to increase the likelihood of a high debt initial financial structure.

5. Empirical Implications and Discussion

Recent empirical work examining debt covenant violations provides a context in which we may evaluate some of the predictions of our model. Summarizing the discussion above, high debt is associated with very restrictive covenants. Restrictive covenants, when violated are renegotiated to be less restrictive in exchange for an increased debt burden. Furthermore, we are more likely to observe high debt and restrictive covenants the weaker is the observationally equivalent pool of firms, the smaller is the cost of renegotiation, the stronger is the economic outlook, and the larger is the firm's debt capacity.

A set of recent studies, Chava and Roberts (2008), Roberts and Sufi (2009), and Nini, Smith, and Sufi (2012), examine the impact of debt covenant violations on a variety of subsequent corporate decisions. The some of the findings presented in these studies are broadly consistent with results developed here. The most directly applicable result from these papers is the reporting by Chava and Roberts (2008) that covenants are initially written to be remarkably tight. They report that for covenants written on the firm's current ratio, the difference between the value of the firm's actual current ratio and the level at which the covenant is violated, relative to the firm-specific standard deviation of the current ratio, has an average value (median) of 1.09 (0.84). For covenants written based on the firm's net worth the average (median) value of this measure of covenant tightness is 0.68 (0.56). These statistics are consistent with our result that firms with significant amounts of debt will have very restrictive covenants.

Chava and Roberts (2008) also report, for example, that violating firms see a significant decline in investment activity immediately after the violation. Furthermore, they find that the impact on investment of covenant violation is concentrated in the firms which have relatively severe agency and asymmetric information problems. Our model indicates that greater asymmetric information is associated with higher leverage and more restrictive covenants *ex ante*. If firms with larger problems associated with asymmetric information have debt with more restrictive covenants then, all else equal, one would expect that these firms will be heavily represented in the set of firms that violate covenants.

Nini, Smith, and Sufi (2012) examine a number of different corporate decisions. Broadly speaking, they find that violation of debt covenants leads to revisions in these decisions that are consistent with at least a greater weight put on debt holder incentives in subsequent decision making. Most interestingly they find that these changes increase firm performance and value. These findings may suggest that, given the conflict of interest between debt holders and equity holders and the state of the firm/market indicated by a covenant violation, creditor influence on the decision making process limits (or prevents) equity holders' ability to engage in value destroying activities and so increases performance and value. This finding is broadly consistent with our model. In the model, the violation of restrictive covenants and their renegotiation separates good and bad firms *ex post*, imposing efficient decision making and so resulting in an upward revision in value. The model is, however, not rich enough to capture the various changes in behavior examined by Nini, Smith, and Sufi.

One interesting aspect of the findings of Nini, Smith, and Sufi concerns the increase in the likelihood of CEO turnover following a covenant violation. The other decisions that

are influenced by creditors after a covenant violation *may* all simply stem from a restriction on the availability of credit via the sources and uses of funds identity. The increase in the likelihood of CEO turnover is not obviously related to a restriction of the supply of credit in the same way as are new debt issuances or investment behavior. This highlights the fact that broad aspects of managerial decision making may be impacted in the renegotiation game that follow the violation of a debt covenant. These findings suggest that models of capital structure choice should seek to capture a wider variety of conflicts of interest and the exercise of creditor influence that extends well beyond what is provided contractually.

A direct relation between existing empirical results and the predictions of this model is represented by results reported in Roberts and Sufi (2009). Roberts and Sufi (Table VIII) show that covenant violations result in a greater reduction in post-violation debt issuance for firms that have high pre-violation leverage, low equity valuations (market-to-book ratios), and that lack an S&P credit rating. Roberts and Sufi interpret this as indicating that when the firm has few available alternate sources of financing given a covenant violation (the lender has more *ex post* bargaining power), the lender will be able to extract more concessions from the firm in exchange for a waiver of the covenant violation. This result is consistent with the predictions of our model concerning the *ex post* renegotiation following a covenant violation. All else equal, the greater the bargaining power of the lender, the greater will be the concessions a good firm provides in order to renegotiate the covenant.

This empirical result also allows for an interesting *ex ante* interpretation. The model presented in the paper suggests that *ex ante* firms are indifferent to the extent of lender's *ex post* bargaining power. This stems from the assumption that the debt market is

competitive when the firm is initially financed. However, if we alter the model to allow for market power for lenders in the debt market at time *0* (as may be plausible for firm/lender pairs for which the firm will have few alternate sources of financing post-violation), we would see that the impact of greater *ex post* bargaining for the lender on the renegotiation of a violated covenant will not be completely priced into the initial debt contract. This will translate to costlier debt from the firm's perspective and so the use of less debt *ex ante*.

Roberts and Sufi (2009) indicate that firms with higher leverage prior to the violation, lower market-to-book-ratios, and that lack a credit rating, see the greatest reduction in net debt issuance after a covenant violation. All else equal, these are characteristics of firms that indicate an increased tension between the interests of debt holders and equity holders. This finding suggests that firms choosing higher leverage or that are faced poorer investment opportunities or greater informational asymmetries will be expected to have more restrictive covenants attached to their debt than firms for which the agency problems are not as pronounced. This interpretation of their results is consistent with our model. Finally, note that in their empirical model explaining the restrictiveness of covenants (Table VIII) the estimated coefficient on the explanatory variable measuring the tangibility of assets is positive (though not statistically significant). To the extent that asset tangibility is related to liquidation value or debt capacity this is also consistent our model (firms with greater debt capacity are more likely to use high debt and restrictive covenants in the initial financial structure).

Lemmon and Zender (2001) and Strebulaev and Yang (2013) examine very low leveraged firms. They find that firms with very low or zero leverage make up a significant

portion of the set of large public firms and that this policy choice is very persistent. They however find this choice inconsistent with the standard tradeoff theory forces. The low debt/unrestrictive covenant equilibrium identified here offers one possible explanation for their empirical findings.

As a final note on empirical implications we consider what happens when standard tradeoff theory forces are considered in the model. These tensions will help determine the firm's leverage within a range between the firm's riskless level of debt and the firm's debt capacity. This is very different from the idea, introduced by Myers (1984), that tradeoff theory forces affect financing choice only at extreme debt levels. In our model, debt in excess of capacity prevents renegotiation of restrictive debt covenants and so results in the possibility of inefficient investment/liquidation decision making. Standard tradeoff forces, therefore should affect the firm's leverage only to the extent which they impact the choice between the low versus the high debt equilibrium. If the forces determining *ex ante* leverage in this model are important for the broad cross section of firms, then the model suggests an adjustment to standard empirical modeling of the tradeoff theory that recognizes an important role for debt capacity.

6. Conclusion

The capital structure decision is examined in a setting in which asymmetric information is the sole friction. A parsimonious model is developed that allows consideration of the level of debt and the associated covenants as part of the tradeoff in the choice between debt and equity financing. The model can be thought of as a simple extension of Myers and Majluf (1984) where the implications for subsequent decision

making of having debt in the capital structure when there is asymmetric information are considered. Incentive problems created by debt financing and asymmetric information are controlled by the use of debt covenants which transfer control of the relevant decision making to the lender in some states of nature. We demonstrate that equilibrium financial capital structure may be derived trading off the benefits of selling debt, given its relatively low information sensitivity, and the costs of debt, derived from the inefficient decision making induced by the presence of debt in the capital structure or implied by the transfer of control to uninformed parties.

The model considers the renegotiation of debt covenants that inefficiently constrain the actions of the firm. It has been shown that debt covenants are commonly written to be very restrictive and commonly renegotiated when violated (Chava and Roberts (2005) and Nini, Smith, and Sufi (2012)). Our results indicate that the use of restrictive debt covenants is value enhancing only when they may be renegotiated to remove the induced *ex post* inefficiencies. We examine the initial capital structure choice and the renegotiation of restrictive covenants that have been violated and highlight a number of testable implications from the model.

Continuing research examines the extent to which the model's conclusions depend on its relatively simple structure. While this model is simple and transparent and the results seem intuitive, it is important to understand how robust the results are to a more complex environment. Because the ultimate goal is to inform empirical tests of capital structure choice, this is an important avenue of continuing research.

Appendix (proofs of the lemmas and propositions)**:**

Some initial definitions, while tedious, will ease the presentation of the proofs of the results. For low levels of debt, $F \in [L, F^L]$, and an unrestricted covenant:

$$S^G(F, w_2) = pGw_1(H-F) + (1-p)Gw_2(H-F)$$
$$S^B(F, w_2) = pBw_1(H-F) + (1-p)(Q-F)$$
$$S^U(F, w_2, \mu = \theta) = p\bar{t}w_1(H-F) + (1-p)[\theta Gw_2(H-F) + (1-\theta)(Q-F)]$$
$$D^G(F, w_2) = pGw_1(F-L) + (1-p)Gw_2(F-L) + L$$
$$D^B(F, w_2) = pBw_1(F-L) + (1-p)(Q-L) + L$$
$$V^G(F, w_2) = pGw_1(H-L) + (1-p)Gw_2(H-L) + L$$
$$V^B(F, w_2) = pBw_1(H-L) + (1-p)(Q-L) + L$$
$$V^U(F, w_2, \mu = \theta) = p\bar{t}w_1(H-L) + (1-p)[\theta Gw_2(H-L) + (1-\theta)(Q-L)] + L$$

Low debt and a restrictive covenant (no renegotiation) provide:

$$S^G(F, w_1) = pGw_1(H-F) + (1-p)(Q-F)$$
$$S^B(F, w_1) = pBw_1(H-F) + (1-p)(Q-F)$$
$$S^U(F, w_1, \mu = \theta) = p\bar{t}w_1(H-F) + (1-p)(Q-F)$$
$$D^G(F, w_1) = pGw_1(F-L) + (1-p)(F-L) + L$$
$$D^B(F, w_1) = pBw_1(F-L) + (1-p)(F-L) + L$$
$$V^G(F, w_1) = pGw_1(H-L) + (1-p)(Q-L) + L$$
$$V^B(F, w_1) = pBw_1(H-L) + (1-p)(Q-L) + L$$
$$V^U(F, w_1, \mu = \theta) = p\bar{t}w_1(H-L) + (1-p)(Q-L) + L$$

High debt, $F \in (F^L, Q]$, and an unrestrictive covenant (no renegotiation) provide:

$$S^G(F, w_2) = pGw_1(H-F) + (1-p)Gw_2(H-F)$$
$$S^B(F, w_2) = pBw_1(H-F) + (1-p)Bw_2(H-F)$$
$$S^U(F, w_2, \mu = \theta) = p\bar{t}w_1(H-F) + (1-p)\bar{t}w_2(H-F)$$
$$D^G(F, w_2) = pGw_1(F-L) + (1-p)Gw_2(F-L) + L$$
$$D^B(F, w_2) = pBw_1(F-L) + (1-p)Bw_2(F-L) + L$$
$$V^G(F, w_2) = pGw_1(H-L) + (1-p)Gw_2(H-L) + L$$
$$V^B(F, w_2) = pBw_1(H-L) + (1-p)Bw_2(H-L) + L$$
$$V^U(F, w_2, \mu = \theta) = p\bar{t}w_1(H-L) + (1-p)\bar{t}w_2(H-L) + L$$

And high debt and a restrictive covenant (no renegotiation) provide:

$$S^G(F, w_1) = pGw_1(H-F) + (1-p)(Q-L)$$
$$S^B(F, w_1) = pBw_1(H-F) + (1-p)(Q-F)$$
$$S^U(F, w_1, \mu = \theta) = p\bar{t}w_1(H-F) + (1-p)(Q-F)$$
$$D^G(F, w_1) = pGw_1(F-L) + (1-p)(F-L) + L$$
$$D^B(F, w_1) = pBw_1(F-L) + (1-p)(Q-L) + L$$

$$V^G(F,w_1) = pGw_1(H-L)+(1-p)(Q-L)+L$$
$$V^B(F,w_1) = pBw_1(H-L)+(1-p)(Q-L)+L$$
$$V^U(F,w_1,\mu=\theta) = p\bar{t}w_1(H-L)+(1-p)(Q-L)+L$$

Proof of Proposition 1: The objective function of the good manager can be written:

$$\frac{\left(S^U(F,w')+D^U(F,w')-I\right)}{S^U(F,w')}S^G(F,w') = (V^U(F,w')-I)\frac{S^G(F,w')}{S^U(F,w')}.$$

Assuming efficient liquidation decision making by all firms (*i.e.* holding the time *1* decision making constant) direct calculation shows the value of the objective function for the good (bad) manager is strictly increasing (decreasing) in *F* from *0* to *Q* and then remains constant for further increases in *F*. ■

Proof of Lemma 1:

Because $Q > Bw_2(H-L)+L$ and $Bw_2 < 1$, the quantity $(Q-F)-Bw_2(H-F)$ is strictly decreasing in *F* for $F \in [L,Q]$ and is strictly positive for $F = L$ and strictly negative for $F = Q$. Define F^L implicitly by

$$(Q-F^L) = Bw_2(H-F^L).$$

This equality indicates the debt level at which the bad manager is indifferent between continuation and liquidation conditional on the realization of a weak market. Direct calculation provides that $F^L = \frac{Q-w_2BH}{1-w_2B} < Q$.

It is efficient for a good manager to continue in both weak and strong markets and for any value of *F* they have the incentive to do so. ■

Proof of Proposition 2: (no renegotiation)

The structure of the problem implies that the strategy space for managers can be reduced to four options; low debt $(F \in [L,F^L])$ and an unrestrictive covenant, high debt $(F \in (F^L,Q])$ and an unrestrictive covenant, low debt and a restrictive covenant, and high debt and a restrictive covenant. The value to the manager of a type *G* firm from choosing low debt and an unrestrictive covenant is given by

$$\frac{pGw_1(H-F)+(1-p)Gw_2(H-F)}{p\bar{t}_\mu w_1(H-F)+(1-p)[\mu Gw_2(H-F)+(1-\mu)(Q-F)]}\times$$
$$\left(p\bar{t}_\mu w_1(H-L)+(1-p)[\mu Gw_2(H-L)+(1-\mu)(Q-L)]+L-I\right) \quad \textbf{(9)}$$

where μ represents the market's beliefs about the probability the firm choosing the associated financial structure is a good firm. Similarly, the value of a given financial structure choice with low debt and an unrestrictive covenant for the manager of a bad firm is given by

$$\frac{pBw_1(H-F)+(1-p)(Q-F)}{p\bar{t}_\mu w_1(H-F)+(1-p)[\mu Gw_2(H-F)+(1-\mu)(Q-F)]}\times$$
$$\left(p\bar{t}_\mu w_1(H-L)+(1-p)[\mu Gw_2(H-L)+(1-\mu)(Q-L)]+L-I\right). \quad \textbf{(10)}$$

In any equilibrium it must be that $\mu = \theta$. Note that, for any μ, equation (9) is strictly increasing in the face value of debt for $F \in [L, F^L]$ while equation (10) is strictly decreasing *F*. Therefore, the market's off the equilibrium path beliefs suggest that for any candidate equilibrium with an unrestrictive covenant and a "low" level of debt $(F \in [L, F^L])$, if the market observes a deviation to another low debt unrestrictive covenant strategy, the market will believe the deviating firm is a good firm if the deviation represents an increase in the initial debt level relative to the candidate equilibrium and will believe it is a bad firm if the deviation represents a decrease in the initial debt level. Therefore, good firms will benefit by deviating to a higher initial debt level (bad firms would also benefit due to the resulting belief they were good firms) and only $F = F^L$, the upper limit of the range, can possibly represent an equilibrium strategy containing low debt and an unrestrictive covenant. At $F = F^L$ equation (9) collapses to

$$\frac{G}{t}\left(p\bar{t}_\mu w_1(H-L) + (1-p)[\mu G w_2(H-L) + (1-\mu)(Q-L)] + L - I \right) \quad \textbf{(11)}$$

and equation (10) becomes

$$\frac{B}{t}\left(p\bar{t}_\mu w_1(H-L) + (1-p)[\mu G w_2(H-L) + (1-\mu)(Q-L)] + L - I \right) \quad \textbf{(12)}$$

For any financial structure with high debt $(F \in (F^L, Q])$ and an unrestrictive covenant the value to the manager of a good firm is given by

$$\frac{pGw_1(H-F) + (1-p)Gw_2(H-F)}{p\bar{t}_\mu w_1(H-F) + (1-p)\bar{t}_\mu w_2(H-F)} \times \left(p\bar{t}_\mu w_1(H-L) + (1-p)\bar{t}_\mu w_2(H-L) + L - I \right) \quad \textbf{(13)}$$

which equals

$$\frac{G}{\bar{t}_\mu}\left(p\bar{t}_\mu w_1(H-L) + (1-p)\bar{t}_\mu w_2(H-L) + L - I \right) \quad \textbf{(14)}$$

for *all* $F \in (F^L, Q]$. Similarly, for a bad firm choosing *any* high debt unrestrictive covenant financial structure the value to the manager is given by

$$\frac{B}{\bar{t}_\mu}\left(p\bar{t}_\mu w_1(H-L) + (1-p)\bar{t}_\mu w_2(H-L) + L - I \right), \quad \textbf{(15)}$$

independently of the initial debt level in the range $F \in (F^L, Q]$.

If a firm selects a financial structure with a restrictive covenant (with either high or low debt) the value to the manager of a good firm is

$$\frac{pGw_1(H-F) + (1-p)(Q-F)}{p\bar{t}_\mu w_1(H-F) + (1-p)(Q-F)}\left(p\bar{t}_\mu w_1(H-L) + (1-p)(Q-L) + L - I \right) \quad \textbf{(16)}$$

and for a bad firm's manager the value is

$$\frac{pBw_1(H-F) + (1-p)(Q-F)}{p\bar{t}_\mu w_1(H-F) + (1-p)(Q-F)}\left(p\bar{t}_\mu w_1(H-L) + (1-p)(Q-L) + L - I \right) \quad \textbf{(17)}$$

Equation (16) is strictly increasing in *F* for debt levels in the interval $[L, Q]$ and equation (17) is strictly decreasing in *F*.

Off the equilibrium path beliefs for a deviation from a candidate equilibrium including a restrictive covenant to any other strategy that includes a restrictive covenant therefore are that the market believes that increases in the initial debt level relative to the equilibrium are undertaken by good firms while reductions in the initial level of debt relative to the equilibrium strategy are undertaken by bad firms. These beliefs imply that for any candidate equilibrium with $F < Q$ combined with a restrictive covenant, good (and bad) firms benefit from a deviation to higher levels of initial debt and a restrictive covenant. Only an initial financial structure with $F = Q$ and a restrictive covenant is possibly an equilibrium. The values for the manager of a good firm and a bad firm are given by

$$\frac{G}{\bar{t}_\mu}\left(p\bar{t}_\mu w_1(H-L)+(1-p)(Q-L)+L-I\right) \qquad \textbf{(18)}$$

and

$$\frac{B}{\bar{t}_\mu}\left(p\bar{t}_\mu w_1(H-L)+(1-p)(Q-L)+L-I\right) \qquad \textbf{(19)}$$

The set of PSPB equilibria for the signaling game without renegotiation can then be established by comparing versions of equations (11), (12), (14), (15), (18), and (19). Consider the candidate equilibrium with $F = F^L$ and an unrestrictive covenant. The equilibrium values for the manager of the manager of a good firm and for a bad firm are given by equations (11) and (12) with $\mu = \theta$. If a deviation to an initial financial structure with a restrictive covenant is observed (equations (18) and (19)), using beliefs given by $\mu = \theta$ direct comparison (because $Gw_2(H-L) > Q-L$) makes it clear that neither good nor bad firms will benefit from such a deviation. The market will then assign passive beliefs of $\mu = \theta$ regarding the probability a deviating firm is a good firm and consequently neither good nor bad firms will deviate from the proposed equilibrium by including a restrictive covenant as part of the initial financial structure.

Similarly, comparing equations (11) and (12) to equations (14) and (15) it is clear, under passive beliefs, that neither good nor bad firms benefit $(Bw_2(H-L) < Q-L)$ from a deviation to an initial financial structure with high debt and an unrestrictive covenant. The market, therefore, assigns beliefs given by $\mu = \theta$ to any such deviation and neither good nor bad firms have an incentive to deviate to a high debt, unrestrictive covenant initial financial structure. Therefore, the low debt $(F = F^L)$ unrestrictive covenant financial structure represents an equilibrium financial structure.

From a candidate equilibrium financial structure including a restrictive covenant (equations (18) and (19)) consider a deviation to low debt $(F = F^L)$ and an unrestrictive covenant. Under passive beliefs, both good firms (equation (18) compared to equation (11)) and bad firms (equation (19) compared to equation (12)) have an incentive to deviate to a low debt unrestrictive covenant financial structure. Off the equilibrium path beliefs are therefore given by $\mu = \theta$ and consequently both types will deviate from the proposed equilibrium.

Similarly from a high debt $(F = Q)$, unrestrictive covenant candidate equilibrium, (equations (14) and (15)) consider a deviation to a low debt $(F = F^L)$ unrestrictive

financial structure. Comparing equation (14) to equation (11) and equation (15) to equation (12) it is clear that under passive beliefs that both types of firms have an incentive to deviate so the off the equilibrium path beliefs resulting from this deviation are given by $\mu = \theta$. Therefore, both types of firm will deviate and the high debt unrestrictive covenant financial cannot represent an equilibrium choice. ■

Proof of Lemma 3: (costless renegotiation of a restrictive covenant)
When the initial financial structure includes a restrictive covenant, following the realization of a weak market, $w = w_2$, it is inefficient to allow the lender to make the liquidation decision. The lender does not know the firm's type and would prefer to liquidate the firm regardless of type. Renegotiation can take place if there are gains to having the set of firms (the lender believes is) attempting to renegotiate continue rather than liquidate in a weak market. For the first case, assume that $\bar{t}w_2(H-L)+L \le Q$. Then the only firms for which it is efficient to renegotiate a restrictive covenant in a weak market are the good firms. In equilibrium, therefore only separating renegotiation offers will be accepted. Any renegotiation offer the good firm will make and the lender will accept must satisfy constraints for participation from good firms and the lender and a separation constraint that indicates bad firms will not make the same offer. Therefore, given an initial debt level F^R and conditional on the realization of a weak market, the following constraints must be satisfied for the new debt level F^N

$$Gw_2(F^N - L) + L \ge F^R \qquad \textbf{(20)}$$

which ensures the lender, believing a good firm has made the offer will accept the new debt level in exchange for waiving the covenant violation (allowing continuation). Good firms must be willing to make the offer of an increased debt level in order to allow the firm to continue rather than liquidate:

$$Gw_2(H - F^N) \ge Q - F^R \ . \qquad \textbf{(21)}$$

And finally bad firms must prefer to liquidate at the initial debt level rather than continue under the new debt level

$$Q - F^R \ge Bw_2(H - F^N) . \qquad \textbf{(22)}$$

Each of these conditions can be solved to find a relation between the new level of debt as a function of the initial debt level, $F^N(F^R)$. When the firm makes a take-it-or-leave-it offer to the lender, renegotiation offers will be those satisfying all three constraints that offer the lowest new debt level for a given initial debt level. Solving equations (20) and (22) for F^N and setting them equal identifies the cutoff point F^{R_L},

$$F^{R_L} = \frac{Gw_2\left((Q-L) - Bw_2(H-L)\right)}{Gw_2 - Bw_2} + L \ . \qquad \textbf{(23)}$$

For initial debt levels $F^R \le F^{R_L}$ the offer a good firm will make is given by equation (20) written as an equality:

$$F^N(F^R) = \frac{F^R - L}{Gw_2} + L \ . \qquad \textbf{(24)}$$

For initial debt levels $F^{R_L} < F^R \le H$ the offer a good firm will make is given by equation (22) written as an equality:

$$F^N(F^R) = H - \frac{Q - F^R}{Bw_2} \quad . \tag{25}$$

These offers will be accepted by the lender and the bad firm does not renegotiate.

If instead $\bar{t}w_2(H-L)+L > Q$, then there is scope for renegotiation of the initial debt level by good firms or good and bad firms (pooling renegotiation). The change in parameter values introduces one change in the discussion above. For initial debt levels $F^R \le F^{R_L}$ separating renegotiation offers will be given by equation (24) and for initial debt levels $F^{R_L} < F^R \le F^{R_H}$, where F^{R_H} is given as

$$F^{R_H} = \frac{\bar{t}w_2\left((Q-L) - Bw_2(H-L)\right)}{\bar{t}w_2 - Bw_2} + L \tag{26}$$

separating renegotiation offers are given by equation (25). For initial debt levels $F^{R_H} < F^R \le Q$, however, renegotiation offers will be pooling offers where both good and bad firms make renegotiation offers. The lender believing that both types pool will accept offers given by

$$F^N(F^R) = \frac{F^R - L}{\bar{t}w_2} + L \tag{27}$$

and both good and bad type firms will offer to renegotiate the violated covenant. ■

Proof of Lemma 4: (costless renegotiation of an unrestrictive covenant)
The proof of lemma 4 closely follows that of lemma 3 and so we sketch the proof. The one difference is that for any initial debt level $F^U \le F^L$ the lender will not accept any offer to renegotiate an unrestrictive covenant as both types of firms already have an incentive to make efficient decisions. For parameter values such that it is efficient for the average firm to continue, $\bar{t}w_2(H-L)+L \ge Q$ there is scope only for bad firms to renegotiate an unrestrictive covenant. In this case, the separating renegotiation offers made by a bad firm in a weak market entail an offer to lower the debt level in exchange for a voluntary liquidation of the firm. If the initial debt level F^U is in the range $F^L < F^U \le F^{U_L}$, where F^{U_L} represents the intersection of the lender's separating participation constraint and the separation constraint,

$$F^{U_L} = \frac{Gw_2(H-L) - (Q-L)}{Gw_2 - Bw_2} + L \ , \tag{28}$$

then separating renegotiation offers will be given by the lender's participation constraint

$$F^N(F^U) = Bw_2(F^U - L) + L \, . \tag{29}$$

If the initial debt level is such that $F^{U_L} \le F^U \le H$ (note that $F^U = H$ only if the value added of the investment is zero in expectation) then separating renegotiation offers are determined by the separation constraint:

$$F^N(F^U) = Q - Gw_2(H - F^U) \, . \tag{30}$$

The lender believing the offer is made by a bad firm accepts the reduction in face value in exchange for the firm being liquidated and good firms continue with the initial debt level. For parameter values such that it is inefficient for the average firm to continue, $\bar{t}w_2(H-L)+L < Q$ there is scope for pooling renegotiation of an unrestrictive covenant.

In this case for initial debt in the range $F^L < F^U \le F^{U_L}$ then renegotiation offers will be separating and given by equation (29). For initial debt levels such that $F^{U_L} \le F^U \le F^{U_H}$

$$F^{U_H} = \frac{Gw_2(H-L)-(Q-L)}{Gw_2 - \bar{t}w_2} + L \qquad \textbf{(31)}$$

acceptable renegotiation offers will be given by equation (30). For initial levels of debt such that $F^{U_H} \le F^U \le H$ renegotiation offers will be pooling offers where both firm types renegotiate. The offers are determined by

$$F^N(F^U) = \bar{t}w_2(F^U - L) + L \ . \qquad \textbf{(32)}$$

The lender believing that both types will renegotiate will accept the offer. This version of the proof assumes that $F^L < F^{U_L}$ which is not necessarily true. In the case $F^L \ge F^{U_L}$ the set of initial debt levels that allow the renegotiation of the unrestrictive covenant by a bad firm is simply truncated so that the case presented is the most complete. ■

Proof of Proposition 3: (costless renegotiation)

As in the proof of proposition 2, in order to prove proposition 3 we first specify the *ex ante* values of the debt, equity, and the firm, taking into account the expectations for any *ex post* renegotiation, for the possible combinations of debt level and covenants. The value to the manager of good and bad firms are then identified and the equilibrium initial financial structure can be identified.

For low levels of debt, $L \le F^R \le F^{R_L}$, combined with a restrictive covenant the informed values of equity, debt and the firm are written (using lemma 3)

$$\begin{aligned} S^G(F^R, w_1) &= pGw_1(H - F^R) + (1-p)Gw_2(H - F^N) \\ &= pGw_1(H - F^R) + (1-p)Gw_2\left(H - \frac{F^R - L}{Gw_2} - L\right) \\ &= pGw_1(H - F^R) + (1-p)Gw_2(H - L) - (1-p)(F^R - L) \end{aligned} \qquad \textbf{(33)}$$

$$\begin{aligned} D^G(F^R, w_1) &= pGw_1(F^R - L) + (1-p)Gw_2(F^N - L) + L \\ &= pGw_1(F^R - L) + (1-p)(F^R - L) + L \end{aligned} \qquad \textbf{(34)}$$

$$V^G(F^R, w_1) = pGw_1(H - L) + (1-p)Gw_2(H - L) + L \qquad \textbf{(35)}$$

$$S^B(F^R, w_1) = pBw_1(H - F^R) + (1-p)(Q - F^R) \qquad \textbf{(36)}$$

$$D^B(F^R, w_1) = pBw_1(F^R - L) + (1-p)(F^R - L) + L \qquad \textbf{(37)}$$

$$V^B(F^R, w_1) = pBw_1(H - L) + (1-p)(Q - L) + L \qquad \textbf{(38)}$$

The uninformed (market) values are found by taking the expectation over the types based on the market's belief concerning the probability a firm is good (μ) or bad (1-μ).

As above, the value of type *t* firm manager's retained equity for this financial structure is given by

$$\frac{S^t(F^R, w_1)}{S^U(F^R, w_1)}\left(V^U(F^R, w_1) - I\right) \qquad \textbf{(39)}$$

It is straightforward to show that the good firm manager's value is strictly increasing in the initial debt level F^R and the bad firm manager's value is strictly decreasing in F^R on the interval $L \le F^R \le F^{R_L}$. This implies that the only financial structure with low debt

and a restrictive covenant that may be an equilibrium is when $F^R = F^{R_L}$. The value of the equity retained by the manager of a type *t* firm under this initial financial structure is

$$\begin{aligned}&\frac{S^t(F^R, w_1)}{S^U(F^R, w_1)}\left(V^U(F^R, w_1) - I\right)\\ &= \frac{t}{\bar{t}}\left(p\bar{t}_\mu w_1(H-L) + (1-p)[\mu G w_2(H-L) + (1-\mu)(Q-L)] + L - I\right)\end{aligned} \tag{40}$$

Note that, for any candidate equilibrium with a lower initial debt level and a restrictive covenant, deviation to $F^R = F^{R_L}$ and a restrictive covenant will, holding market beliefs at $\mu = \theta$, will strictly benefit good firms which implies that the market's belief upon seeing such a deviation is that the deviation was done by a good firm. Thus both types of firms will wish to deviate from the candidate equilibrium.

For high debt $F^{R_L} < F^R \le Q$ if it is inefficient for an average firm to continue (or for debt levels $F^{R_L} < F^R \le F^{R_H}$ when it is efficient for an average firm to continue) the informed values of the equity, debt, and the firm are written

$$\begin{aligned}S^G(F^R, w_1) &= pGw_1(H - F^R) + (1-p)Gw_2(H - F^N)\\ &= pGw_1(H - F^R) + (1-p)Gw_2\left(\frac{Q - F^R}{Bw_2}\right)\end{aligned} \tag{41}$$

$$\begin{aligned}D^G(F^R, w_1) &= pGw_1(F^R - L) + (1-p)Gw_2(F^N - L) + L\\ &= pGw_1(F^R - L) + (1-p)\left(H - \frac{Q - F^R}{Bw_2} - L\right) + L\end{aligned} \tag{42}$$

$$V^G(F^R, w_1) = pGw_1(H-L) + (1-p)Gw_2(H-L) + L \tag{43}$$

$$S^B(F^R, w_1) = pBw_1(H - F^R) + (1-p)(Q - F^R) \tag{44}$$

$$D^B(F^R, w_1) = pBw_1(F^R - L) + (1-p)(F^R - L) + L \tag{45}$$

$$V^B(F^R, w_1) = pBw_1(H-L) + (1-p)(Q-L) + L \tag{46}$$

The uninformed (market) values are found by taking the expectation over the types based on the market's belief concerning the probability a firm is good (μ) or bad (1-μ). The value of a type *t* firm manager's retained equity for this financial structure is given by

$$\frac{S^t(F^R, w_1)}{S^U(F^R, w_1)}\left(V^U(F^R, w_1) - I\right) \tag{47}$$

It is straightforward to show that the good and bad firm manager's value is independent of F^R and each initial financial structure with high debt and a restrictive covenant will provide a value for the equity retained by the manager of a good firm as given in equation (40).

For debt levels $F^{R_H} < F^R \le Q$ when it is efficient for an average firm to continue, the informed values of the equity, debt, and the firm are written

$$\begin{aligned}S^G(F^R, w_1) &= pGw_1(H - F^R) + (1-p)Gw_2(H - F^N)\\ &= pGw_1(H - F^R) + (1-p)Gw_2\left(\frac{Q - F^R}{\bar{t}w_2}\right)\end{aligned} \tag{48}$$

$$D^G(F^R, w_1) = pGw_1(F^R - L) + (1-p)Gw_2(F^N - L) + L$$
$$= pGw_1(F^R - L) + (1-p)\left(H - \frac{Q - F^R}{\bar{t}w_2} - L\right) + L \quad \textbf{(49)}$$

$$V^G(F^R, w_1) = pGw_1(H - L) + (1-p)Gw_2(H - L) + L \quad \textbf{(50)}$$

$$S^B(F^R, w_1) = pBw_1(H - F^R) + (1-p)Bw_2(F^N - L)$$
$$= pBw_1(H - F^R) + (1-p)Bw_2(H - L) - (1-p)Bw_2\left(\frac{F^R - L}{\bar{t}w_2}\right) \quad \textbf{(51)}$$

$$D^B(F^R, w_1) = pBw_1(F^R - L) + (1-p)Bw_2(F^N - L) + L$$
$$= pBw_1(F^R - L) + (1-p)Bw_2\left(\frac{F^R - L}{\bar{t}w_2}\right) + L \quad \textbf{(52)}$$

$$V^B(F^R, w_1) = pBw_1(H - L) + (1-p)Bw_2(H - L) + L \quad \textbf{(53)}$$

The uninformed (market) values are found by taking the expectation over the types based on the market's belief concerning the probability a firm is good (μ) or bad (1-μ). The value of a type *t* firm manager's retained equity for this financial structure is given by

$$\frac{S^t(F^R, w_1)}{S^U(F^R, w_1)}\left(V^U(F^R, w_1) - I\right) \quad \textbf{(54)}$$

The good firm manager's value is independent of F^R over this range of debt levels and each initial financial structure with high debt and a restrictive covenant will provide a value for the equity retained by the manager of a good firm as given in equation (40).

For low levels of debt, $F^L < F^U \le F^{U_L}$, combined with an unrestrictive covenant the informed values of equity, debt and the firm are written (using lemma 4)

$$S^G(F^U, w_2) = pGw_1(H - F^U) + (1-p)Gw_2(H - F^U) \quad \textbf{(55)}$$

$$D^G(F^U, w_2) = pGw_1(F^U - L) + (1-p)Gw_2(F^U - L) + L \quad \textbf{(56)}$$

$$V^G(F^U, w_2) = pGw_1(H - L) + (1-p)Gw_2(H - L) + L \quad \textbf{(57)}$$

$$S^B(F^U, w_2) = pBw_1(H - F^U) + (1-p)(Q - F^N)$$
$$= pBw_1(H - F^U) + (1-p)\left((Q - L) - Bw_2(F^U - L)\right) \quad \textbf{(58)}$$

$$D^B(F^U, w_2) = pBw_1(F^U - L) + (1-p)(F^N - L) + L$$
$$= pBw_1(H - F^U) + (1-p)Bw_2(F^U - L) + L \quad \textbf{(59)}$$

$$V^B(F^U, w_2) = pBw_1(H - L) + (1-p)(Q - L) + L \quad \textbf{(60)}$$

The uninformed (market) values are found by taking the expectation over the types based on the market's belief concerning the probability a firm is good (μ) or bad (1-μ). The value of a type *t* firm manager's retained equity for this financial structure is given by

$$\frac{S^t(F^R, w_1)}{S^U(F^R, w_1)}\left(V^U(F^R, w_1) - I\right) \quad \textbf{(61)}$$

The good (bad) firm manager's value is strictly decreasing (increasing) in F^U. For any candidate equilibrium with a level of debt in the interval $F^L < F^U \le F^{U_L}$ combined with an unrestricted covenant, holding the market's belief's fixed at $\mu = \theta$, good (bad) firm

managers strictly benefit from a deviation to an initial financial structure with $F^U = F^L$ combined with an unrestrictive covenant.

For levels of debt $F^{U_L} < F^U \le F^{U_H}$ (when it is inefficient for an average firm to continue) or $F^{U_L} < F^U \le H$ (when it is efficient for an average firm to continue) combined with an unrestrictive covenant the informed values of equity, debt and the firm are written (using lemma 4)

$$S^G(F^U, w_2) = pGw_1(H - F^U) + (1-p)Gw_2(H - F^U) \quad \textbf{(62)}$$

$$D^G(F^U, w_2) = pGw_1(F^U - L) + (1-p)Gw_2(F^U - L) + L \quad \textbf{(63)}$$

$$V^G(F^U, w_2) = pGw_1(H - L) + (1-p)Gw_2(H - L) + L \quad \textbf{(64)}$$

$$\begin{aligned} S^B(F^U, w_2) &= pBw_1(H - F^U) + (1-p)(Q - F^N) \\ &= pBw_1(H - F^U) + (1-p)Gw_2(H - F^U) \end{aligned} \quad \textbf{(65)}$$

$$\begin{aligned} D^B(F^U, w_2) &= pBw_1(F^U - L) + (1-p)(F^N - L) + L \\ &= pBw_1(H - F^U) + (1-p)\Big((Q - L) - Gw_2(H - F^U)\Big) + L \end{aligned} \quad \textbf{(66)}$$

$$V^B(F^U, w_2) = pBw_1(H - L) + (1-p)(Q - L) + L \quad \textbf{(67)}$$

The uninformed (market) values are found by taking the expectation over the types based on the market's belief concerning the probability a firm is good (μ) or bad (1-μ). The value of a type *t* firm manager's retained equity for this financial structure is given by

$$\frac{S^t(F^R, w_1)}{S^U(F^R, w_1)}\Big(V^U(F^R, w_1) - I\Big) \quad \textbf{(68)}$$

The good (and bad) firm manager's value is independent of F^U over the relevant range. Therefore, all of the relevant financial structures or none of them will be equilibria in the signaling game for initial choice of financial structure.

For levels of debt $F^{U_H} < F^U \le H$ (when it is inefficient for an average firm to continue) combined with an unrestrictive covenant the informed values of equity, debt and the firm are written (using lemma 4)

$$\begin{aligned} S^G(F^U, w_2) &= pGw_1(H - F^U) + (1-p)(Q - F^N) \\ &= pGw_1(H - F^U) + (1-p)\Big((Q - L) - \bar{t}w_2(F^U - L\Big) \end{aligned} \quad \textbf{(69)}$$

$$\begin{aligned} D^G(F^U, w_2) &= pGw_1(F^U - L) + (1-p)(F^N - L) + L \\ &= pGw_1(F^U - L) + (1-p)\bar{t}w_2(F^U - L) + L \end{aligned} \quad \textbf{(70)}$$

$$V^G(F^U, w_2) = pGw_1(H - L) + (1-p)(Q - L) + L \quad \textbf{(71)}$$

$$\begin{aligned} S^B(F^U, w_2) &= pBw_1(H - F^U) + (1-p)(Q - F^N) \\ &= pBw_1(H - F^U) + (1-p)\Big((Q - L) - \bar{t}w_2(F^U - L\Big) \end{aligned} \quad \textbf{(72)}$$

$$\begin{aligned} D^B(F^U, w_2) &= pBw_1(F^U - L) + (1-p)(F^N - L) + L \\ &= pBw_1(F^U - L) + (1-p)\bar{t}w_2(F^U - L) + L \end{aligned} \quad \textbf{(73)}$$

$$V^B(F^U, w_2) = pBw_1(H - L) + (1-p)(Q - L) + L \quad \textbf{(74)}$$

The uninformed (market) values are found by taking the expectation over the types based on the market's belief concerning the probability a firm is good (μ) or bad (1-μ). The value of a type *t* firm manager's retained equity for this financial structure is given by

$$\frac{S^t(F^R, w_1)}{S^U(F^R, w_1)}\left(V^U(F^R, w_1) - I\right) \quad \textbf{(75)}$$

The good (bad) firm manager's value is decreasing (increasing) in F^U over the relevant range. Therefore only an initial debt level equal to F^{U_H} is a candidate equilibrium for this range of initial debt levels combined with an unrestrictive covenant.

Establishing the set of equilibria with costless renegotiation is now straightforward. For each of the remaining candidate equilibria (low debt $(F^U = F^L)$ and an unrestrictive covenant, high debt $(F^{U_L} \le F^U \le H)$ and an unrestrictive covenant (for the separating and the pooling renegotiation cases), and high debt $(F^{R_L} \le F^R \le Q)$ combined with a restrictive covenant (for both the separating and pooling renegotiation cases) compare the equilibrium value to the manager of a good and/or a bad firm to the value from deviation to an alternate initial financial structure. Applying off the equilibrium path beliefs as described in the text and the process followed in the proof of proposition 2, only the low debt unrestrictive covenant or the set of high debt restrictive covenant financial structures survive as equilibria of the signaling game. In each of the other cases deviation by the good firm to the low debt unrestrictive covenant strategy results in a strict gain implying these candidates cannot be equilibria of the signaling game. For the two types of equilibria identified, deviations are either strictly improving for bad firms (implying a market belief $\mu = 0$) or neutral for both firm types (implying a market belief $\mu = \theta$) and therefore neither firm has an incentive to deviate. ■

Proof of Lemma 5: (costly renegotiation of a restrictive covenant)

For low initial debt levels, $L \le F^R \le F^{R_L}$, where

$$F^{R_L} = \frac{Gw_2\left((Q-L) - Bw_2(H-L)\right)}{Gw_2 - Bw_2} + L + \frac{Gw_2 c}{Gw_2 - Bw_2}\ , \quad \textbf{(76)}$$

combined with a restrictive covenant, renegotiation offers will be determined by the lender's participation constraint

$$F^N(F^R) = \frac{F^R - L}{Gw_2} + L\ . \quad \textbf{(77)}$$

The separation constraint is also satisfied under these conditions and so when a weak market is realized, good firms make renegotiation offers as specified in equation (77), bad firms do not attempt to renegotiate and so will be liquidated due to the covenant violation, and the lender, believing a good firm has made the renegotiation offer will accept the offer to waive the violation of the covenant.

For high initial debt levels, $F^{R_L} \le F^R \le Q$, combined with a restrictive covenant, renegotiation offers will be determined by the separation constraint,

$$F^N(F^R) = H - \frac{Q - F^R}{Bw_2} - \frac{c}{Bw_2}\ . \quad \textbf{(78)}$$

The lender's participation constraint is also satisfied under these conditions so that again bad firms do not attempt to renegotiate following the realization of a weak market and the lender, believing a good firm has made the renegotiation offer will accept the offer to waive the violation of the covenant. ■

Proof of Lemma 6: (costly renegotiation of an unrestrictive covenant)

For low initial debt levels, $F^L < F^U \le F^{U_L}$, where

$$F^{U_L} = \frac{Gw_2(H-L)-(Q-L)}{Gw_2 - Bw_2} + L + \frac{c}{Gw_2 - Bw_2} \text{ ,} \tag{79}$$

combined with an unrestrictive covenant, renegotiation offers will be determined by the lender's participation constraint

$$F^N(F^U) = Bw_2(F^U - L) + L \text{ .} \tag{80}$$

The separation constraint is also satisfied under these conditions so good firms will continue under the original debt contract and the lender, believing that a bad firm has made the offer to liquidate the firm in exchange for a reduced debt level will accept the offer.

For high initial debt levels, $F^{U_L} < F^U \le H$, combined with an unrestrictive covenant, renegotiation offers will be determined by the separation constraint

$$F^N(F^U) = Q - Gw_2(H - F^U) - c \text{ .} \tag{81}$$

The lender's participation constraint is also satisfied under these conditions so good firms will continue under the original debt contract and the lender, believing that a bad firm has made the offer to liquidate the firm in exchange for a reduced debt level will accept the offer.

Proof of Proposition 4: (costly renegotiation)

For the initial financial structure with $F^U = F^L$ combined with an unrestrictive covenant, there is no renegotiation and the value to the manager of a good and a bad firm are as given in equations (11) and (12).

Using lemma 5, for low initial values of debt $L \le F^R \le F^{R_L}$ (where F^{R_L} is defined in equation (76)) combined with a restrictive covenant, the informed values of equity, debt and the firm for the good firm are written

$$\begin{aligned} S^G(F^R, w_1) &= pGw_1(H - F^R) + (1-p)(Gw_2(H - F^N) - c) \\ &= pGw_1(H - F^R) + (1-p)\left(Gw_2\left(H - \frac{F^R - L}{Gw_2} - L \right) - c \right) \\ &= pGw_1(H - F^R) + (1-p)Gw_2(H - L) - (1-p)(F^R - L) - (1-p)c \end{aligned} \tag{82}$$

$$\begin{aligned} D^G(F^R, w_1) &= pGw_1(F^R - L) + (1-p)Gw_2(F^N - L) + L \\ &= pGw_1(F^R - L) + (1-p)(F^R - L) + L \end{aligned} \tag{83}$$

$$V^G(F^R, w_1) = pGw_1(H - L) + (1-p)Gw_2(H - L) + L - (1-p)c \tag{84}$$

Because bad firms do not renegotiate, the relevant values are given by equations (36) – (38). The uninformed (market) values are found by taking the expectation over the types based on the market's belief concerning the probability a firm is good (μ) or bad (1-μ). The value of a type *t* firm manager's retained equity for this financial structure is given by

$$\frac{S^t(F^R, w_1)}{S^U(F^R, w_1)}\left(V^U(F^R, w_1) - I\right) \tag{85}$$

which is strictly increasing (decreasing) in the level of initial debt for good (bad) firms over the interval $L \le F^R \le F^{R_L}$.

Using lemma 5, for high initial values of debt $F^{R_L} \le F^R \le Q$ combined with a restrictive covenant, the informed values of equity, debt and the firm for the good firm are written

$$\begin{aligned} S^G(F^R, w_1) &= pGw_1(H - F^R) + (1-p)\left(Gw_2(H - F^N) - c\right) \\ &= pGw_1(H - F^R) + (1-p)\left(Gw_2\left(\frac{Q - F^R + c}{Bw_2}\right) - c\right) \end{aligned} \quad \textbf{(86)}$$

$$\begin{aligned} D^G(F^R, w_1) &= pGw_1(F^R - L) + (1-p)Gw_2(F^N - L) + L \\ &= pGw_1(F^R - L) + (1-p)Gw_2\left((H - L) - \frac{Q - F^R + c}{Bw_2}\right) + L \end{aligned} \quad \textbf{(87)}$$

$$V^G(F^R, w_1) = pGw_1(H - L) + (1-p)Gw_2(H - L) + L - (1-p)c \quad \textbf{(88)}$$

Because bad firms do not renegotiate in equilibrium, the relevant values are again given by equations (36) – (38). The uninformed (market) values are found by taking the expectation over the types based on the market's belief concerning the probability a firm is good (μ) or bad (1-μ). The value of a type *t* firm manager's retained equity for this financial structure is given by

$$\frac{S^t(F^R, w_1)}{S^U(F^R, w_1)}\left(V^U(F^R, w_1) - I\right) \quad \textbf{(89)}$$

which is strictly increasing (decreasing) in the level of initial debt for good (bad) firms over the interval $F^{R_L} \le F^R \le Q$. For any F^R in the interval $L \le F^R < Q$ the value of the retained equity for the manager of a good (bad) firm will strictly increase for any increase in F^R, holding market beliefs at $\mu = \theta$, the only candidate equilibrium that includes the use of a restrictive covenant is at the firm's debt capacity, $F^R = Q$.

Using lemma 6, for low initial values of debt $F^L < F^R \le F^{U_L}$, where

$$F^{U_L} = \frac{Gw_2(H - L) - (Q - L) + c}{Gw_2 - Bw_2} + L, \quad \textbf{(90)}$$

combined with an unrestrictive covenant, the informed values of equity, debt and the firm for the bad firm are written

$$\begin{aligned} S^B(F^U, w_2) &= pBw_1(H - F^U) + (1-p)(Q - F^N) - (1-p)c \\ &= pBw_1(H - F^U) + (1-p)\left((Q - L) - Bw_2(F^U - L)\right) - (1-p)c \end{aligned} \quad \textbf{(91)}$$

$$\begin{aligned} D^B(F^U, w_2) &= pBw_1(F^U - L) + (1-p)(F^N - L) + L \\ &= pBw_1(H - F^U) + (1-p)Bw_2(F^U - L) + L \end{aligned} \quad \textbf{(92)}$$

$$V^B(F^U, w_2) = pBw_1(H - L) + (1-p)(Q - L) + L - (1-p)c. \quad \textbf{(93)}$$

In equilibrium, the good firm does not renegotiate so the relevant values are given by equations (55), (56), and (57). The uninformed (market) values are found by taking the expectation over the types based on the market's belief concerning the probability a firm is good (μ) or bad (1-μ). The value of a type *t* firm manager's retained equity for this financial structure is given by

$$\frac{S^t(F^R, w_1)}{S^U(F^R, w_1)}\left(V^U(F^R, w_1) - I\right) \tag{94}$$

which is strictly decreasing (increasing) in the initial debt level for good (bad) firms over the interval $F^L \le F^U \le F^{U_L}$.

Using lemma 6, for high initial values of debt, $F^{U_L} \le F^R < H$, combined with an unrestrictive covenant, the informed values of equity, debt and the firm for the bad firm are written

$$\begin{aligned} S^B(F^U, w_2) &= pBw_1(H - F^U) + (1-p)(Q - F^N) - (1-p)c \\ &= pBw_1(H - F^U) + (1-p)\left(Gw_2(H - F^U) + c\right) - (1-p)c \\ &= pBw_1(H - F^U) + (1-p)Gw_2(H - F^U) \end{aligned} \tag{95}$$

$$\begin{aligned} D^B(F^U, w_2) &= pBw_1(F^U - L) + (1-p)(F^N - L) + L \\ &= pBw_1(H - F^U) + (1-p)\left((Q - L) - Gw_2(H - F^U) - c\right) + L \end{aligned} \tag{96}$$

$$V^B(F^U, w_2) = pBw_1(H - L) + (1-p)(Q - L) + L - (1-p)c \tag{97}$$

The associated values for the good firm are again by equations (62), (63), and (64). The uninformed (market) values are found by taking the expectation over the types based on the market's belief concerning the probability a firm is good (μ) or bad (1-μ). The value of a type *t* firm manager's retained equity for this financial structure is given by

$$\frac{S^t(F^R, w_1)}{S^U(F^R, w_1)}\left(V^U(F^R, w_1) - I\right). \tag{98}$$

In this case the values for the good and bad manager are independent of the initial debt level. Therefore the only candidate equilibrium that includes an unrestrictive covenant has an initial debt level that is marginally above F^L (i.e. the "smallest" initial debt level that results in the renegotiation of an unrestrictive covenant). For any other initial debt level, the good firm strictly benefits (holding beliefs at $\mu = \theta$) deviating to a lower initial debt level, off the equilibrium path beliefs are therefore that such deviations are from a good firm, implying both types of firms will deviate.

There are then three candidate equilibria in the case of costly renegotiation; low initial debt and an unrestrictive covenant that will not be renegotiated $(F^U = F^L)$, low initial debt and an unrestrictive covenant that may be renegotiated $(F^U = F^L + \varepsilon)$, and high initial debt combined with a restrictive covenant. Consider the financial structure with low initial debt and an unrestrictive covenant that may be renegotiated. Holding beliefs at $\mu = \theta$, good firms strictly benefit from a deviation to the financial structure with low debt and an unrestrictive covenant that will not be renegotiated. Such a deviation will result in the off the equilibrium path belief that a good firm is deviating. Both firm types will then wish to deviate implying this is not an equilibrium.

For the other two candidate equilibria, depending on parameter values, each may represent a PSPB equilibrium of the signaling game. Making substitutions in to equation (89) to determine the value of the good firm manager's retained equity and comparing this to equation (11) results in the inequality provided in the proposition. Consider the high debt/restrictive covenant candidate. If the inequality in equation (8) holds then, assuming beliefs are represented by $\mu = \theta$, the good firm will be strictly worse off from

any deviation implying deviation results in the belief that the deviating firm is bad and no type will deviate from the candidate equilibrium. If the stated inequality is violated, then, assuming beliefs are represented by $\mu = \theta$, good firms strictly benefit from deviating to the low debt/unrestrictive covenant financial structure and this deviation will be seen as having been done by a good firm. The result is that both types will wish to deviate. If the inequality is reversed, a similar argument shows that the low debt/unrestrictive covenant (which will never be renegotiated) financial structure is an equilibrium of the signaling game. ■

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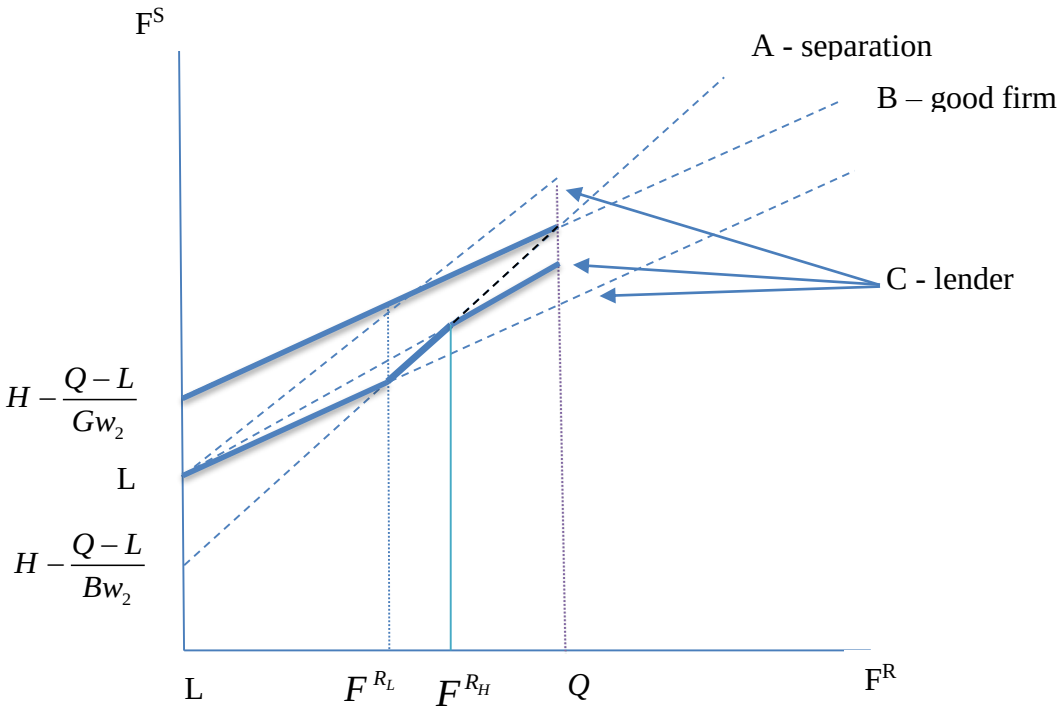
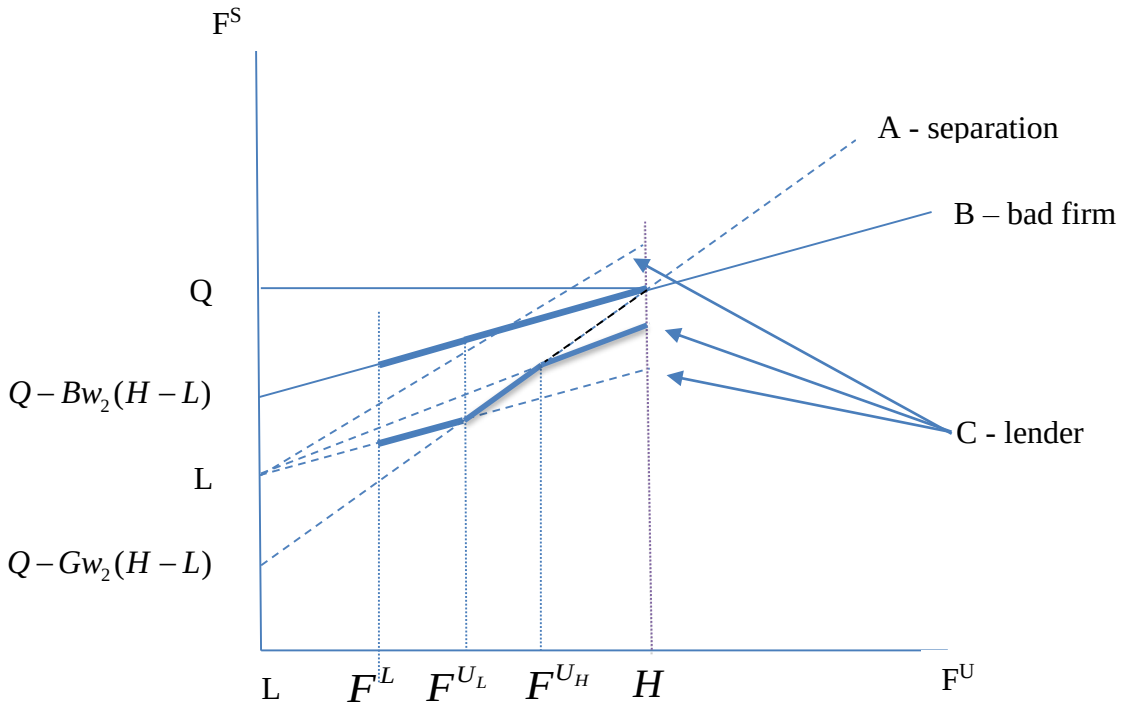
Figure 1: Costless Renegotiation of Restrictive Covenants.



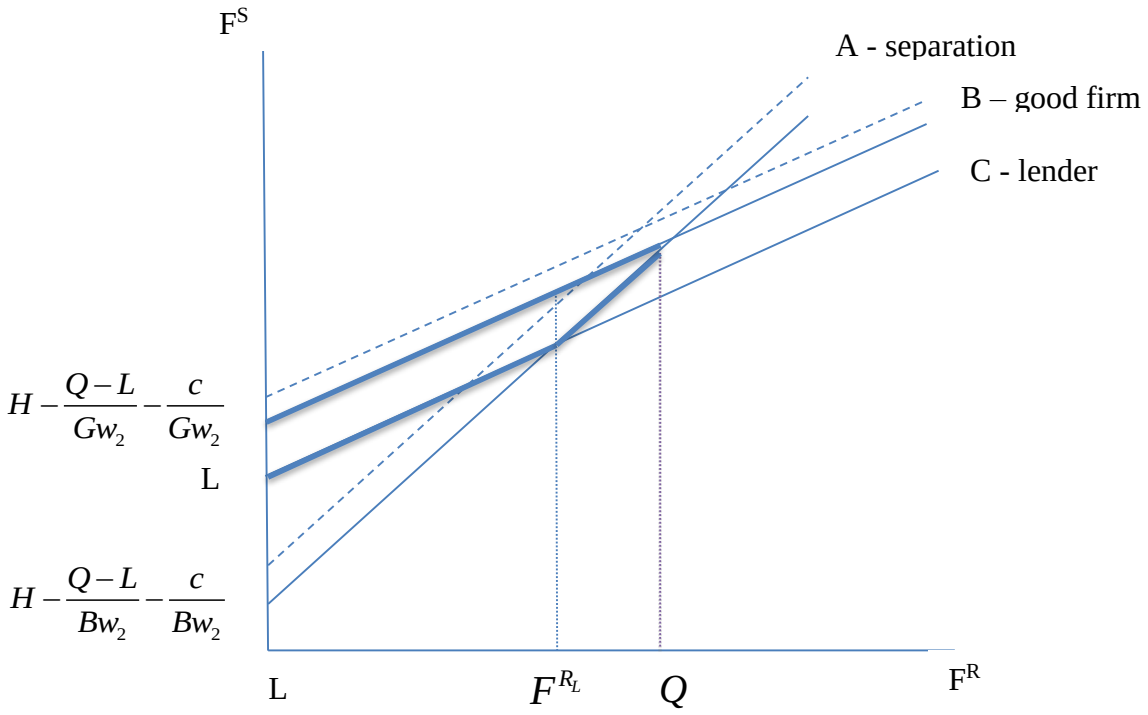
Line "A" represents the "separation" constraint, and identifies the necessary renegotiation offer of a new face value, for each initial level of face value with a restrictive covenant, F^R, for which the bad firm will be indifferent between mimicking the offer and accepting liquidation with a debt level F^R. For any renegotiation offer by a good firm in a weak market to separate good firms from bad the offer must include a face value greater than or equal to this constraint. Line B represents the maximal offer, F^S, of an increase in debt burden that the good firm is willing to make for each initial debt level, F^R. The three lines "C" represent the minimal increase in debt level the lender is willing to accept in exchange for waiving the covenant when the lender believes it is a (1) good firm making the offer, (2) an average firm making the offer and it is efficient for an average firm to continue, and (3) an average firm making the offer and it is inefficient for an average firm to continue. The shaded triangle represents the renegotiation region and its bottom edge represents the set of optimal choices

Figure 2: Costless Renegotiation of Unrestrictive Covenants.



Line "A" illustrates a "separation" constraint, and identifies the necessary renegotiation offer of a new lower face value the bad firm may make, for each initial level of face value with a restrictive covenant, F^U, for which the good firm not mimic. Line B represents the minimum offer, F^S, of a decrease in debt burden that the bad firm is willing to make in exchange for a voluntary liquidation for each initial debt level, F^U. The three lines "C" represent the minimal increase in debt level the lender is willing to accept in exchange for waiving the covenant when the lender believes it is a (1) good firm making the offer, (2) an average firm making the offer and it is efficient for an average firm to continue, and (3) an average firm making the offer and it is inefficient for an average firm to continue. The shaded triangle represents the renegotiation region and its top edge represents the set of optimal choices from the good firm's perspective.

Figure 3: Costly renegotiation of restrictive covenants.



This figure presents a comparison of the renegotiation of restrictive covenants with costly renegotiation to the case of costless renegotiation. The constraints are as defined in lemma 5. The figure shows that the separation constraint is now shifted downward, allowing the good firm to capture more of the gains from any separating renegotiation than was possible in a costless renegotiation. This increase in the good firm's benefit to renegotiation allows, for some parameter values, that it is superior *ex ante* for the good firm to select a high debt level and a restrictive covenant rather than a low debt level and an unrestrictive covenant.