CHAPTER 5: INTRODUCTION TO RISK, RETURN, AND THE HISTORICAL RECORD

PROBLEM SETS

- 1. The Fisher equation predicts that the nominal rate will equal the equilibrium real rate plus the expected inflation rate. Hence, if the inflation rate increases from 3% to 5% while there is no change in the real rate, then the nominal rate will increase by 2%. On the other hand, it is possible that an increase in the expected inflation rate would be accompanied by a change in the real rate of interest. While it is conceivable that the nominal interest rate could remain constant as the inflation rate increased, implying that the real rate decreased as inflation increased, this is not a likely scenario.
- 2. If we assume that the distribution of returns remains reasonably stable over the entire history, then a longer sample period (i.e., a larger sample) increases the precision of the estimate of the expected rate of return; this is a consequence of the fact that the standard error decreases as the sample size increases. However, if we assume that the mean of the distribution of returns is changing over time but we are not in a position to determine the nature of this change, then the expected return must be estimated from a more recent part of the historical period. In this scenario, we must determine how far back, historically, to go in selecting the relevant sample. Here, it is likely to be disadvantageous to use the entire dataset back to 1880.
- 3. The true statements are (c) and (e). The explanations follow.

Statement (c): Let σ = the annual standard deviation of the risky investments and σ_1 = the standard deviation of the first investment alternative over the two-year period. Then:

$$\sigma_1 = \sqrt{2} \times \sigma_2$$

Therefore, the annualized standard deviation for the first investment alternative is equal to:

$$\frac{\sigma_1}{2} = \frac{\sigma}{\sqrt{2}} < \sigma$$

Statement (e): The first investment alternative is more attractive to investors with lower degrees of risk aversion. The first alternative (entailing a sequence of two identically distributed and uncorrelated risky investments) is riskier than the second alternative (the risky investment followed by a risk-free investment). Therefore, the first alternative is more attractive to investors with lower degrees of risk aversion. Notice, however, that if you mistakenly believed that 'time diversification' can reduce the total risk of a sequence of risky investments, you would have been tempted to conclude that the first alternative is less risky and therefore more attractive to more risk-averse investors. This is clearly not the case; the two-year standard deviation of the first alternative is greater than the two-year standard deviation of the second alternative.

4. For the money market fund, your holding period return for the next year depends on the level of 30day interest rates each month when the fund rolls over maturing securities. The one-year savings deposit offers a 7.5% holding period return for the year. If you forecast that the rate on money market instruments will increase significantly above the current 6% yield, then the money market fund might result in a higher HPR than the savings deposit. The 20-year Treasury bond offers a yield to maturity of 9% per year, which is 150 basis points higher than the rate on the one-year savings deposit; however, you could earn a one-year HPR much less than 7.5% on the bond if long-term interest rates increase during the year. If Treasury bond yields rise above 9%, then the price of the bond will fall, and the resulting capital loss will wipe out some or all of the 9% return you would have earned if bond yields had remained unchanged over the course of the year.

- 5. a. If businesses reduce their capital spending, then they are likely to decrease their demand for funds. This will shift the demand curve in Figure 5.1 to the left and reduce the equilibrium real rate of interest.
 - b. Increased household saving will shift the supply of funds curve to the right and cause real interest rates to fall.
 - c. Open market purchases of U.S. Treasury securities by the Federal Reserve Board are equivalent to an increase in the supply of funds (a shift of the supply curve to the right). The equilibrium real rate of interest will fall.
- 6. a. The "Inflation-Plus" CD is the safer investment because it guarantees the purchasing power of the investment. Using the approximation that the real rate equals the nominal rate minus the inflation rate, the CD provides a real rate of 1.5% regardless of the inflation rate.
 - b. The expected return depends on the expected rate of inflation over the next year. If the expected rate of inflation is less than 3.5% then the conventional CD offers a higher real return than the Inflation-Plus CD; if the expected rate of inflation is greater than 3.5%, then the opposite is true.
 - c. If you expect the rate of inflation to be 3% over the next year, then the conventional CD offers you an expected real rate of return of 2%, which is 0.5% higher than the real rate on the inflation-protected CD. But unless you know that inflation will be 3% with certainty, the conventional CD is also riskier. The question of which is the better investment then depends on your attitude towards risk versus return. You might choose to diversify and invest part of your funds in each.
 - d. No. We cannot assume that the entire difference between the risk-free nominal rate (on conventional CDs) of 5% and the real risk-free rate (on inflation-protected CDs) of 1.5% is the expected rate of inflation. Part of the difference is probably a risk premium associated with the uncertainty surrounding the real rate of return on the conventional CDs. This implies that the expected rate of inflation is less than 3.5% per year.
- $\begin{aligned} \text{7.} \quad & E(r) = [0.35 \times 44.5\%] + [0.30 \times 14.0\%] + [0.35 \times (-16.5\%)] = 14\% \\ & \sigma^2 = [0.35 \times (44.5 14)^2] + [0.30 \times (14 14)^2] + [0.35 \times (-16.5 14)^2] = 651.175 \\ & \sigma = 25.52\% \end{aligned}$

The mean is unchanged, but the standard deviation has increased, as the probabilities of the high and low returns have increased.

8. Probability distribution of price and one-year holding period return for a 30-year U.S. Treasury bond (which will have 29 years to maturity at year's end):

Economy	Probability	YTM	Price	Capital Gain	Coupon Interest	HPR
Boom	0.20	11.0%	\$ 74.05	-\$25.95	\$8.00	-17.95%
Normal Growth	0.50	8.0%	\$100.00	\$ 0.00	\$8.00	8.00%
Recession	0.30	7.0%	\$112.28	\$12.28	\$8.00	20.28%

9. $E(q) = (0 \times 0.25) + (1 \times 0.25) + (2 \times 0.50) = 1.25$

 $\sigma_q = [0.25 \times (0 - 1.25)^2 + 0.25 \times (1 - 1.25)^2 + 0.50 \times (2 - 1.25)^2]^{1/2} = 0.8292$

- 10. (a) With probability 0.9544, the value of a normally distributed variable will fall within two standard deviations of the mean; that is, between -40% and 80%.
- 11. From Table 5.3 and Figure 5.6, the average risk premium for the period 1926-2009 was: (11.63% 3.71%) = 7.92% per year
 Adding 7.92% to the 3% risk-free interest rate, the expected annual HPR for the S&P 500 stock portfolio is: 3.00% + 7.92% = 10.92%

CHAPTER 5: INTRODUCTION TO RISK, RETURN, AND THE HISTORICAL RECORD

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	Mean	Standard Deviation	Skewness	Kurtosis
1926 - 2005	12.15%	20.26%	-0.3605	-0.0673
1976 - 2005	13.85%	15.68%	-0.4575	-0.6489
1926 – 1941	6.39%	30.33%	-0.0022	-1.0716
	BONDS			
		BON	NDS	
	Mean	BON Standard Deviation	NDS Skewness	Kurtosis
1926 - 2005	Mean 5.68%	BON Standard Deviation 8.09%	NDS Skewness 0.9903	Kurtosis
1926 – 2005 1976 – 2005	Mean 5.68% 9.57%	BON Standard Deviation 8.09% 10.32%	NDS Skewness 0.9903 0.3772	Kurtosis 1.6314 -0.0329

12. The average rates of return and standard deviations are quite different in the sub periods:

The most relevant statistics to use for projecting into the future would seem to be the statistics estimated over the period 1976-2005, because this later period seems to have been a different economic regime. After 1955, the U.S. economy entered the Keynesian era, when the Federal government actively attempted to stabilize the economy and to prevent extremes in boom and bust cycles. Note that the standard deviation of stock returns has decreased substantially in the later period while the standard deviation of bond returns has increased.

13. a
$$r = \frac{1+R}{1+i} - 1 = \frac{R-i}{1+i} = \frac{0.80 - 0.70}{1.70} = 0.0588 = 5.88\%$$

b. $r \approx R - i = 80\% - 70\% = 10\%$

Clearly, the approximation gives a real HPR that is too high.

- 14. From Table 5.2, the average real rate on T-bills has been: 0.70%
 - a. T-bills: 0.72% real rate + 3% inflation = 3.70%
 - b. Expected return on large stocks:
 3.70% T-bill rate + 8.40% historical risk premium = 12.10%
 - c. The risk premium on stocks remains unchanged. A premium, the difference between two rates, is a real value, unaffected by inflation.
- 15. Real interest rates are expected to rise. The investment activity will shift the demand for funds curve (in Figure 5.1) to the right. Therefore the equilibrium real interest rate will increase.

CHAPTER 5: INTRODUCTION TO RISK, RETURN, AND THE HISTORICAL RECORD

		STOCK		PUT	
State of the	Drobability	Ending Price +	UDD	Ending	LIDD
Economy	FIODADIIIty	Dividend	11F K	Value	11F K
Excellent	0.25	\$ 131.00	31.00%	\$ 0.00	-100%
Good	0.45	\$ 114.00	14.00%	\$ 0.00	-100%
Poor	0.25	\$ 93.25	-6.75%	\$ 20.25	68.75%
Crash	0.05	\$ 48.00	-52.00%	\$ 64.00	433.33%

16. a. Probability Distribution of the HPR on the Stock Market and Put:

Remember that the cost of the index fund is \$100 per share, and the cost of the put option is \$12.

b. The cost of one share of the index fund plus a put option is \$112. The probability distribution of the HPR on the portfolio is:

State of the		Ending Price +		
State of the	Probability	Put +	HPR	
Economy		Dividend		
Excellent	0.25	\$ 131.00	17.0%	=(131 - 112)/112
Good	0.45	\$ 114.00	1.8%	=(114 - 112)/112
Poor	0.25	\$ 113.50	1.3%	=(113.50-112)/112
Crash	0.05	\$ 112.00	0.0%	=(112 - 112)/112

c. Buying the put option guarantees the investor a minimum HPR of 0.0% regardless of what happens to the stock's price. Thus, it offers insurance against a price decline.

17. The probability distribution of the dollar return on CD plus call option is:

State of the	Probability	Ending Value	Ending Value	Combined
Economy	, second second	of CD	of Call	Value
Excellent	0.25	\$ 114.00	\$16.50	\$130.50
Good	0.45	\$ 114.00	\$ 0.00	\$114.00
Poor	0.25	\$ 114.00	\$ 0.00	\$114.00
Crash	0.05	\$ 114.00	\$ 0.00	\$114.00

CHAPTER 5: INTRODUCTION TO RISK, RETURN, AND THE HISTORICAL RECORD

CFA PROBLEMS

- 1. The expected dollar return on the investment in equities is \$18,000 compared to the \$5,000 expected return for T-bills. Therefore, the expected risk premium is \$13,000.
- 2. $E(r) = [0.2 \times (-25\%)] + [0.3 \times 10\%] + [0.5 \times 24\%] = 10\%$
- 3. $E(r_x) = [0.2 \times (-20\%)] + [0.5 \times 18\%] + [0.3 \times 50\%] = 20\%$ $E(r_y) = [0.2 \times (-15\%)] + [0.5 \times 20\%] + [0.3 \times 10\%] = 10\%$
- 4. $\sigma_x^2 = [0.2 \times (-20 20)^2] + [0.5 \times (18 20)^2] + [0.3 \times (50 20)^2] = 592$ $\sigma_x = 24.33\%$ $\sigma_y^2 = [0.2 \times (-15 - 10)^2] + [0.5 \times (20 - 10)^2] + [0.3 \times (10 - 10)^2] = 175$ $\sigma_y = 13.23\%$
- 5. $E(r) = (0.9 \times 20\%) + (0.1 \times 10\%) = 19\% \rightarrow $1,900 \text{ in returns}$
- 6. The probability that the economy will be neutral is 0.50, or 50%. *Given* a neutral economy, the stock will experience poor performance 30% of the time. The probability of both poor stock performance and a neutral economy is therefore:

 $0.30 \times 0.50 = 0.15 = 15\%$

7. $E(r) = (0.1 \times 15\%) + (0.6 \times 13\%) + (0.3 \times 7\%) = 11.4\%$