Debt and Capacity Commitments

J. Chris Leach, Nathalie Moyen, and Jing Yang*

ABSTRACT

In capital-intensive industries, firms face complicated multi-staged financing, investment, and production decisions under the watchful eye of existing and potential industry rivals. In various representations of this environment, we show that a first-mover advantage in debt weakly dominates a first-mover advantage in capacity. Without a first-mover advantage in debt, the incumbent may suffer a dead-weight loss. When both the entrant and incumbent deploy debt prior to capacity, a first-mover in capacity benefits from softer competition. With a long-purse debt cost, leading in debt still remains advantageous.

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1. Introduction

It is a well-known theoretical notion that a firm's capacity investment can affect its own, and a rival's, production choices. Since the influential work of Spence (1977), capacity and other forms of investment are recognized as deterrents to potential entrants. Dixit (1980) analyzes how capacity can commit an incumbent to a subsequent aggressive production strategy. As an incumbent’s capacity costs become “sunk” by production time, they effectively lower the relevant marginal cost of production, thereby giving the incumbent a cost advantage. It then follows that the incumbent produces at a level higher than the entrant. Other papers discussing the effect of a firm’s capacity on its product market include Haruna (1996), Kirman and Masson (1986), Kulatilaka and Perotti (1998), Reynolds (1991), Rosenbaum (1989), and Zhang (1993).

As capacity investments can be financed by debt, it is natural to consider how the all-equity framework of Dixit (1980) extends to the situation where both the incumbent and the entrant choose between debt and equity financing when demand is uncertain. We examine the case where both financial (debt) and real (capacity) strategies are chosen with the foresight that they may effectively commit an incumbent to produce aggressively and thereby influence a rival’s production. In doing so, we focus on the Brander and Lewis (1986) and Maksimovic (1988) debt aggression channel, but we introduce a channel related to a long purse later in Section 4.

In the industrial organization literature, the usual notion of deterrence is keeping a potential entrant from entering.\(^1\) Of more general interest than this binary decision, however, are producer interactions where equilibrium production outcomes reflect a broader range of production adjustments than just dominance and capitulation. Bulow, Geanakoplos and Klemperer’s (1985) analysis of strategic substitutes and strategic complements recognizes such production adjustments. In dealing with two factors influencing the intensity of strategic substitutability, we consider interactions and tradeoffs in strategic positioning with debt and capacity.

We show that debt and capacity can be, but will not always be, factor substitutes in influencing a rival’s output. We demonstrate that, generally, the two factors interact to intensify strategic substitution through their joint production of convexity in the equity payoff. Importantly, however, there are limitations. When production is constrained by the capacity in place, simultaneously adding debt above a specific threshold level cannot increase production. We construct a measure of a rival’s quantity displaced through debt and capacity positioning and demonstrate that it exhibits Leontief regions where the debt and capacity are not factor substitutes.

In equilibrium, the incumbent produces at capacity and uses risky debt to further discourage a

rival’s production. In the presence of risky debt, the optimal strategy in this simple setup results in Stackelberg leadership production quantities. Stackelberg leadership can also be achieved when the firm commits only with debt and buys capacity at the time of production. In contrast, Stackelberg leadership may not be attainable when the firm commits only with capacity as in Dixit (1980).

We enrich the basic two-factor model of incumbent positioning by allowing the entrant to also commit with debt and capacity. In this multi-stage environment, we use numerical methods to quantify the effects of different sequences of firms’ debt and capacity commitments on output and profitability. When both the incumbent and entrant can use debt prior to any capacity choices, the leverage-induced increase in total production depresses the price in the marketplace to such an extent that both firms are worse off than under the Cournot-Nash outcome without commitment. This result extends the Brander and Lewis (1986) prisoner’s dilemma characterization of debt commitments to an environment with sequential decisions.

If, subsequent to debt choices, the incumbent also leads in capacity, then there is some improvement as the incumbent’s profit reaches the Nash level. In fact, firms weakly benefit when the incumbent deploys capacity first. The incumbent foresees the entrant decision policy and can control the price erosion in the marketplace in a way that the entrant cannot, because the entrant by definition remains under the influence of the incumbent’s first-mover advantage in debt. As a result of this control on the market price, the incumbent earns higher profits than under any other capacity deployment sequence, while the entrant’s profits remain unchanged.

When considering a long-purse cost of debt alongside the deterrence benefit, we see that the incumbent de-leverages and loses some market share. With a higher cost, the market price increases to such an extent that the softer competition improves the profitability of both the incumbent and the entrant.

This paper proceeds as follows: Section 2 introduces the basic incumbent two-factor positioning
model, analyzes the production subgame, and characterizes the incumbent’s first-stage optimal choices of debt and capacity. In Section 3, we examine how the solution to the basic model changes when the entrant can also pre-commit with debt and capacity. Section 4 introduces a cost of debt and provides some comparative static results. Section 5 concludes.

2. Model of Incumbent Two-Factor Commitments

In this Section, we present the basic model where only the incumbent is able to strategically influence the product market outcomes. We consider the decisions of the incumbent and the entrant over two stages. In the first stage, the incumbent maximizes firm value by choosing debt and capacity with the foresight that they influence the firms’ subsequent production decisions. In particular, an incumbent’s ability to commit to aggressive production preempts at least a portion of a new rival’s production. In the second stage, the incumbent and the entrant take the incumbent’s debt and capacity choices as given and maximize their equity values by choosing how much to produce. The following timeline illustrates these choices:

<table>
<thead>
<tr>
<th>First Stage</th>
<th>Second Stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incumbent chooses:</td>
<td></td>
</tr>
<tr>
<td>debt (F^I)</td>
<td>production (q^I)</td>
</tr>
<tr>
<td>capacity (k^I)</td>
<td></td>
</tr>
<tr>
<td>Entrant chooses:</td>
<td></td>
</tr>
<tr>
<td>debt (F^E)</td>
<td></td>
</tr>
<tr>
<td>capacity (k^E)</td>
<td></td>
</tr>
<tr>
<td>production (q^E)</td>
<td></td>
</tr>
<tr>
<td>Nature chooses:</td>
<td></td>
</tr>
<tr>
<td>shock (\tilde{z} \in [\underline{Z}, \bar{Z}])</td>
<td></td>
</tr>
</tbody>
</table>

Our approach emphasizes the strategic addition of capacity and subsequent inherited production constraints. The sequencing enables the incumbent to use debt and capacity to influence the production subgame equilibrium. The critical feature here, and in Dixit (1980), is that the incumbent’s debt and capacity choices are observable prior to production, whereas the entrant’s are
not. Consequently, the entrant’s production choice can be adapted to the incumbent’s debt and capacity, explicitly introducing the possibility of complementarity and substitutability in achieving strategic positioning. In contrast, the incumbent does not observe the entrant’s debt and capacity choices prior to production and therefore cannot adapt its production.

For simplicity, we assume that both the incumbent and the entrant are risk neutral, the riskless rate is zero, and the inverse demand function is linear: \( p(q^I, q^E) = a + \tilde{z} - b(q^I + q^E) \), where \( q^I \) and \( q^E \) are the incumbent and entrant second-stage production levels, and the demand shock \( \tilde{z} \in [\tilde{Z}, \tilde{Z}] \) is drawn after the production decisions.\(^2\)

### 2.1. SECOND STAGE: THE ENTRANT’S BEST REPLY

We first characterize the entrant’s best reply. The entrant maximizes its equity value subject to fairly pricing its debt issue. As is well known, the maximand reduces to the total firm value:

\[
\Pi^E = (a + E[\tilde{z}] - b(q^I + q^E)) q^E - r k^E,
\]

where \( r \), the one-period unit cost of capacity \( k^E \), satisfies \( a > r > 0 \). This equation is just the usual linear-demand duopolist’s profit.

The entrant maximizes its total value by choosing debt, capacity, and production simultaneously subject to the capacity constraint \( q^E \leq k^E \).\(^3\) The entrant’s best reply has the usual functional form:

\[
q^E(q^I) = \frac{a + E[\tilde{z}] - b q^I - r}{2b}.
\]

The entrant’s capacity is equal to its production, and its capital structure is irrelevant as there

\(^2\)Our basic insights are not confined to the case of linear demand although it provides a useful tractability.

\(^3\)We have simplified the entrant’s problem to making concurrent production and financing decisions. In an expanded setting, with incumbent commitments followed by entrant commitments then production, the incumbent still retains the advantage over the entrant. These results are presented in Section 3.
is no interaction with risk or production, no tax benefit of debt, no default cost, no information asymmetry, etc.

2.2. SECOND STAGE: THE INCUMBENT’S BEST REPLY

There are two inherited characteristics from the first stage that must be incorporated into the incumbent’s second-stage equity flow. First, the incumbent’s existing debt financing is assumed irreversible. The debt proceeds $B^I$ are distributed in the first stage and the face value $F^I$ must be paid in the second stage. Second, the incumbent’s one-period maximum capacity commitment $k^I$ is irreversible prior to production and leads to a zero marginal cost for all feasible production levels. Due to the markets for buying, selling, renting, and swapping capacity, we consider capacity commitments involving only one period ahead. The incumbent pays $rk^I$ to enable any capacity $k^I$ the incumbent would want to have the option of using, whether ex post the capacity is utilized or not. The incumbent must constrain production to capacity.

For the inherited debt and capacity, we can write the incumbent’s second-stage expected equity flow as:

$$
\int_{\hat{Z}^I}^{\overline{Z}^I} \{(a + \hat{z} - b(q^I + q^E)) q^I - rk^I - F^I\} d\Phi(\hat{z}),
$$

(3)

where $r$ is the incumbent’s marginal capacity cost, and $\hat{Z}^I$ is the incumbent’s default point, assumed to satisfy $\hat{Z}^I \in [\underline{Z}, \overline{Z}]$ and defined by:

$$
\left(a + \hat{Z}^I - b(q^I + q^E)\right) q^I - rk^I - F^I = 0.
$$

(4)

In contrast to the entrant’s second-stage problem, note that the incumbent’s debt proceeds have already been paid out at this stage and those proceeds do not affect the equityholders’ production decision. The incumbent is also constrained in its production choice. The incumbent maximizes its equity value by choosing production subject to a maximum possible production (at capacity) inherited from the first stage.
To solve for the incumbent’s best reply, we assume for the remainder of our analysis that $\hat{Z} \sim U[Z, \bar{Z}]$. It is convenient to denote the incumbent’s unconstrained best reply by $q^{I,u}(q^{E})$. Then, the best reply is

$$q^{I}(q^{E}) = \begin{cases} q^{I,u}(q^{E}) = \frac{2a - 2bq^{E} + \hat{Z} + \hat{k}^{I}}{2b}, & \text{when } q^{I,u}(q^{E}) \leq k^{I} \\ k^{I}, & \text{when } q^{I,u}(q^{E}) \geq k^{I}. \end{cases}$$

(5)

2.3. SECOND STAGE: THE PRODUCTION SUBGAME EQUILIBRIA

While the best replies specified in Equations (2) and (5) are functions of the incumbent’s default point $\hat{Z}^{I}$ and capacity $k^{I}$ – suggesting debt and capacity positioning – the incumbent’s default point remains itself a function of productions $q^{I}$ and $q^{E}$. Substituting in the definition for the incumbent’s default point, we solve for the best replies. We obtain a quadratic in $q^{I,u}$, leading to the optimal production policies:

$$q^{I,*} = \begin{cases} q^{I,u} = \frac{a - E[\hat{Z}] + r + 2\bar{Z} + R(k^{I}, F^{I})}{10b}, & \text{when } q^{I,u} \leq k^{I} \\ k^{I}, & \text{when } q^{I,u} \geq k^{I} \end{cases}$$

(6)

$$q^{E,*} = \begin{cases} q^{E,u} = \frac{9a + 11E[\hat{Z}] - 11r - 2\bar{Z} - R(k^{I}, F^{I})}{20b}, & \text{when } q^{I,u} \leq k^{I} \\ q^{E,c} = \frac{a + E[\hat{Z}] - bk^{I} - \bar{Z}}{2b}, & \text{when } q^{I,u} \geq k^{I}, \end{cases}$$

(7)

where $R(k^{I}, F^{I}) = \sqrt{(a - E[\hat{Z}] + r + 2\bar{Z})^2 + 40b(F^{I} + rk^{I})} > 0$.

From these second-period production policies, we can directly see that debt commitments ($F^{I}$) and capacity commitments ($rk^{I}$) can be perfect substitutes in increasing the convexity of the equity payoff. As Brander and Lewis (1986) point out, equityholders facing larger liabilities (from debt or capacity commitments) produce more aggressively. However, because production is constrained by the capacity in place, adding debt commitments above a specific threshold level cannot increase production. At that point, only adding capacity commitments can increase production.

The best that the incumbent can achieve with debt and capacity positioning when facing an entrant would be Stackelberg production:
LEMMA 1. The Stackelberg production levels are:

\[ q^{I,S} = \frac{a + E[\tilde{z}] - r}{2b} \]  

(8)

\[ q^{E,S} = \frac{a + E[\tilde{z}] - r}{4b} \]  

(9)

Another reference point is simultaneous Cournot-Nash production which confers no advantage to the incumbent. When we collapse our two-stage game into one simultaneous move for both firms, both firms exhibit capital structure irrelevance and produce at capacity.

LEMMA 2. The simultaneous Cournot-Nash production levels are:

\[ q^{I,N} = q^{E,N} = \frac{a + E[\tilde{z}] - r}{3b} \]  

(10)

We adopt simultaneous Cournot-Nash as the positioning-free benchmark, and define the entrant’s displaced production, \( \Delta = -(q^E - q^{E,N}) \), the incumbent’s expanded production, \( X = q^I - q^{I,N} \), and the overall market expansion, \( \Sigma = X - \Delta = (q^I + q^E) - (q^{I,N} + q^{E,N}) \) for given production levels \( q^I \) and \( q^E \).

The entrant’s best reply (2) implies that the incumbent reduces the entrant’s production by half as much as it expands its own production, \( \Delta = X/2 \), and the market expands by an amount equal to the entrant’s displaced production, \( \Sigma = \Delta \). Consequently, the entrant’s displaced production, \( \Delta \), is a sufficient descriptor. We are interested in the displacement production function in terms of inherited first-stage choices of debt \( (F^I) \) and capacity \( (k^I) \):

\[ \Delta(k^I, F^I) = \begin{cases} 
-\frac{7a - 13E[\tilde{z}] + 13r + 6Z + 3R(k^I, F^I)}{6b}, & \text{when } q^{I,u} \leq k^I \\
-\frac{a - E[\tilde{z}] + r + 3bk^I}{6b}, & \text{when } q^{I,u} \geq k^I.
\end{cases} \]  

(11)

Figure 1 displays two iso-displacement curves \( \Delta(k^I, F^I) \), where the leftmost curve graphs a lower level of displacement. There are two regions of interest. In the diagonal region of the iso-displacement curve labeled “Perfect Substitutes Segment,” the entrant perceives a dollar-for-dollar tradeoff between incumbent commitments to debt \( (F^I) \) and capacity costs \( (rk^I) \). Both
commitments are perfect substitutes in shifting the default point \( \hat{Z}_I \), i.e., the kink in the convex equity payoff for the firm with risky debt. In the vertical region labeled “Leontief Segment,” the marginal contribution of additional debt is zero. Additional debt increases the convexity-related incentive to produce, but the incumbent’s production is constrained at capacity. There is no substitutability between debt and capacity commitments in this region. Figure 2 maps the incumbent production policy of Equation (6) corresponding to these two segments.

Figure 1 also illustrates that, in contrast to Dixit (1980), the presence of risky debt insures that capacity increases are always a credible threat to increase production. In our model, as in Dixit, capacity is required to produce. In Dixit’s equity-only case, if the incumbent considers an increase from equilibrium capacity, the extra capacity has no mechanism to become a credible threat to increase production. In other words, capacity does not always have a positive marginal displacement product in Dixit’s context. Credibility arises from the additional role the capacity costs play in determining the incumbent’s default point. Both promised future debt payments and committed future capacity payments increase equityholders’ default point \( \hat{Z}_I \). In the presence of risky debt, debt and capacity payments are perfect substitutes in discouraging a rival’s production through their influence on convexity in the equity payoff function. Committed capacity costs have the additional benefit that they expand available capacity. Figure 1 indicates that, holding the level of risky debt constant, an increase in capacity always leads to higher incumbent production and lower entrant production.

In the first stage, debt is fairly priced without deadweight costs while excess capacity is costly. Excess capacity is equivalent to money-burning. Consequently, debt has a cost advantage as a factor.\(^4\) This suggests that the incumbent in equilibrium will choose zero excess capacity and

\(^4\)When we include a cost to debt financing, the entrant chooses zero debt and the incumbent chooses less debt than characterized in the baseline two-factor model. Along these lines, Section 4 presents results derived with a long-purse cost of debt.
increase gearing to produce any remaining desirable positioning. An equilibrium with zero excess capacity would be situated at the kink of an iso-displacement curve in Figure 1. Importantly, rather than establishing zero excess capacity in equilibrium because additional capacity is not a credible threat to increase production as in Dixit (1980), we find zero excess capacity because debt is a cheaper factor in producing convexity-related positioning.

2.4. FIRST STAGE: THE INCUMBENT’S DEBT AND CAPACITY COMMITMENTS

To confirm the intuition gained from examining the iso-displacement curves, we roll the production subgame equilibrium policies back into the first-stage optimization problem where the incumbent chooses debt and capacity. This is the mechanism through which the incumbent is granted first-mover advantages: the incumbent sees how the production equilibrium is affected by its choices of debt $F^I$ and capacity $k^I$.

In order to discuss debt financing, we adopt a three-tiered capital structure. In order to preserve the Brander and Lewis (1986) limited liability equity, we must have an unlimited liability security in the capital structure. Therefore, we introduce risky unlimited liability debt $B^I$ (and its associated face value $F^I$) for strategic debt positioning. We simplify the analysis by treating capacity costs $rk^I$ as financed with senior risk-free debt $D^I$ (and its associated face value $G^I$).\(^5\)

The incumbent’s first stage problem is to maximize the equity flow over the two stages

$$B^I + D^I - rk^I + \int_{Z^I} \{p(q^I, q^E, \tilde{z})q^I - F^I - G^I\} \partial \Phi(\tilde{z})$$

subject to the fair bond-pricing equations

$$B^I = \int_{Z^I} F^I \partial \Phi(\tilde{z}) + \int_{Z^I} \{p(q^I, q^E, \tilde{z})q^I - G^I\} \partial \Phi(\tilde{z})$$

\(^5\)With riskless capacity financing, we sidestep the complication of debt priority in default among the two types of debt and, importantly, the need to keep track of two non-linear default points per firm.
\[ D^I = \int_Z^Z G^I \partial \Phi(\hat{z}) \] (14)

and the financing requirement of capacity costs \( D^I = r k^I \). Substituting constraints (13) and (14) in the maximand (12), the incumbent’s equity flow over the two stages coincides with the total firm value:

\[ \Pi^I = (a + E[\hat{z}] - b(q^I + q^E)) q^I - r k^I. \] (15)

**PROPOSITION 1.** In the presence of risky debt where \( \hat{Z}^I \in (Z, Z) \), equilibrium in the two-stage game is characterized by:

\[ q^{I,*} = k^{I,*} = \frac{a + E[\hat{z}] - r}{2b} \] (16)

\[ q^{E,*} = \frac{a + E[\hat{z}] - r}{4b} \] (17)

\[ \hat{F}^I > F^{I,*} \geq \frac{(3a + 7E[\hat{z}] - 11r - 4Z)(a + E[\hat{z}] - r)}{8b}, \] (18)

where \( \hat{F}^I \) is the debt level corresponding to \( \hat{Z}^I = Z \).

**Proof.** See the Appendix 6.1.

When the incumbent wants to shift the default point, issuing additional debt is the least expensive approach, as long as the capacity constraint is not binding. Capacity, however, is expensive. The optimizing incumbent builds capacity only to make its production feasible. Debt is then expanded to achieve the desired level of convexity. Additional debt above this level achieves the same capacity-constrained output and provides no additional benefit or cost.\(^6\) Equilibrium production coincides with the Stackelberg level.

### 2.5. INCUMBENT CAPACITY-ONLY POSITIONING

\(^6\)By assuming zero interest rates, we focus on debt financing of production activities. Even in the presence of positive interest rates, debt would still be mostly neutral to value other than through production if we incorporated the revenues from non-production debt in our firm’s cash flow function.
Similar to Dixit (1980), this restriction focuses on the incumbent’s first-mover advantage in capacity. We recognize that, in Dixit (1980), the incumbent can deploy capacity in two stages and the entrant deploys capacity only in the second stage when both duopolists produce. In equilibrium, Dixit’s incumbent deploys all capacity in the first stage. “We have seen above that at all points that are ever going to be observed without or with entry, the established firm will be producing an output equal to its chosen pre-entry capacity.” (p. 100). Consequently, Dixit’s second-stage capacity investment option is “out-of-the-money” and corresponds to our single-stage capacity investment structure considered here: \( k^I, \{q^I, q^E\} \), where braced variables are chosen simultaneously.

A similar analysis to what we used for our two-factor model indicates that the second-stage production policies are:

\[
q^{I,k} = \begin{cases} 
q^{I,k,u} = \frac{a + E[\bar{z}] + r}{3b}, & \text{when } q^{I,k,u} \leq k^{I,k} \\
k^{I,k}, & \text{when } q^{I,k,u} \geq k^{I,k}
\end{cases}
\]  

\[\]  

\[
q^{E,k} = \begin{cases} 
q^{E,k,u} = \frac{a + E[\bar{z}] - 2r}{3b}, & \text{when } q^{I,k,u} \leq k^{I,k} \\
q^{E,k,c} = \frac{a + E[\bar{z}] - bk^{I,k} - r}{2b}, & \text{when } q^{I,k,u} \geq k^{I,k}
\end{cases}
\]  

where \( q^{I,k,u} \) is the unconstrained incumbent’s production; \( q^{E,k,u} \) is the associated entrant’s production; and \( q^{E,k,c} \) is the entrant’s production when the incumbent is constrained. Solving for the first-stage equilibrium indicates that:

**Proposition 2.** Capacity-only positioning achieves Stackelberg leadership if and only if capacity costs are expensive enough, \( 5r \geq a + E[\bar{z}] \).

*Proof.* See Appendix 6.2. \( \Box \)

As detailed in the appendix, all equilibria involve production at capacity (zero excess capacity) \( q^{I,k} = k^{I,k} \) because capacity is costly. In the case when the first-stage capacity constraint is not binding, the unconstrained production is \( q^{I,k,u} = \frac{a + E[\bar{z}] + r}{3b} \). This production level is the same
as Stackelberg \((\frac{a+E[\tilde{z}]+r}{3b} = \frac{a+E[\tilde{z}]-r}{2b})\) if and only if the following parameter condition applies:

\[ a + E[\tilde{z}] = 5r. \]

In the case when capacity is insufficient to produce at the unconstrained level, the constraint \(k^{I,k} < q^{I,k,u} = \frac{a+E[\tilde{z}]+r}{3b}\) is binding. The constrained production level is Stackelberg \(q^{I,k} = k^{I,k} = \frac{a+E[\tilde{z}]-r}{2b}\). Putting this constrained production level into the constraint, we have \(\frac{a+E[\tilde{z}]-r}{2b} < \frac{a+E[\tilde{z}]+r}{3b}\) if and only if \(5r > a + E[\tilde{z}]\).

Taking the two cases together, the incumbent achieves Stackelberg leadership if and only if capacity costs are expensive enough, \(5r \geq a + E[\tilde{z}]\). The parameter condition has an intuitive interpretation. When capacity costs are expensive enough, the capacity constraint is binding and therefore it can be used as a pre-commitment mechanism. As the parameter condition need not be satisfied, Stackelberg leadership may not always be achieved when the incumbent leads in capacity but not in debt.

**COROLLARY 1.** A first-mover in capacity need not achieve Stackelberg leadership.

**COROLLARY 2.** Even though an incumbent is a first-mover in capacity, the option to be also a first-mover in debt can be valuable.

Without debt, a first-mover option in capacity remains valuable. Displacement is strictly positive \((\Delta^k > 0)\) and the incumbent is better off than under a simultaneous Cournot-Nash production game. While Dixit’s (1980) capacity first-mover option is valuable for an all-equity-financed firm, it may not achieve Stackelberg leadership. Adding debt leadership strictly increases the incumbent’s firm value when capacity costs are not expensive enough, \(5r < a + E[\tilde{z}]\).

We contrast the incumbent capacity-only positioning with incumbent debt-only positioning. In this case, the model effectively reduces to a Brander and Lewis (1988) setting without entrant debt:

\[ F^I, \{q^I, q^E\}. \] Even though the incumbent has no first-mover advantage in capacity, it still achieves
Stackelberg leadership by increasing its gearing to the unique optimal level

\[ F^{I,F} = \frac{(3a + 7E[\tilde{z}] - 3r - 4\tilde{Z}) (a + E[\tilde{z}] - r)}{8b}. \]  

(21)

Hence when the incumbent is a first-mover in debt, the option also to be a first-mover in capacity is worthless.

The debt level \( F^{I,F} \) is higher than the two-factor debt threshold, \( \min\{F^{I,*}\} \), by \( 2rk^{I,S} \). When positioning only with debt, the incumbent chooses capacity at production time, so that capacity costs can no longer affect convexity. A higher debt level is therefore needed to maintain the same convexity. In fact, the debt-only unique optimal level is exactly the amount of debt needed to replicate the best reply with first-stage capacity costs.

3. Introducing Entrant Commitments and Sequencing Debt-then-Capacity

In this Section, we examine more realistic models where debt is chosen before capacity is put in place. We also introduce the entrant’s ability to pre-commit with debt and capacity.\(^7\) The models may include up to five decision stages, including debt commitments \( F^I, F^E \), capacity commitments \( k^I, k^E \), and productions \( \{q^I, q^E\} \). Importantly, when both the entrant and incumbent have optimal debt structures, the non-linearity caused by the entrant’s default point is intertwined with the non-linearity of the incumbent’s default point in a way that prohibits closed-form solutions. We solve the models numerically using a four-dimensional grid space for debt and capacity of the incumbent and entrant.

The numerical method requires a choice of model parameters. We set the demand intercept to \( a = 4 \) and the slope to \( b = 1 \), and the per unit capacity cost to \( r = 0.05 \). We set the demand shock support to \( Z = -0.9 \) and \( \tilde{Z} = 1.1 \) so that the incumbent and entrant equilibrium cash flows \((a + \tilde{z} - b(q^I + q^E))q^i - rk^i\) are always positive at the worst shock \( Z \). This insures that the firm

\(^7\) We are indebted to the referee for suggesting this line of inquiry as well as the one in the next Section.
never defaults on its debt-financed capacity costs $G^I$, consistent with the simplifying assumption in Equation (14). The firm defaults only for large enough unlimited liability debt face values $F^I$, as considered in Equation (13).

We discretize the capacity grid by increments of 0.01 over a range of $[0, 2.2]$, where the upper limit is well above Stackelberg. The debt level capacity grid is set to $[0, 4.2]$ and includes every increment of 0.05. The capacity/production grid is finer than the debt grid because any approximation error would compound when solving the model backwards from the production game. We optimize over 0.35 billion different combinations of debt and capacity commitments of the entrant and the incumbent $(F^I, F^E, k^I, k^E)$.

We present eight different models of debt and capacity commitments. The models differ by their sequencing of incumbent and entrant debt and capacity choices. We build up the intuition in steps to reach the full model, where the incumbent first chooses its debt $F^I$, then the entrant chooses its debt $F^E$, followed by the capacity choices of the incumbent $k^I$ and entrant $k^E$, and finally the quantity choices: $\{q^I, q^E\}$.

3.1. NUMERICAL APPROXIMATION OF THE ANALYTICAL RESULTS: $\{F^I, k^I\}, \{q^I, q^E\}$

In the basic model of the previous Section, the incumbent simultaneously pre-commits with debt and capacity. Here we compare the analytical solution of Equations (16) to (18) with the given parameter values to its numerical grid point approximation to assess the magnitude of the error of the grid search method. Given the computational limitation placed by the curse of dimensionality, Table I shows that the approximation performs relatively well.

We also note that the lower and upper bounds for the optimal debt level $F^I,*$ in Equation (18) are close to each other. This arises because the limited liability effect induces the incumbent to increase its debt commitment substantially. As a result, the incumbent finds itself not so far
from the default point. In the analytical two-factor deterrence model, debt deterrence is cheaper (capacity is costly), which explains why incumbent capacity is set equal to the optimal Stackelberg production while debt is set to meet a minimum threshold that achieves the desired convexity. Additional debt above the threshold achieves the same capacity-constrained output and provides no additional benefit or cost. Additional debt beyond the point where equityholders default still does not trigger any cost in our model as we have no (deadweight) bankruptcy cost. Proposition 1, however, restricts the solution to well-defined cases where the upper bound does not trigger default with probability one. Solving for the upper bound using Equations (4), (16), (17) and the parameter values, the upper bound is exactly $\hat{F}^I = 4.08$.

3.2. SEQUENTIAL COMMITMENTS: $F^I, k^I, \{q^I, q^E\}$

With sequential commitments, the incumbent maximizes the total firm value (15) by first choosing how much debt to issue $F^I$. Once the debt is raised, the incumbent chooses capacity $k^I$ with the interests of equityholders in mind as in Equation (3). In contrast to the basic model above, the capacity decision is made to benefit equity value, not total firm value. A sequential maximization of debt-then-capacity would lead to results identical to the simultaneous maximization of debt-and-capacity when there is no uncertainty realization between two choices and when the objective function remains the same. Here, however, the objective function changes: capacity is chosen to maximize total firm value in the simultaneous case but chosen to maximize the equity value in the sequential case. The convexity provided by the equity payoff changes only the equilibrium range of debt levels, as seen in Table II.

Not surprisingly, the incumbent still achieves Stackelberg production outcomes with sequential commitments. However, its debt level is contained in a tighter range. The aggressiveness provided by the debt commitment when equityholders choose production compounds when equityholders
also choose capacity. The additional convexity can make the incumbent overproduce from the perspective of the total firm value. At the very high debt levels, the incumbent would (subgame perfectly) have the incentive to commit to a capacity beyond the Stackelberg level, which would at that point be optimal for equityholders but not for the firm as a whole. Foreseeing this, the incumbent while issuing fairly-priced debt (and therefore maximizing total firm value) backs away from the higher leverage.

3.3. ENTRANT CAPACITY COMMITMENT: $F^I, k^E, \{q^I, q^E\}$

We introduce the entrant’s ability to influence the production outcome by considering a capacity commitment. Here, and in all numerical models, we preserve the incumbent’s lead in debt and therefore do not change its first-stage decision. In the second stage, it is now the entrant who chooses capacity $k^E$ to maximize its total value (1). In the third stage, the incumbent chooses production to maximize its equity flow (3) and the entrant chooses production to maximize its total value.

The entrant capacity commitment provides some strategic positioning benefit. Compared to the sequential incumbent-only commitments above, the incumbent’s production in equilibrium is slightly below the Stackelberg outcome. Table III highlights the comparison.

Through its capacity commitment, the entrant commits to a higher production at any given level of incumbent debt compared to the previous models of incumbent-only commitments. Solving backwards to the first-stage, the incumbent could choose a higher debt level to keep its production level the same as in the case with no entrant commitment. The same production of the incumbent combined with a higher entrant production, however, would depress the price $p = a + \tilde{z} - b(q^I + q^E)$ too much. The incumbent optimally backs away from using too much debt and achieves the highest profit by decreasing production and therefore increasing the price per unit produced. With the
entrant capacity fixed, the incumbent chooses the price and quantity combination that maximizes its expected profit. Not surprisingly, when moving away from models of incumbent-only commitments to a model with an entrant (capacity) commitment, the entrant’s profit improves.

It is also worthwhile to note that, when the incumbent can pre-commit only with debt, the optimal debt level collapses from a range to a unique point. In contrast to the two-factor model where the capacity constraint limits the incumbent’s ability to produce more through debt positioning, the debt-only positioning involves no such capacity constraint. When debt rises, there is always a convexity effect providing the incumbent the incentive to increase production. Debt above the threshold causes the incumbent to add too much capacity at production time and thereby overproduce.

3.4. ENTRANT DEBT COMMITMENT: $F^I, F^E, \{q^I, q^E\}$

The entrant can alternatively influence the product market game through a debt commitment. The second stage therefore describes the entrant’s choice of debt $F^E$ to maximize total value. In the third stage, both the incumbent and the entrant choose productions to maximize their respective equity flows (3) and (1).

Following an incumbent’s capacity commitment, an entrant’s debt commitment still achieves a significant amount of strategic positioning. Compared to the entrant’s capacity commitment, the entrant’s debt commitment is more effective. As shown in Table IV, the entrant earns higher profits (1.21 > 1.09).

The entrant debt commitment model can be viewed as a sequential Brander and Lewis (1986) formulation. Brander and Lewis (1986) examine a staged debt-then-production game where both firms choose debt levels in the first stage and then condition on each other’s debt levels when choosing production levels in the second stage. Instead of having two firms choosing their debt
levels simultaneously, we designate the incumbent firm to choose first and the entrant to adapt to it.

Our sequential results are put in perspective by comparing them to the Brander and Lewis (1988) simultaneous debt commitment model and the Nash quantity game without any commitment. As is well known, the Brander and Lewis equilibrium falls in the trap of the prisoners’ dilemma, where both firms collect far lower profits than in the Nash equilibrium without commitment ($\Pi_{BL} = 1.67 < \Pi^N = 1.82$).

More to our point, the results highlight the benefits of being a leader in debt. In contrast to the simultaneous debt choices in Brander and Lewis, the incumbent earns higher profits while the entrant earns significantly lower profits ($\Pi^I = 1.79 > \Pi_{BL} = 1.67 > \Pi^E = 1.21$).

When comparing the sequential debt equilibrium with the Nash outcome, we note that both the incumbent and the entrant are worse off. This result extends the Brander and Lewis notion that debt pre-commits firms to be too aggressive. When the entrant is able to lever up following the incumbent’s debt choice, it depresses the incumbent’s profits to below the Nash level.

Our results thus far underscore the aggressiveness of debt. When only the incumbent has access to debt, the incumbent benefits tremendously from the aggression in the product market. However, when the entrant also uses debt then both firms lose compared to a framework without commitments.

3.5. SIMULTANEOUS CAPACITY CHOICES: $F^I$, $\{k^I, k^E\}$, $\{q^I, q^E\}$

In this sequence, we examine how the simultaneity of the capacity decisions affects the results. After the incumbent chooses its total-value-maximizing debt level, a simultaneous capacity game ensues. It is worth noting that, in this second stage, the capacity decisions of the incumbent and the entrant are driven by different motives: the incumbent maximizes equity value (3) while the
entrant maximizes total value (1). This makes the levered-incumbent more aggressive.

In Table V, we expect the entrant to influence productions through its capacity commitment, although to a lesser extent than when only the entrant chooses capacity in the second stage. The incumbent increases its debt aggressiveness and gains market power, although less than Stackelberg. The entrant’s profit is smaller when the incumbent simultaneously chooses capacity, but greater than the case without any entrant commitment.

3.6. THE FULL MODEL: $F^I, F^E, \{k^I, k^E\}, \{q^I, q^E\}$

All necessary intuition should now be in place to understand the full model with debt and capacity commitments from the two firms. Here, the incumbent commits to its total-value-maximizing debt level, followed by the entrant’s total-value-maximizing debt commitment. Because the capacity commitments are simultaneous and chosen after the incumbent and entrant debt commitments, they do not influence the product market positioning beyond the debt commitments. The full model produces solutions in Table VI that are identical to the entrant’s debt commitment model discussed previously.

3.7. INCUMBENT-THEN-ENTRANT CAPACITY CHOICES: $F^I, F^E, k^I, k^E, \{q^I, q^E\}$

We examine the intermediate capacity decisions and allow the incumbent’s equityholders to choose capacity before the entrants’ equityholders. Compared to the model with simultaneous capacity choices, Table VII shows that the incumbent earns higher profits by choosing capacity first. The incumbent foresees the entrant’s capacity commitment. As a result, the incumbent has greater control over the price at which the total production will be sold through its first-mover advantages in both debt and capacity. Because the combined production of the entrant and incumbent decreases, the market price increases $p = a + \tilde{z} - b(q^I + q^E)$. The incumbent earns higher profits because of both its higher production and the higher price. The entrant is able to
maintain its same level of profit as in the model with simultaneous capacity choices, because the higher market price fortuitously compensates for its lower production.

3.8. ENTRANT-THEN-INCUMBENT CAPACITY CHOICES: $F^I, F^E, k^E, k^I, \{q^I, q^E\}$

Conversely, when the entrant’s equityholders choose capacity before the incumbents’ equityholders, the entrant’s positioning could be strengthened if it were not for the incumbent’s aggressive first-mover advantage in debt. The incumbent, foreseeing the entrant’s first-mover advantage in capacity responds aggressively with its debt.

Unlike above for the incumbent in Section 3.7, the entrant cannot completely control the price at which the total production will be sold. This is because the entrant is a second-mover in debt, and therefore always remains under the influence of the incumbent. Compared to the simultaneous capacity model, Table VIII shows that the entrant produces the same quantity but the incumbent produces 0.02 additional units due to its increased aggression in debt $F^I$ from 3.10 to 3.25. As a result, the total quantity brought to market increases, which depresses the price. The aggression produces a prisoner’s dilemma: because of the lower market price, both the incumbent’s and the entrant’s profits are lower than in the simultaneous capacity model. This result echoes the Brander and Lewis (1986) intuition whereby both firms earn lower profits with aggressive debt positioning.

3.9. SUMMARY OF THE NUMERICAL RESULTS

In all eight numerical sets of results, we have defined the incumbent as the firm that chooses its leverage first. The incumbent is able to achieve higher than Nash profits only when the entrant does not have access to debt commitments. When firms strategically position themselves (simultaneously or sequentially) with debt, both firms collect much lower profits than under the Nash outcome without any commitment. The prisoner’s dilemma characterization of the Brander and Lewis
result pervades all sequential formulations with incumbent and entrant debt irrespective of capacity commitments.

Of the eight numerical models, the last three are more likely to represent the data, where all possible debt and capacity commitments are considered. When debt and capacity commitments are used to stake out market power, a dominating outcome can be achieved by letting the incumbent also have the first-mover advantage in capacity. The incumbent and entrant naturally benefit by letting the incumbent control the price erosion caused by aggressive positioning, and a softer competition ensues. The natural coordination is beneficial for firms, but not for consumers who are charged a higher price.

When the entrant has the first-mover advantage in capacity, subsequent to an incumbent first-mover advantage in debt, a prisoner’s dilemma arises. Both the incumbent and entrant are hurt by the incumbent’s aggressive debt anticipating the entrant’s first-mover advantage in capacity. The aggressive production in the marketplace erodes the price. From a public policy perspective, customers would favor such a mandate forcing the entrant to deploy capacity early.

4. Adding a Long-Purse Cost of Debt

We enrich the above eight models of debt and capacity commitments to include the long-purse argument according to which debt is perceived as a sign of weakness. As before, the incumbent and entrant choose their commitments, then they simultaneously compete in the product market. Going forward, the incumbent and entrant find themselves in an industry with continuation profits of $X$, net of any proportional recapitalization cost $x$ when the high debt causes default. The recapitalization cost captures the long-purse-style argument where debt may weaken one’s position. For simplicity, we assume that recapitalization reduces debt to zero. The incumbent and entrant each decide to discontinue their operations upon default when $X - xF^i < 0$, for $i = I, E$. 

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This reduced-form representation of a long-purse argument is simplified. The continuation value is assumed to be fixed rather than specified as a proportion of the prior choices of production. We assume that the continuation value in an established, post-deterrence industry converges to a more competitive setting where strategic interactions no longer play a prominent role. The continuation value can be viewed in terms of rents which, in equilibrium, cover the entry cost of newcomers into the industry. When aggressive deterrence is allowed to persist into the continuation value, the benefit of deterrence would be magnified relative to the long-purse argument. In future research, it would be interesting to examine the dynamic evolution among industry rivals.

The continuation value $X$ provides financial slack that is capitalized by stakeholders. The equity value of firm $i = I, E$ is therefore redefined as:

\[
\int_{\tilde{Z}^i} \left\{ (a + \tilde{z} - b(q^i + q^j)) q^i - rk^i - F^i + X \right\} d\Phi(\tilde{z}), \tag{22}
\]

and the risky unlimited liability debt $B^i$ in Equation (13) as:

\[
B^i = \int_{\tilde{Z}^i} F^i \partial \Phi(\tilde{z}) + \int_{Z} \tilde{Z}^i \left\{ p(q^i, q^j, \tilde{z}) q^i - G^i + \max\{0, X - xF^i\} \right\} \partial \Phi(\tilde{z}). \tag{23}
\]

The long-purse cost through $x$ plays a role only when default occurs. When a firm’s leverage is not high enough to cause default, the firm can count on continuing operations. However, when positioning requires a large debt, the firm also faces the probability of foregoing operations when $X - xF^i < 0$.

The two features of the long-purse argument, the continuation value $X$ and the recapitalization cost $x$, produce different numerical results although the basic trade-offs defining the equilibrium remain the same. Table IX presents the numerical results with parameter values of $X = 0.25$ and $x = 0.1$ for the case of the full sequential model with simultaneous capacity choices $F^I, \{k^I, k^E\}, \{q^I, q^E\}$.

As expected, a cost of debt generates a softer competition. The incumbent debt level decreases
substantially from 3.10 to 1.35, with leverage $B^I/\Pi^I$ also reducing from 0.98 to 0.60.\(^8\) The entrant adapts to the decreased aggression and increases its debt level slightly from 0.60 to 0.85, which corresponds to a slight increase in market leverage $B^E/\Pi^E$ from 0.43 to 0.44. As a result, the entrant increases its production while the incumbent reduces it. The softer competition increases the market price from 0.93 to 1.18. The higher price combined with benefit of the continuation value generates larger firm values, especially for the entrant.

4.1. COMPARATIVE STATICS

In this Section, we examine how the results change with different values for the model parameters of interest: the long-purse cost of debt $x$, the continuation value $X$, the demand level $a$, the demand elasticity to output $b$, and the capacity cost $r$. We discuss the results within the context of the full sequential model with simultaneous capacity choices $F^I$, $\{k^I, k^E\}$, $\{q^I, q^E\}$ considering both debt deterrence and the long-purse argument.\(^9\) We maintain the benchmark set of parameters ($a = 4$, $b = 1$, $r = 0.05$, $x = 0.1$, $X = 0.25$, $Z = -0.9$, $Z = 1.1$) and change only one value at a time.

The long-purse effect is represented by the proportional recapitalization cost $x$. When the incumbent aggressively positions with a high debt level $F^I$, it may subsequently suffer from discontinuing operations when the continuation value $X$ is lower than the recapitalization costs $xF^I$. In Table X, the incumbent debt level $F^I$ decreases from 2.90 to 1.35 (and the market debt decreases from 0.94 to 0.60) when the recapitalization cost increases from $x = 0.01$ to the benchmark value of $x = 0.10$. With significantly less incumbent debt positioning, the market share of the incumbent decreases while that of the entrant increases. The higher recapitalization cost increases the market price from 0.98 to 1.18, consistent with a softer competition.

\(^8\)Not surprisingly, leverage in models without any cost to debt (e.g., in Tables I to VIII) is very high ($\geq 0.98$).

\(^9\)The comparative static results are qualitatively unchanged when considering other sequences of incumbent and entrant commitments rather than the full model with simultaneous capacity choices.
Under the long-purse argument, equityholders can count on receiving a continuation value $X$ if they do not default on their liabilities. When in default, debtholders can count on receiving less: the greater of nothing or the continuation value net of recapitalization costs, i.e., $\max\{0, X - xF\}$. Because both the incumbent and the entrant receive a continuation value, it is the entrant that effectively has the ability to readjust its debt positioning in the face of the greater financial slack. The incumbent is relatively disadvantaged by the already large debt recapitalization costs. Large recapitalization costs reduce the net continuation value of debtholders in default and therefore the total firm value. The entrant, not saddled with a large debt, leverages up to take advantage of the financial slack provided by the larger continuation value without much of an impact on its recapitalization costs. When the continuation value increases from $X = 0.10$ to the benchmark value of $X = 0.25$, Table XI shows that the entrant debt increases from 0.45 to 0.85 (and its leverage from 0.32 to 0.44). Facing a more aggressive entrant, the incumbent decreases its debt $F^I$ from 2.95 to 1.35 (and its leverage from 0.94 to 0.60). As a result, the entrant increases its production and the incumbent decreases it. Firm values from both the entrant and the incumbent increase, reflecting the higher continuation value.

Firms operating in industries with a higher demand $a$ use more debt. In Table XII, the incumbent’s debt $F^I$ increases from 1.25 to 1.35 when the demand level increases from $a = 3.6$ to the benchmark value of $a = 4$. Similarly the entrant’s debt $F^E$ increases from 0.60 to 0.85. The incumbent leverage, however, is lower. This is because firms respond to the higher demand by producing more, generating greater profits. The increase in incumbent profits overwhelms the debt level increase such that the incumbent leverage ratio decreases slightly from 0.63 to 0.60. In contrast, the entrant leverage increases from 0.38 to 0.44.

We expect to observe roughly opposite results in Table XIII when considering firms in industries with a more inelastic demand, i.e., a higher demand slope $b$. An increase in the demand
slope from $b = 0.9$ to the benchmark value of $b = 1$ does decrease the production quantities and profits. In contrast to the demand level $a$, however, the demand slope $b$ is multiplied by the total quantity produced in forming the market price $p = a + \tilde{z} - b(q^I + q^E)$, which adds another layer of strategic interactions between the incumbent and the entrant. Nonetheless, the incumbent’s debt $F^I$ decreases from 1.55 to 1.35 (and leverage from 0.62 to 0.60). Similarly, the entrant reduces its aggressiveness from a debt of 0.95 to 0.85 (and leverage from 0.45 to 0.44).

As a last parameter of interest, we examine the effects of the proportional capacity cost $r$ in Table XIV. Firms in industries with higher capacity costs $r$ use less debt, because higher costs allow the firm to decrease the amount of debt used for positioning. An increase in the capacity cost from $r = 0.01$ to the benchmark value of $r = 0.05$ decreases the incumbent’s debt from 1.50 to 1.35 (and the incumbent leverage from 0.64 to 0.60). Higher costs naturally reduce profits, except for the entrant with its steady production benefiting from the higher price.

5. Conclusion

Our model considers debt and capacity as factors of strategic positioning toward rivals. In the basic two-factor model of incumbent positioning, we show that the debt face value and capacity costs can be perfect substitutes in increasing the convexity of the equity payoff. Our equilibrium is characterized by production at capacity and debt set to achieve Stackelberg leadership. The incumbent is also able to achieve full leadership with debt commitments, but not always with capacity commitments alone. With capacity-only, the incumbent may suffer a deadweight loss. In the two-factor model of incumbent positioning, the first-mover option in debt is valuable while the first-mover option in capacity is worthless.

When considering entrant commitments, we show that the entrant gains more market share by positioning with debt than with capacity. However, both levered firms are worse off compared to
the Nash outcome without any commitment. This result extends to an environment with sequential commitments the Brander and Lewis notion that debt makes firms too aggressive. In practice, the value destruction in both firms may be contained as entrants typically face much greater external financing constraints than incumbents, except perhaps in time periods with easy access to credit or in industries with high collateral values.

In considering capacity commitments, we show that the leveraged-incumbent and leveraged-entrant naturally coordinate to have the incumbent build capacity first. This capacity sequence allows the incumbent to control the price at which the production will be sold in the product market such that profits are weakly dominating compared to alternative deployment sequences. Without a doubt, consumers are hurt by paying the higher price. Further analysis could examine whether or not it is socially optimal to force the entrant to deploy capacity early. Evaluating such policy concerns may represent an interesting avenue for further research.

In another line of pursuit, one may examine mechanisms allowing the incumbent and entrant to collude tacitly through avoiding aggressive debt commitments. Such mechanisms would help avoid the prisoner's dilemma brought on by debt commitments. Further analysis could explore whether such legal mechanisms exist and are in use.

One force limiting the aggressive use of debt is the long-purse cost. When debt weakens firms' positions, competition softens and profits increase. Our cross-sectional analysis suggests that strategic debt levels, balancing the cost with the deterrence benefit, vary across industries. For example, firms should be more levered in industries with more elastic demand.

6. Appendix

6.1. PROOF OF PROPOSITION 1

In solving for the first-stage debt and capacity commitments, we recognize that the incum-
bent’s choices are common knowledge prior to production by specifying the functional dependencies $q^I(k^I, F^I)$ and $q^E(k^I, F^I)$ under the assumption that debt is risky, $\hat{Z} \in (\mathbb{Z}, \overline{\mathbb{Z}})$. The incumbent’s optimal production policy (6) is not differentiable in $k^I$ at a critical point. Figure 2 presents a graph of the unconstrained policy, the constraint, and the combination yielding the non-differentiable optimal policy specified in (6), for a given debt level $F^I$. We proceed by considering first-stage debt and capacity choices separately for: the excess capacity region, $k^I \geq q^{I,u}(k^I, F^I)$, which contains the rightmost flat segment; and the exhausted capacity region, $k^I \leq q^{I,u}(k^I, F^I)$, which contains the leftmost $45^o$ segment.\(^{10}\)

Case 1: Excess Capacity Region ($k^I \geq q^{I,u}(k^I, F^I)$)

For the excess capacity region, we want to assure that the optimal unconstrained production policy $q^{I,*} = q^{I,u}$ remains active. Accordingly, we must constrain the optimization to choices that generate excess capacity, $k^I \geq q^{I,u}(k^I, F^I)$. From Equation (15), we can write the incumbent’s Lagrangian for this region as:

$$\begin{align*}
(a - b(q^{I,u}(k^I, F^I) + q^{E,u}(k^I, F^I))) q^{I,u}(k^I, F^I) - rk^I + \lambda_1 (k^I - q^{I,u}(k^I, F^I)).
\end{align*}$$

The Karush-Kuhn-Tucker first order conditions for debt $F^I$ and capacity $k^I$ are:

$$\begin{align*}
\frac{2(2a + 3E[\hat{z}] - 5\lambda_1 + 2r - \overline{Z}) - R(k^I, F^I)}{5R(k^I, F^I)} &= 0 \quad \text{(FOC for } F^I) \quad (25) \\
\frac{2r(2a + 3E[\hat{z}] - 5\lambda_1 + 2r - \overline{Z}) + (5\lambda_1 - 6r)R(k^I, F^I)}{5R(k^I, F^I)} &= 0 \quad \text{(FOC for } k^I) \quad (26) \\
\lambda_1 &\geq 0 \quad (27) \\
q^{I,u}(k^I, F^I) &\geq k^I \quad (28) \\
\lambda_1 (k^I - q^{I,u}(k^I, F^I)) &\geq 0. \quad (29)
\end{align*}$$

Subcase 1a: Excess Capacity Region with $k^I > q^{I,u}(k^I, F^I)$ (interior)\(^{10}\) Even though non-differentiability makes a unified analysis infeasible, continuity of the optimal policy allows us to treat the point of non-differentiability with its adjacent regions.
For this region, it must be that $\lambda_1=0$. The first order conditions for $F^I$ and $k^I$ imply the following:

$$R(k^I, F^I) = 2 \left( 2a + 3E[\tilde{z}] + 2r - Z \right)$$

(30)

$$R(k^I, F^I) = \frac{1}{3} \left( 2a + 3E[\tilde{z}] + 2r - Z \right).$$

(31)

For general cases of the parameters $a$, $r$, and $Z$, there is no $R(k^I, F^I)$ that solves both equations as they are parallel lines. This contradiction indicates that there is no solution with excess capacity.

This is our analogue to Dixit’s (1980) no excess capacity result.

Subcase 1b: Excess Capacity Region with $k^I = q_{I,u}^I(k^I, F^I)$ (boundary)

For this region, there is no a priori restriction on the multiplier $\lambda_1$ other than non-negativity. However, we do have $k^I = q_{I,u}^I(k^I, F^I)$. Using the incumbent’s unconstrained production policy in (6) and the first order conditions, we can solve for the candidate equilibrium of debt, capacity, and productions:

$$F^I = \frac{(3a + 7E[\tilde{z}] - 11r - 4Z)(a + E[\tilde{z}] - r)}{8b}$$

(32)

$$k^I = \frac{a + E[\tilde{z}] - r}{2b}$$

(33)

$$q^I = \frac{a + E[\tilde{z}] - r}{2b}$$

(34)

$$q^E = \frac{a + E[\tilde{z}] - r}{4b},$$

(35)

where $\lambda_1 = r$.

Case 2: Exhausted Capacity Region ($k^I \leq q_{I,u}^I(k^I, F^I)$)

For the exhausted capacity region, we want to assure that the optimal capacity production policy $q^{I,*} = k^I$ remains active. Accordingly, we must constrain the optimization to choices that exhaust capacity, $k^I \leq q_{I,u}^I(k^I, F^I)$. From Equation (15), we can write the incumbent’s Lagrangian for this region as:

$$\left(a + E[\tilde{z}] - b \left( k^I + \frac{a - bk^I - r}{2b} \right) \right) k^I - rk^I + \lambda_2(q_{I,u}^I(k^I, F^I) - k^I).$$

(36)
It is important to note, particularly if contrasting with (24), that we have substituted in \( k^I \) for \( q^I \) but put \( q^{I,u}(k^I, F^I) \) in the constraint. The unconstrained production quantity \( q^{I,u}(k^I, F^I) \) is what must remain infeasible if we are analyzing equilibria where the production is constrained.

The multiplier reflects that an unconstrained incumbent would wish to produce at least as much as the capacity \( (k^I \leq q^{I,u}(k^I, F^I)) \). The Karush-Kuhn-Tucker first order conditions for debt \( F^I \) and capacity \( k^I \) are:

\[
\frac{2\lambda_2}{R(k^I, F^I)} = 0 \quad \text{(FOC for } F^I) \tag{37}
\]

\[
a + E[\tilde{z}] - r - 2bk^I + \lambda_2 \left(-2 + \frac{4r}{R(k^I, F^I)}\right) = 0 \quad \text{(FOC for } k^I) \tag{38}
\]

\[
\lambda_2 \geq 0 \tag{39}
\]

\[
q^{I,u}(k^I, F^I) \geq k^I \tag{40}
\]

\[
\lambda_2(q^{I,u}(k^I, F^I) - k^I) = 0. \tag{41}
\]

Subcase 2a: Exhausted Capacity Region with \( k^I < q^{I,u}(k^I, F^I) \) (interior)

For this region, it must be that \( \lambda_2 = 0 \). The first order condition for \( k^I \) immediately implies:

\[
k^I = \frac{a + E[\tilde{z}] - r}{2b} \tag{42}
\]

\[
q^I = \frac{a + E[\tilde{z}] - r}{2b} \tag{43}
\]

\[
q^E = \frac{a + E[\tilde{z}] - r}{4b}. \tag{44}
\]

The constraint \( k^I < q^{I,u}(k^I, F^I) \) implies a lower bound which debt must strictly exceed:

\[
F^I > \frac{(3a + 7E[\tilde{z}] - 11r - 4\Xi)(a + E[\tilde{z}] - r)}{8b}. \tag{45}
\]

This is precisely the same debt level that showed up in subcase 1b. It must be strictly exceeded so that debt provides enough convexity-related production incentive to make the unconstrained production level infeasible within the capacity constraint.
Subcase 2b: Exhausted Capacity Region with $k^I = q^{I,u}(k^I, F^I)$ (boundary)

As for subcase 1b, there is no a priori restriction on the multiplier $\lambda_2$ in this region other than non-negativity, and we do have the restriction $k^I = q^{I,u}(k^I, F^I)$. Using the incumbent’s unconstrained production policy in (6) and the first order conditions, we solve to the same candidate equilibrium of debt, capacity, and productions:

\begin{align*}
F^I &= \frac{(3a + 7E[\tilde{z}] - 11r - 4\bar{Z})(a + E[\tilde{z}] - r)}{8b} \tag{46} \\
k^I &= \frac{a + E[\tilde{z}] - r}{2b} \tag{47} \\
q^I &= \frac{a + E[\tilde{z}] - r}{2b} \tag{48} \\
q^E &= \frac{a + E[\tilde{z}] - r}{4b} \tag{49}
\end{align*}

where $\lambda_2 = 0$.

Together subcases 1a, 1b, 2a, and 2b combine to specify the equilibria of Equation (16). When we substitute these optimal policies for $k^I$, $q^I$, and $q^E$ into the implicit definition of $\hat{Z}^I$, we find that the risky debt assumption $\hat{Z}^I \in (\bar{Z}, \bar{Z})$ (where $\hat{Z}^I = \frac{a + 5E[\tilde{z}] - 5r - 2\bar{Z}}{2}$) holds if and only if the exogenous parameters satisfy $a + 5E[\tilde{z}] - 5r \in [2\bar{Z} - 2\bar{Z}, 4\bar{Z}]$. The condition $a + 5E[\tilde{z}] - 5r > 2\bar{Z} - 2\bar{Z}$ guarantees that there are enough committed future payments ($F^I + r k^I$) to assure that equityholders face some possibility of having to default and therefore that any debt would be risky. In that sense, when capacity costs are not expensive enough on their own, debt convexity must be large enough. The other condition $a + 5E[\tilde{z}] - 5r < 4\bar{Z}$ is merely to insure that default is not certain.

6.2. PROOF OF PROPOSITION 2

In solving the capacity-only model, the analysis of the first-stage equilibrium proceeds through cases around the non-differentiable kink similar to the two-factor model. Subcase 1a, the excess capacity region where $k^{I,k} > q^{I,k,u}$, still yields a contradiction. This is Dixit’s (1980) result of no excess capacity.
Subcases 1b and 2b, where capacity is equal to the unconstrained production, \( k_{I,k} = q_{I,k,u} \), by definition give the candidate equilibrium capacity and productions of \( k_{I,k} = q_{I,k,u} = \frac{a+E[\hat{z}] + r}{3b} \) and \( q_{E,k,u} = \frac{a+E[\hat{z}] - 2r}{3b} \). For this candidate equilibrium, the entrant’s displaced production is: \( \Delta k = \frac{r}{3b} > 0 \). It will never be optimal to deviate from Stackelberg because it is the best the incumbent can do. Setting the production \( q_{I,k,u} = \frac{a+E[\hat{z}] + r}{3b} \) equal to the Stackelberg level yields the restriction \( 5r = a + E[\hat{z}] \).

Subcase 2a, the exhausted capacity region where \( k_{I,k} < q_{I,k,u} \), achieves Stackelberg leadership \( k_{I,k} = q_{I,k} = \frac{a+E[\hat{z}] - r}{2b} \) but it is an equilibrium if and only if \( k_{I,k} = \frac{a+E[\hat{z}] - r}{2b} < q_{I,k,u} = \frac{a+E[\hat{z}] + r}{3b} \). This yields the following condition on parameters: \( 5r > a + E[\hat{z}] \).

Together subcases 1b, 2a, and 2b lead to the restriction \( 5r \geq a + E[\hat{z}] \). Stackelberg leadership with capacity-only positioning is achieved only when \( 5r \geq a + E[\hat{z}] \). Otherwise, capacity-only positioning is inferior to two-factor positioning.

To shed light on why we have \( 5r \geq a + E[\hat{z}] \) when \( k_{I,k} \leq q_{I,k,u} \), we return to the incumbent’s second-stage unconstrained production choice. Without the capacity constraint, the incumbent’s second-stage production solves the problem:

\[
\max_{q^l} (a + E[\hat{z}] - b(q^l + q^E)) q^l - rk^l. \tag{50}
\]

The first order condition,

\[
(a + E[\hat{z}] - b(q^l + q^E)) - bq^l = 0, \tag{51}
\]

involves two effects: one more unit of production increases the incumbent’s revenues by the market price \( (a + E[\hat{z}] - b(q^l + q^E)) \), but the increased production also decreases the price on the entire production \( -bq^l \). In the exhausted capacity region, the first-stage capacity solves the problem:

\[
\max_{k^l} \left( a + E[\hat{z}] - b \left( k^l + \frac{a - bk^l - r}{2b} \right) \right) k^l - rk^l. \tag{52}
\]
The first order condition,
\[
\left( a + E[\tilde{z}] - b \left( k^I + \frac{a - bk^I - r}{2b} \right) \right) - b \left( 1 - \frac{1}{2} \right) k^I - r = 0,
\] (53)
involves three effects. First, as in the unconstrained second-stage optimization, an additional unit increases the incumbent’s revenues by the price \( a + E[\tilde{z}] - b \left( k^I + \frac{a - bk^I - r}{2b} \right) \). Second, it also directly decreases the price on all production \( -bk^I \). However, in this first-stage capacity optimization, the incumbent incorporates the entrant’s second-stage production best reply. That is, the incumbent knows that an increase in its capacity by one unit will lead to a production subgame equilibrium where the entrant decreases its production by a half unit. Therefore, relative to the incumbent’s second-stage unconstrained production problem, the incumbent choosing capacity in the first period within the exhausted capacity region perceives less price erosion from the total production (with the term \( b \left( \frac{1}{2} \right) k^I \)). The final effect of a unit increase in capacity is its marginal cost \( -r \). To remain in the exhausted capacity region where capacity is less than the unconstrained production level \( k^I < q_{I,k,u}^I \), it must be the case that the additional marginal benefit of a unit increase in capacity \( b \left( \frac{1}{2} \right) k^I \) is less than or equal to the additional marginal cost \( r \):
\[
\begin{align*}
 b \left( \frac{1}{2} \right) k^I - r & \leq 0 \\
 \iff b \left( \frac{a + E[\tilde{z}] - r}{2b} \right) - r & \leq 0 \\
 & \iff 5r \geq a + E[\tilde{z}].
\end{align*}
\] (54) (55) (56)

When capacity costs are expensive enough, the constraint can be binding and capacity can serve as a commitment to aggressive production.

References


Table 1. Numerical Approximation

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Table VII. Incumbent-then-Entrant Capacity Choices

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Table IX. Adding Long-Purse Features

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Figure 1: Iso-Displacement Curves $\Delta(k_i, F_1)$

**Leontief Segment:**
(i) debt above the threshold has zero marginal displacement product; (ii) capacity is at floor for given level of displacement

**Perfect Substitutes Segment:**
(i) equal tradeoff between debt dollars and capacity cost dollars; (ii) capacity lies in the “collared” region

- Debt $F_1$
- Threshold Debt Level
- Increasing Displacement
- Floor Capacity Costs
- Ceiling Capacity Costs
- Capacity Costs $r_k$
Figure 2: Incumbent’s Optimal Production Policy $q_t^*$ for a Given Debt Level $F_t$

$q_t = q_t^U$
$q_t = k_t$
$q_t^* = \begin{cases} q_t^U & \text{when } q_t^U \leq k_t \\ k_t & \text{when } q_t^U \geq k_t \end{cases}$

exhausted capacity region

excess capacity region

$q_t$

$k_t$