2. Futures and Forward Markets

2.3. Hedging Strategies

Should you Hedge?

Yes:

- Concentrate on main business, hedge away the risk associated with interest rates, exchange rates, etc.

Should you Hedge?

No:

- Treasurer may have a hard time explaining a hedging loss.
- Shareholders can make their own hedging decisions.
- It may increase risk to hedge when competitors do not.

Profit Margins, Risk, and Competitors

- When all companies but me do not hedge, p varies with S.

S	competitors $(p - S)$	me $(p - F)$
$\uparrow$	constant	$\uparrow$
$\downarrow$	constant	$\downarrow$

- When all companies but me hedge, p does not vary with S.

S	competitors $(p - F)$	me $(p - S)$
$\uparrow$	constant	$\downarrow$
$\downarrow$	constant	$\uparrow$

Basis Risk

- When you need to close out your futures contract before delivery $t < T$
- When the asset to be hedged S is different from the asset underlying the futures contract S^*
- $b = (S^* - F) + (S - S^*)$
- $S^*_T - F_T = 0$

Long Hedge

Close at T:

- $\text{flow}_T = -K + F_T - S_T$

Close at $t < T$:

- $\text{flow}_T = -K + F_t$
- $\text{flow}_t = -S_t$
- $b = S - F$

Long Cross Hedge $S^* \neq S$

Close at T:

- $\text{flow}_T = -K + F_T - S_T$
- $b = S - S^*$

Close at $t < T$:

- $\text{flow}_T = -K + F_t$
- $\text{flow}_t = -S_t$
- $b = (S^* - F) + (S - S^*)$

Short Hedge

Close at T:

- $\text{flow}_T = +K - F_T + S_T$

Close at $t < T$:

- $\text{flow}_T = +K - F_t$
- $\text{flow}_t = +S_t$
- $b = S - F$

Short Cross Hedge $S^* \neq S$

Close at T:

- $\text{flow}_T = +K - F_T + S_T$
- $b = S - S^*$

Close at $t < T$:

- $\text{flow}_T = +K - F_t$
- $\text{flow}_t = +S_t$
- $b = (S^* - F) + (S - S^*)$

How to Choose a Contract?

- Choose a delivery time T as close as possible as t since $F_T = S_T$ (but choose $T \geq t$)

- Choose an underlying asset S^* such that $Corr(S^*, S)$ or $Corr(F, S)$ is high

Rolling a Hedge Forward

- If need longer time-to-delivery than available
- If longer time-to-delivery futures contracts are illiquid
- Incur risk when rolling the hedge forward

For example, you want to hedge your sale of oil in two years.

Delivery	Short	Close
Aug this year	today $30	Jul this year $33
Feb next year	Jul this year $34	Jan next year $27
Aug next year	Jan next year $25	Jul next year $29
Feb in two years	Jul next year $32	Jan in two years $34

Optimal Hedge Ratio h

- $h = \frac{N_F}{N_S}$,
  where N_F is the number of units hedged and N_S is the number of units expected to sell on the spot market at T.
- Short Hedge Profit:

$$\begin{aligned}
\Pi_T &= (-S_T + K)N_F + S_T N_S \\
&= (-F_T + F_0)N_F + S_T N_S \\
&= S_0 N_S + (S_T - S_0)N_S - (F_T - F_0)N_F \\
&= S_0 N_S + \Delta S N_S - \Delta F N_F \\
&= S_0 N_S + N_S(\Delta S - h\Delta F)
\end{aligned}$$

- $\min_h Var(\Pi_T) \leftrightarrow \min_h Var(\Delta S - h\Delta F)$

$$\begin{aligned}
Var(\Delta S - h\Delta F) &= \sigma_S^2 + h^2\sigma_F^2 - 2hCov(\Delta_S, \Delta_F) \\
&= \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F \\
&= \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F + \rho^2\sigma_S^2 - \rho^2\sigma_S^2 \\
&= \sigma_S^2 - \rho^2\sigma_S^2 + (h\sigma_F - \rho\sigma_S)^2
\end{aligned}$$

- Optimal hedge ratio: $h = \rho\frac{\sigma_S}{\sigma_F} = \frac{Cov(\Delta_S, \Delta_F)}{\sigma_F^2}$

Optimal Number of Contracts

- $N_F = hN_S$ is the number of units hedged
- Q_F is the contract size

$$N = \frac{N_F}{Q_F} = \frac{hN_S}{Q_F}$$

Hedging a Stock Portfolio

CAPM: $E[r_P] - r = \beta(E[r_A] - r)$

$$N = \beta \frac{P}{A}$$

P is the current value of the portfolio

A is the current value of the stocks underlying one futures contract

Example of a Stock Portfolio Hedge

You want to hedge the exposure of your stock portfolio to market risk (where the market is proxied by the S&P 500 index) over the next four months. The value of your portfolio is \$50,000,000 and its β is 1.2. You use five-month S&P 500 futures contract to hedge your portfolio. One futures contract delivers \$250 times the index value. The value of the S&P 500 index is 1,000 and its dividend yield is three percent per annum. The risk-free interest rate is six percent per annum.

(a) How many contracts do you short?

After four months, the price of the S&P 500 index drops to 900.

(b) What is the value of your futures position?

(c) Using the CAPM equation $E[r_p] - r = \beta(E[r_m] - r)$ where r_p is the portfolio rate of return, r is the riskfree rate, and r_m is the market rate of return, what is the expected value of your portfolio? (Simplify your calculations by continuing to use continously compounded rates in the CAPM equation.)

(d) What is the expected value of your total position?

Changing β to β_F

$$N = (\beta_F - \beta)\frac{P}{A}$$

Homework

1. (Hull 3.16) The standard deviation of monthly changes in the spot price of live cattle is 1.2 (in cents per pound). The standard deviation of monthly changes in the futures price of live cattle for the closest contract is 1.4. The correlation between the futures price changes and the spot price changes is 0.7. It is now October 15. A beef producer is committed to purchasing 200,000 pounds of live cattle on November 15. The producer wants to use the December live-cattle futures contracts to hedge its risk. Each contract is for the delivery of 40,000 pounds of cattle. What strategy should the beef producer follow?

2. (Baby Hull 3.21, Papa Hull 3.24) It is July 16. A company has a portfolio of stocks worth $100 million. The beta of the portfolio is 1.2. The company would like to use the CME December futures contract on the S&P 500 to change the beta of the portfolio to 0.5 during the period July 16 to November 16. The index is currently 1000, and each contract is on $250 times the index.
(a) What position should the company take?
(b) Suppose that the company changes its mind and decides to increase the beta of the portfolio from 1.2 to 1.5. What position in futures contracts should it take?

3. (Baby Hull 3.22, Papa Hull 3.23) The following table gives data on monthly changes in the spot price and the futures price for a certain commodity. Use the data to calculate the minimum variance hedge ratio.

Spot Price Change	Futures Price Change
+0.50	+0.56
+0.61	+0.63
−0.22	−0.12
−0.35	−0.44
+0.79	+0.60
+0.04	−0.06
+0.15	+0.01
+0.70	+0.80
−0.51	−0.56
−0.41	−0.46