

CHAPTER 8

Index Models

INVESTMENTS | BODIE, KANE, MARCUS

Advantages of the Single Index Model

- Reduces the number of inputs for diversification
- Easier for security analysts to specialize

Single Factor Model

$$r_i = E(r_i) + \beta_i m + e_i$$

β_i response of an individual security's return to the common factor m .

Beta measures systematic risk.

m a common macroeconomic factor that affects all security returns. The S&P 500 is often used as a proxy for m .

e_i firm-specific surprises

Single-Index Model

- Regression Equation:

$$R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t)$$

- The expectation of the residual term e_i is zero, so the expected return-beta relationship is:

$$E(R_i) = \alpha_i + \beta_i E(R_M)$$

Single-Index Model

Risk and covariance:

- Variance - Systematic risk and Firm-specific risk, assume noise is uncorrelated:

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma^2(e_i)$$

- Covariance - product of betas x market index risk:

$$\text{Cov}(r_i, r_j) = \beta_i \beta_j \sigma_M^2$$

Single-Index Model - Correlation

Product of correlations with the market index:

$$\text{Corr}_{i,j}(r_i, r_j) = \text{Cov}_{i,j}(r_i, r_j) / (\sigma_i \sigma_j)$$

$$\text{Corr}_{i,j}(r_i, r_j) = \frac{\beta_i \beta_j \sigma_M^2}{\sigma_i \sigma_j} \times \frac{\sigma_M^2}{\sigma_M \sigma_M} =$$

$$\frac{\beta_i \sigma_M^2}{\sigma_i \sigma_M} \frac{\beta_j \sigma_M^2}{\sigma_j \sigma_M} = \text{Corr}(r_i, r_M) \times \text{Corr}(r_j, r_M)$$

Questions to test your intuition

- What is the stock's $E(r)$ if $(r_M - r_f) = 0$?
- What is the responsiveness of the stock to market movements relative to r_f ?
- What is the stock-specific component of return (not driven by the market)?
- What is the variance attributable to uncertainty of the market?
- And that attributable to firm-specific events?

Index Model and Diversification

- Consider an Equally weighted portfolio and take the expected return R_P as the average:

$$\begin{aligned} R_P &= \frac{1}{n} \sum_{i=1}^n R_i = \frac{1}{n} \sum_{i=1}^n (\alpha_i + \beta_i R_M + e_i) \\ &= \frac{1}{n} \sum_{i=1}^n \alpha_i + \frac{1}{n} \sum_{i=1}^n \beta_i R_M + \frac{1}{n} \sum_{i=1}^n e_i \\ R_P &= \alpha_P + \beta_P R_M + e_P \end{aligned}$$

Index Model and Diversification

- The portfolio variance by definition:

$$\sigma_P^2 = \beta_P^2 \sigma_M^2 + \sigma^2(e_P)$$

where the market component comes from the portfolio's sensitivity to the market:

$$\beta_P = \frac{1}{n} \sum_{i=1}^n \beta_i$$

and the non-systemic component $\sigma^2(e_P)$ is the contribution of all the stocks in the portfolio.

Index Model and Diversification

- Variance of the non-systemic component of an equally weighted portfolio is (we assume all the stock-specific components are uncorrelated):

$$\sigma^2(e_P) = \sum_{i=1}^n \left(\frac{1}{n}\right)^2 \sigma^2(e_i) = \frac{1}{n} \bar{\sigma}^2(e)$$

- When n gets large, $\sigma^2(e_P)$ becomes negligible and firm specific risk can be diversified away.

Figure 8.1 The Variance of an Equally Weighted Portfolio with Risk Coefficient β_p

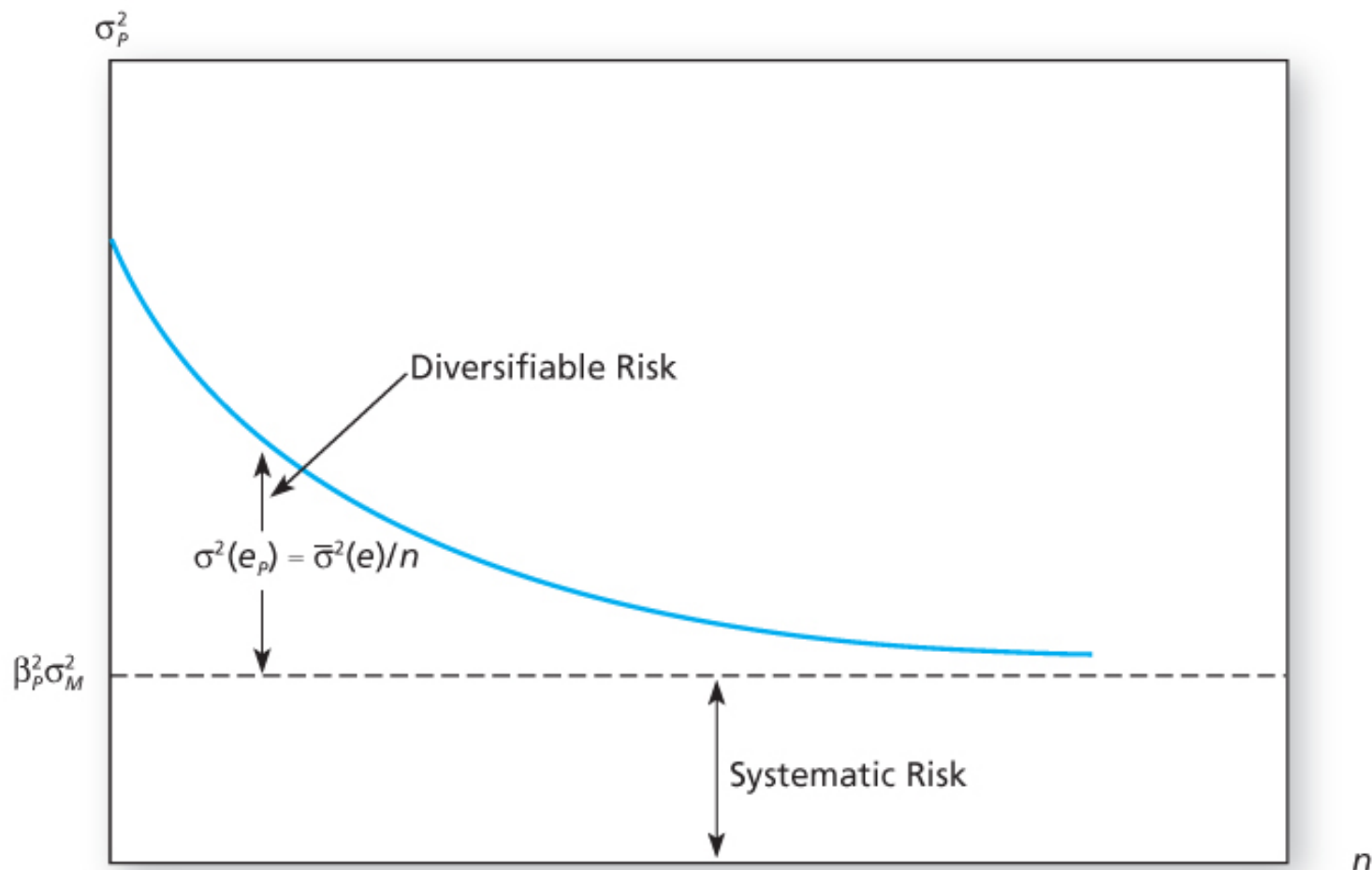


Figure 8.2 Excess Returns on HP and S&P 500

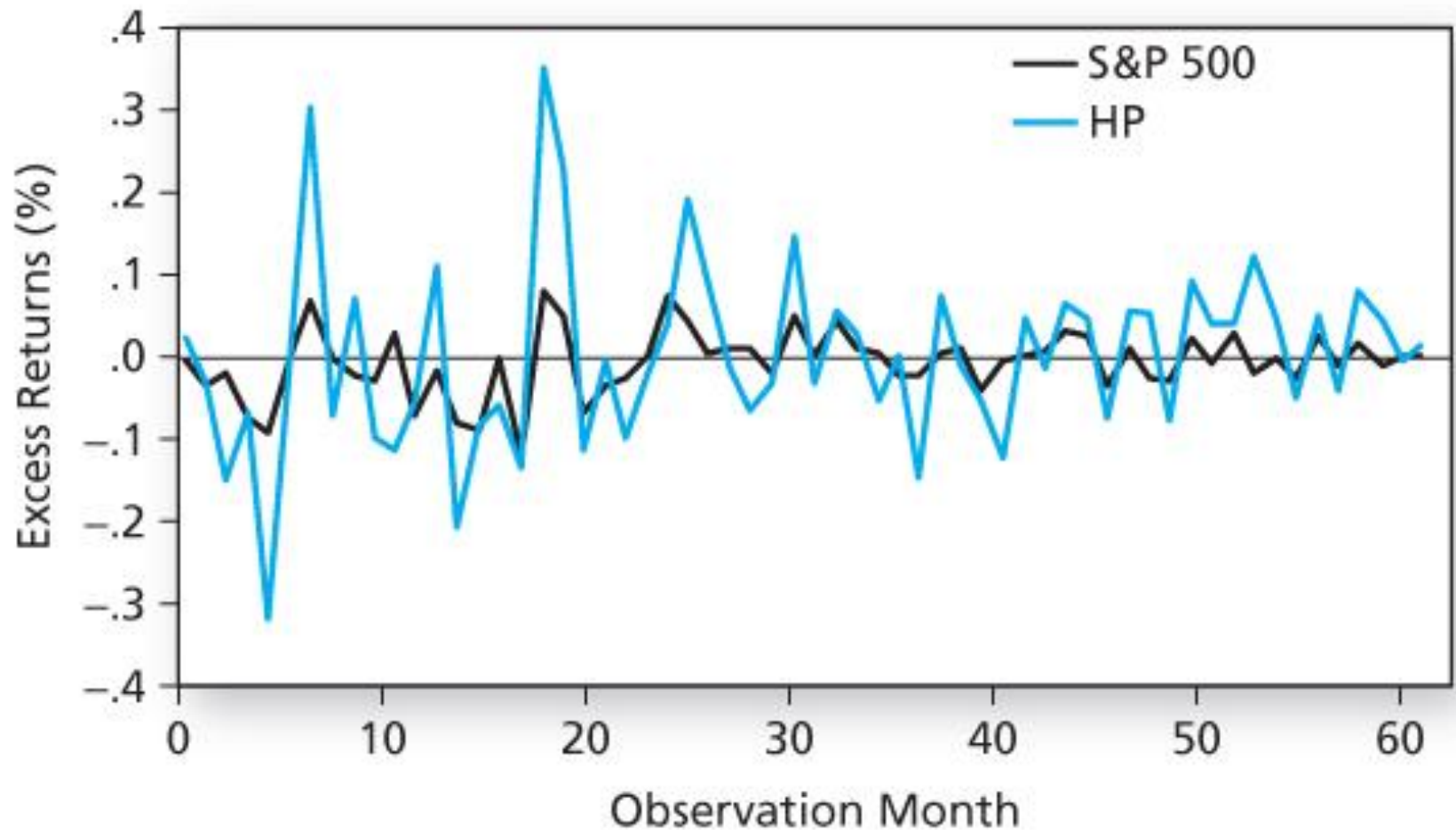


Figure 8.3 Scatter Diagram of HP, the S&P 500, and HP's Security Characteristic Line (SCL)

$$R_{HP}(t) = \alpha_{HP} + \beta_{HP} R_{SP500}(t) + e_{HP}(t)$$

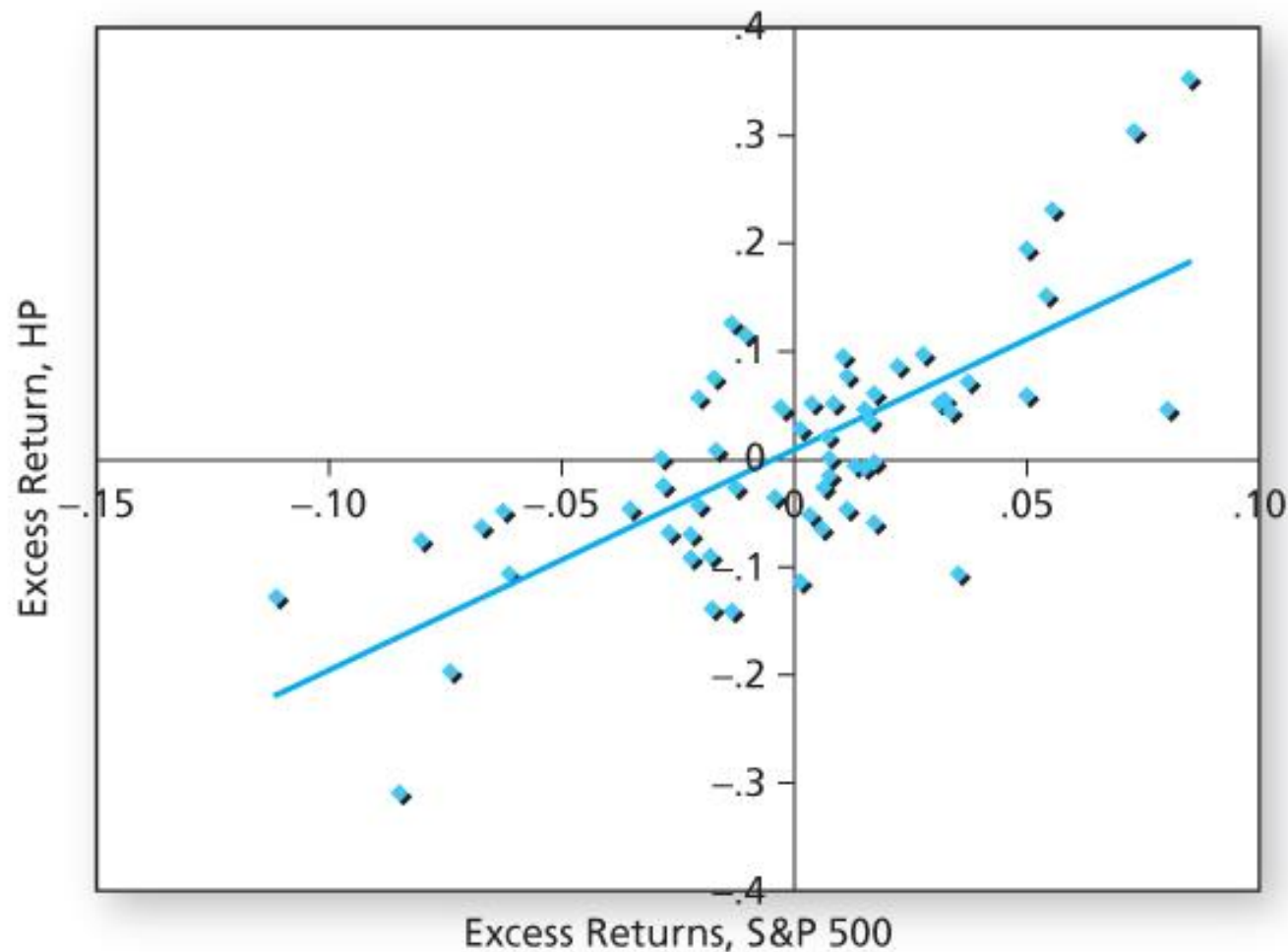


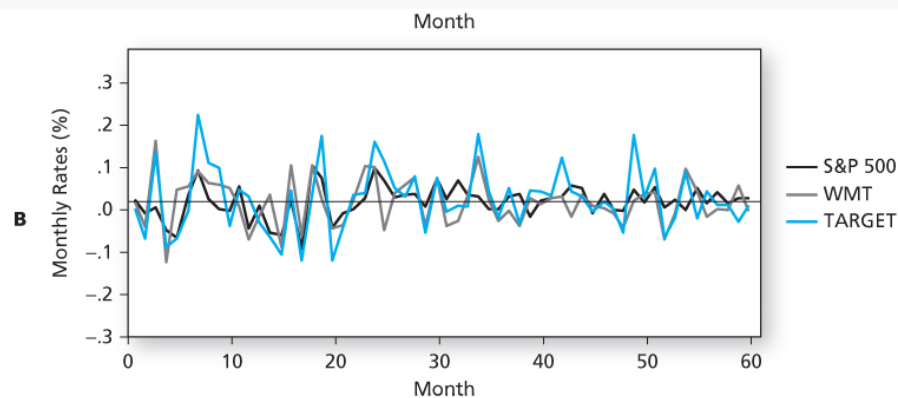
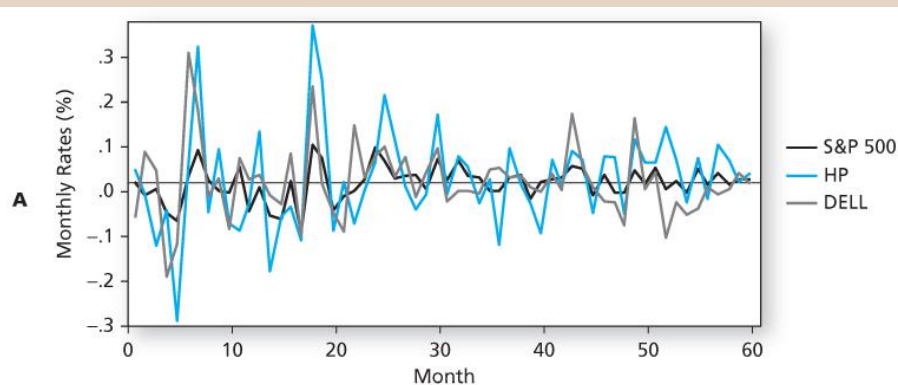
Table 8.1 Excel Output: Regression Statistics for the SCL of Hewlett-Packard

Regression Statistics				
Multiple R	.7238	← correlation		
R -square	.5239	← explanatory power		
Adjusted R -square	.5157			
Standard error	.0767			
Observations (monthly)	60			
ANOVA				
	df	SS	MS	
Regression	1	.3752	.3752	
Residual	58	.3410	.0059	
Total	59	.7162		
	Coefficients	Standard Error	t -Stat	p -Value
Intercept	$\alpha \rightarrow$ 0.0086	.0099	0.8719	.3868
S&P 500	$\beta \rightarrow$ 2.0348	.2547	7.9888	.0000

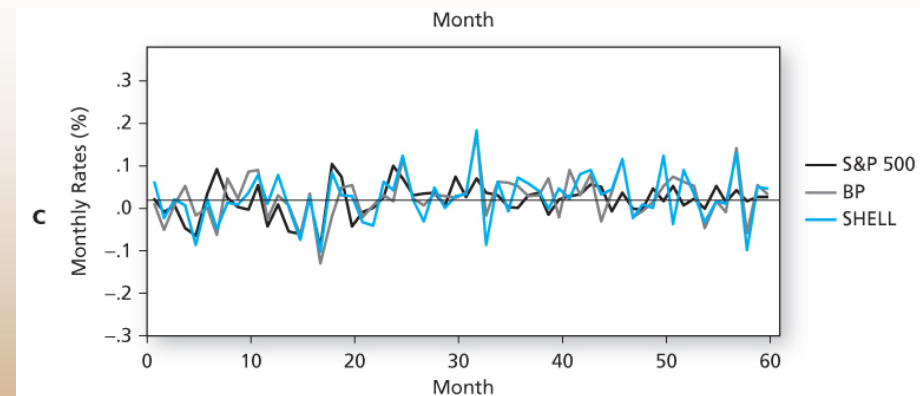
Table 8.1 Interpretation

- Correlation of HP with the S&P 500 is 0.7238
- The model explains about 52% of the variation in HP
- HP's alpha is 0.86% per month (10.32% p.a.), but it is not statistically significant
- HP's beta is 2.0348, but the 95% confidence interval is +/- ~2 standard errors, which is quite wide

Figure 8.4 Excess Returns on Portfolio Assets



- Study pairs of securities vs the market to estimate correlations
- Compute stats to measure correlations



Study portfolio stats – 1

Panel 1: Risk Parameters of the Investable Universe (annualized)					
	SD of Excess Return	Beta	SD of Systematic Component	SD of Residual	Correlation with the S&P 500
S&P 500	0.1358	1.00	0.1358	0	1
HP	0.3817	2.03	0.2762	0.2656	0.72
DELL	0.2901	1.23	0.1672	0.2392	0.58
WMT	0.1935	0.62	0.0841	0.1757	0.43
TARGET	0.2611	1.27	0.1720	0.1981	0.66
BP	0.1822	0.47	0.0634	0.1722	0.35
SHELL	0.1988	0.67	0.0914	0.1780	0.46
Panel 2: Correlation of Residuals					
	HP	DELL	WMT	TARGET	BP
HP	1				
DELL	0.08	1			
WMT	-0.34	0.17	1		
TARGET	-0.10	0.12	0.50	1	
BP	-0.20	-0.28	-0.19	-0.13	1
SHELL	-0.06	-0.19	-0.24	-0.22	0.70

Study portfolio stats – 2

Panel 3: The Index Model Covariance Matrix

		S&P 500	HP	DELL	WMT	TARGET	BP	SHELL
	Beta	1.00	2.03	1.23	0.62	1.27	0.47	0.67
S&P 500	1.00	0.0184	0.0375	0.0227	0.0114	0.0234	0.0086	0.0124
HP	2.03	0.0375	0.1457	0.0462	0.0232	0.0475	0.0175	0.0253
DELL	1.23	0.0227	0.0462	0.0842	0.0141	0.0288	0.0106	0.0153
WMT	0.62	0.0114	0.0232	0.0141	0.0374	0.0145	0.0053	0.0077
TARGET	1.27	0.0234	0.0475	0.0288	0.0145	0.0682	0.0109	0.0157
BP	0.47	0.0086	0.0175	0.0106	0.0053	0.0109	0.0332	0.0058
SHELL	0.67	0.0124	0.0253	0.0153	0.0077	0.0157	0.0058	0.0395

Cells on the diagonal (shadowed) equal to variance

formula in cell C26 = $B4^2$

Off-diagonal cells equal to covariance

formula in cell C27 = $C\$25 * \$B27 * \$B\4^2

multiplies beta from row and column by index variance

Panel 4: Macro Forecast and Forecasts of Alpha Values

	S&P 500	HP	DELL	WMT	TARGET	BP	SHELL
Alpha	0	0.0150	-0.0100	-0.0050	0.0075	0.012	0.0025
Risk premium	0.0600	0.1371	0.0639	0.0322	0.0835	0.0400	0.0429

Example: build optimal portfolio

Panel 5: Computation of the Optimal Risky Portfolio									
	S&P 500	Active Pf A	HP	DELL	WMT	TARGET	BP	SHELL	Overall Pf
$\sigma^2(e)$			0.0705	0.0572	0.0309	0.0392	0.0297	0.0317	
$\alpha/\sigma^2(e)$		0.5505	0.2126	-0.1748	-0.1619	0.1911	0.4045	0.0789	
$w^0(i)$		1.0000	0.3863	-0.3176	-0.2941	0.3472	0.7349	0.1433	
$[w^0(i)]^2$			0.1492	0.1009	0.0865	0.1205	0.5400	0.0205	
α_A		0.0222							
$\sigma^2(e_A)$		0.0404							
w_A^0		0.1691							
$w^*(\text{Risky portf})$	0.8282	0.1718							
Beta	1	1.0922	2.0348	1.2315	0.6199	1.2672	0.4670	0.6736	1.0158
Risk premium	0.06	0.0878	0.1371	0.0639	0.0322	0.0835	0.0400	0.0429	0.0648
SD	0.1358	0.2497							0.1422
Sharpe ratio	0.44	0.35							0.46

Alpha and Security Analysis

1. Use **Macroeconomic analysis** to estimate risk premium and risk of the market index.
2. Use **statistical analysis** to estimate the beta coefficients of all securities and their residual variances, $\sigma^2(e_i)$

Alpha and Security Analysis

3. Use **numerical methods** to establish the expected return of each security *independently of security analysis*.
4. Use **security analysis** to develop your own forecast of the expected returns for each security.