Corporate Cash Holdings and Credit Line Usage*

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Abstract:
We investigate the factors driving the unprecedented rise in corporate liquidities since the 1970s. We find that an economy wide reduction in the cost of holding liquidities and an increase in risk best explain the rise in cash holdings and the widespread use of credit lines. The structural estimation results shed light on two widely-acknowledged motives for holding cash. The precautionary motive and the liquidity motive translate risk exposure into cash holdings. Our results however do not suggest that firms have become more prudent over time. It is higher liquidity needs that has forced firms to hold more cash and use more credit lines.

Keywords: Dynamic capital structure, cash holdings, precautionary savings, corporate liquidity.

JEL Classifications: G31, G32, G35, E21, E22

Short Title: Corporate Cash and Credit Lines

* Manuscript submission: 3 October 2013; Major revision: 19 March 2015.

1 We thank Gian Luca Clementi, Adlai Fisher, Rick Green, Denis Gromb, Burton Hollifield, Lars-Alexander Kuehn, Erwan Morellec, Norman Schürhoff, and Toni Whited for helpful comments and discussions. We would also like to thank Jesus Fernandez-Villaverde, anonymous referees, and seminar participants at the American Finance Association meetings, Carnegie-Mellon University, the Federal Reserve Bank of Dallas, French Finance Association meetings, HEC Lausanne, INSEAD, KU Leuven, the Northern Finance Association meetings, the T2M annual conference, the University of British Columbia Summer Finance Conference, the University of Colorado at Boulder, the University of Wisconsin at Milwaukee, Victoria University Wellington, and Wilfrid-Laurier University for helpful comments.
1 Introduction

North American firms increasingly use liquidity instruments to manage the risk they face. Bates, Kahle, and Stulz (2009) document a large and steady increase in cash holdings as a proportion of total assets for US listed firms. Alongside the unprecedented level of cash liquidities, Sufi (2009) documents the widespread use of credit lines. To gain some perspective, consider the period starting in 1995 and ending in 2006 just before the crisis. For this period, long-term debt represents 22 percent of total assets for the average firm in our sample of listed firms, while cash holdings represents almost 21 percent of total assets. In fact, 45 percent of firms have on average more cash than debt.

The literature highlights that, because of financial frictions, firms must rely on liquidity instruments to manage risk. This suggests that the larger importance of liquidities is attributable to three possible causes. Firms may rely more on liquidity instruments because they face more risk, because the cost of using liquidities has decreased, or because financial frictions have increased. Bates et al. (2009) conclude that the large cash increase is attributable to higher cash flow risk for listed firms.

Building on Bates et al. (2009), we identify the mechanism by which the increase in risk leads to an increase in liquidities for North American listed firms. Table 1 documents more volatile sales and more volatile Operating Expenses, in accord with Dichev and Tang (2008). Interestingly, the rise in the volatility of Operating Expenses is itself attributable to General, Selling, and Administrative Expenses, an item that is mostly unrelated to the scale of operations.

TABLE 1 HERE

We use a structural approach to investigate the underlying mechanisms by which increases in risk explain the large increase in corporate cash holdings amidst prevalent credit lines and other liquidity management tools. As is standard, we model the sale revenue risk by a shock to total factor productivity (TFP) observed by the firm at the beginning of the year. In line with the data, we model another source of risk as a time-varying fixed cost unrelated to the production scale of the firm. In addition, we recognize that shocks occur throughout the year, and we allow this second shock to be realized during the year.

\(^2\)Using a random sample of 300 COMPUSTAT firms, Sufi (2009) documents that 85 percent of firms have access to a credit line between 1996 and 2003, and their line usage amounts to about six percent of total assets on average. For firms with a Standard and Poor’s credit rating, 94.5 percent of them have a credit line where usage represents 4.7 percent of total assets.
The structural model embeds two mechanisms by which more risk leads to more cash holdings. First, the liquidity mechanism emphasizes the greater flexibility of cash and credit lines over the firm’s other instruments. The model recognizes that it is more costly for firms to sell off assets, take back distributed dividends, or raise new debt to cover an adverse mid-year shock than to use accumulated liquidities. Instead, firms transfer funds between those interest-earning assets and cash at the beginning of each decision period in anticipation of their liquidity requirements during that period. Thus, one possible mechanism explaining the rise in cash is the increase in mid-year risk which would escalate the need for liquidities. Such a liquidity mechanism is featured in the seminal work of Miller and Orr (1966), where firms must manage cash inventories to face immediate liquidity needs. The liquidity mechanism is also similar to that discussed in Telyukova and Wright (2008), where liquidity needs yield a motive for consumers to accumulate liquidities.

Second, the precautionary mechanism emphasizes the role of taxes on distributions and costs to issuing equity. Firms not only smooth payouts to avoid extreme taxes and issuing costs, but may also behave prudently and accumulate cash holdings to self-insure against future adverse shocks. In this sense, the firm may accumulate precautionary holdings over and above those required by immediate funding needs discussed above. In this second possible mechanism explaining the cash increase, the increase in firms’ idiosyncratic risk underscores the need to self-insure. This precautionary mechanism is similar to that discussed in Leland (1968) and Carroll and Kimball (2008), where a convex marginal utility generates prudence and yields a motive to accumulate precautionary cash holdings.

To summarize, both the Miller and Orr (1966) liquidity motive and the Leland (1968) precautionary motive operate through financial frictions. Our liquidity mechanism focuses on the flexibility of cash and credit lines. A liquidity firm is a firm that is exposed to significant risk after its capital budgeting and financing decisions have been planned.\textsuperscript{3} Our precautionary motive focuses on payout taxes and equity issuing costs. A prudent firm is a firm that wants to avoid instances where it has to raise a large amount of equity and must therefore pay large issuing costs.\textsuperscript{4}

We know from Graham and Harvey (2001) that preserving financial flexibility is a top concern of CFOs when making capital structure choices. Accordingly, our paper offers a dynamic theory of cash accumulation and credit line usage in a framework that also characterizes firms’ investment and financial decisions, including those related to debt, equity, and dividends. Importantly, the

\textsuperscript{3}For example, airlines typically face significant competition and fuel price volatility, and as such the setting may represent a good illustration of the liquidity motive for holding cash and managing risk carefully.

\textsuperscript{4}All firms behave prudently to some extent, as they smooth dividends and avoid paying large issuing costs.
model assumes a much greater flexibility of cash and credit lines over other instruments, and this implies that cash is not negative debt. The model also features the main characteristics of credit lines, such as covenants, credit limits, and bounds to the substitutability between cash and credit lines.5

Our quantitative analysis shows that the model produces a reasonable description of firms' behavior, including matching the increase in cash holdings and the widespread use of credit lines. Our analysis highlights the lower cost to using liquidities and the large increase in risk, but no heightened financial frictions over the sample period. An important financial innovation over that time was the progressive adoption of sweep accounts starting in the mid-1990s, as documented in Anderson (2003). Cash became much more attractive because money market sweep accounts effectively raised its real rate of return. We find evidence that this economy-wide innovation played a role in explaining the increase in cash holdings. This is consistent with the decade dummy variable results in Bates et al. (2009) suggesting that some of the increase in cash holdings in the 2000s is unrelated to changes in firm characteristics. This innovation by itself, however, would have led to a counterfactual substitution away from credit line usage.

Our quantitative results indicate that the increased TFP volatility leads to a large increase in cash holdings but again a counterfactual substitution away from credit lines. The key increase in risk originates from the large increase in mid-year volatility, which produces a large increase in cash holdings and widespread credit line use.6

Our model is related to a large literature on corporate cash holdings. Indeed, there are many influential complementary papers examining other motives for holding liquidities. For example, Armenter and Hnatkovska (2011) focus on the role of taxation, Lyandres and Palazzo (2015) and Schroth and Szalay (2010) examine the role of R&D intensity and competition, and Nikolov and Whited (2014) quantify agency conflicts in understanding cash accumulation behavior.7 Others

5These characteristics are described in Acharya et al. (2014), Agarwal et al. (2004), Berger and Udell (1995), Disatnik et al. (2014), Ham and Melnik (1987), Shockley and Thakor (1997), and Sufi (2009), among others.

6Our paper underscores the need to understand better why mid-year shocks have become more volatile over time. As a first step toward that line of inquiry, our paper emphasizes that such volatility is important in explaining the large increase in liquidities. For example, in the context of R&D expenses, our paper would suggest that firms hold more liquidities because R&D-related expenses have become more volatile, not because firms incur more of those expenses. There may well exist other motives outside our model to hold liquidities in response to an increase in the level of R&D expenses, but our paper focuses on the volatility of such expenses.

7The relationship between higher cash holdings (or lower cash value) and higher agency costs is also well documented empirically in Dittmar and Mahrt-Smith (2007), Dittmar et al. (2003), Faulkender and Wang (2006), Harford
focus on the relationship with financial constraints, including Acharya et al. (2007), Almeida et al. (2004), Bolton et al. (2011), Han and Qiu (2007), Hugonnier et al. (2015), Morellec et al. (2013) and Riddick and Whited (2009). Another strand of the theoretical corporate liquidity literature centers on credit lines, including Boot et al. (1987), DeMarzo and Fishman (2007), DeMarzo and Sannikov (2006), Holmstrom and Tirole (1998), Martin and Santomero (1997), Sannikov (2007), and Tchisty (2006). Finally, the financial flexibility model of Gamba and Triantis (2008) is related to ours in that a firm can finance its investment through debt issues, equity issues, and internal funds, where only internal funds do not trigger transaction costs. We contribute to the above literature by identifying which economic factors, in the context of precautionary and liquidity motives, are responsible for the large increase in cash holdings amidst the widespread use of credit lines.

2 The Model

We study how a firm manages its cash holdings and credit line in an otherwise standard dynamic model of financial and investment decisions. The firm does not consider cash as negative debt. Instead, cash may serve two purposes. It may provide self-insurance against future adverse shocks and it may provide liquidity to meet current needs. A line of credit may also provide liquidity to meet current needs, but only if the firm has not yet violated its financial covenant. In this sense, the extent to which cash holdings and lines of credit can be substitute is limited.

To operationalize these roles, we recognize that the firm faces shocks throughout the year. Knowing the current realization of TFP, the firm chooses how much to invest, how much cash to save, how much debt to issue, how much dividend to pay out (or how much equity to raise). During the year, however, the firm faces another shock. When this mid-year shock is worse than expected, the firm cannot scale back its investment commitments, take back its distributed dividend, or go back to external markets with more favorable issuing conditions. Instead, mid-year shocks are met with cash or a line of credit if available.

2.1 The Firm

The firm, acting in the interest of shareholders, maximizes the discounted expected stream of payouts \( D_t \) taking into account taxes and issuing costs. When payouts are positive, shareholders pay taxes on the distributions according to a tax schedule \( T(D) \). The schedule recognizes that firms (1999), Harford et al. (2008), Keefe and Kieschnick (2011), Lins et al. (2010), and Pinkowitz et al. (2006), and Yun (2009), among others.
can minimize taxes for smaller payouts by distributing them in the form of share repurchases. Firms, however, have no choice but to trigger the dividend tax for larger payouts. Following Hennessy and Whited (2007), the tax treatment of payouts is captured by a schedule that is increasing and convex:

\[ T(D_t) = \tau_D D_t + \frac{\tau_D}{\phi} \exp(-\phi D_t) - \frac{\tau_D}{\phi}, \]

where \( \phi > 0 \) controls the convexity of \( T(D) \) and \( 0 < \tau_D < 1 \) is the tax rate. When payouts are negative, shareholders send cash infusions into the firm as in the case of an equity issue. The convex schedule \( T(D) \) also captures the spirit of Altinkilic and Hansen (2000), where equity issuing costs are documented to be increasing and convex.

Net payouts are

\[ U(D_t) = D_t - T(D_t). \]

This function is increasing \( U'(D) = 1 - \tau_D + \tau_D \exp(-\phi D) > 0 \), concave \( U''(D) = -\phi \tau_D \exp(-\phi D) < 0 \), and its third derivative is positive \( U'''(D) = \phi^2 \tau_D \exp(-\phi D) > 0 \). As a result of taxes and issuing costs, the net payout function characterizes the firm behavior as if it is risk averse and prudent. In fact, the parameter \( \phi \) is the coefficient of absolute prudence: \( \phi = -U'''(D)/U''(D) \).

The firm faces two sources of risk. The first source of risk comes from the stochastic TFP. Revenues \( Y_t \) are generated by a decreasing returns to scale function of the capital stock \( K_t \):

\[ Y_t = \exp(z_t) K_t^\alpha, \]

where \( z_t \) is the current realization of TFP and \( 0 < \alpha < 1 \) denotes capital intensity. TFP follows the autoregressive process

\[ z_t = \rho_z z_{t-1} + \sigma_z \epsilon_{zt}, \]

where \( \epsilon_{zt} \) is the innovation to TFP, \( 0 < \rho_z < 1 \) denotes its persistence, and \( \sigma_z > 0 \), its volatility. The innovations \( \epsilon_{zt} \) are independently and identically distributed random variables drawn from a standard normal distribution: \( \epsilon_{zt} \sim N(0,1) \).

The second source of risk enters additively into the sources and uses of funds equation and is given by

\[ F_t = \bar{F} + f_t, \]
where $\bar{F}$ is the predictable level and $f_t = \sigma_f \epsilon_{ft}$ is the innovation, where $\sigma_f > 0$. The innovations are assumed to be independent of the TFP innovations and drawn from a uniform distribution: $\epsilon_{ft} \sim U[-1, 1]$.

The firm chooses how much to invest $I_t$, how much cash to hold $M_{t+1}$, how much debt to raise $B_{t+1}$, how much credit line to use $L_{t+1}$, and how much to pay out (or how much equity to issue) $D_t$. The sources and uses of funds define the firm’s liquidities at the beginning of the next year:

\[
M_{t+1} = Y_t - F_t - I_t + B_{t+1} + L_{t+1} - D_t - (1 + r)B_t - (1 + \xi)L_t + (1 + \iota)M_t - T^C_t - \Omega^K_t - \Omega^B_t,
\]

where $T^C_t$ represents corporate taxes, while $\Omega^K_t$ and $\Omega^B_t$ denote adjustment costs to capital and debt. The interest rates $r$, $\xi$, and $\iota$ are associated with debt, credit line, and cash holdings, where it is assumed that $\iota < r < \xi$.

Corporate taxes are imposed on revenues after depreciation, interest payments, and interest income:

\[
T^C_t = \tau_C (Y_t - F_t - \delta K_t - rB_t - \xi L_t + \iota M_t),
\]

where $0 < \tau_C < 1$ is the corporate tax rate.

Capital accumulates as follows:

\[
K_{t+1} = I_t + (1 - \delta)K_t,
\]

where $0 < \delta < 1$ denotes the depreciation rate. The firm faces quadratic capital adjustment costs:

\[
\Omega^K_t = \frac{\omega_K}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t,
\]

where $\omega_K \geq 0$.

The firm also faces quadratic costs to varying the debt level away from its long-run level $\bar{B} > 0$:

\[
\Omega^B_t = \frac{\omega_B}{2} (B_{t+1} - \bar{B})^2,
\]

where $\omega_B \geq 0$. The after-tax discount factor is $\beta = 1/(1 + (1 - \tau_r)r)$, where $\tau_r \geq 0$ is the personal tax rate on interest income. Because individuals pay taxes on interest income at a lower rate than corporations deduct their interest payment ($\tau_r < \tau_C$), debt financing is tax-advantaged. To counter this benefit of debt financing, the convex cost in equation (10) bounds the debt level. In this sense, deviation costs play a role similar to a collateral constraint.

In contrast to investment, debt issuance, and equity issuance, the firm does not incur any cost when changing its cash holdings or credit line. Credit lines, however, cannot always substitute for
cash holdings. Credit line loans may not exceed a preset upper limit $\bar{L}$, generating the constraint:

$$0 \leq L_{t+1} \leq \bar{L}.$$ 

Credit lines also come with covenants requiring the firm to keep in good standing (as described below).

2.2 The Intertemporal Problem

At the beginning of the year, the firm makes decisions knowing the current realization of the TFP $z_t$, but not the realization of the shock $f_t$, which we label the mid-year shock. During the year, the mid-year shock may trigger a need for liquidity. Of course, foreseeing all this, the firm may already have invested less, paid out less in dividends, or raised more funds externally so that its liquidities (cash plus credit line) can cover the shock. As a result, the stock of cash at the end of the year, $M_{t+1}$, is equal to the firm’s choice of cash savings $S_t$ at the beginning of the year plus the used line of credit $L_{t+1}$ net of the after-tax mid-year shock:

$$M_{t+1} = S_t + L_{t+1} - (1 - \tau_C)\sigma_f \epsilon_{ft},$$

where the cash saving is

$$S_t = (1 - \tau_C)\left( Y_t - \bar{F} - \delta K_t - rB_t - \xi M_t - \Delta K_{t+1} + \Delta B_{t+1} + M_t - L_t - \Omega^K_t - \Omega^B_t - D_t \right)$$

and $\Delta K_{t+1} = K_{t+1} - K_t$ and $\Delta B_{t+1} = B_{t+1} - B_t$.

When the firm has sufficiently high cash flows, it has access to its line of credit. In these circumstances, the firm must nevertheless set aside enough cash to cover the gap between the worst possible mid-year shock and the upper limit on the credit line. Because the firm makes its cash saving decision $S_t$ without knowing the realization of the mid-year innovation $\epsilon_{ft}$, the firm must save enough to cover all possible realizations. The juxtaposition of the end-of-the-year cash holdings in equation (11) and the non-negativity constraint $M_{t+1} \geq 0$ requires that $S_t + \bar{L} - (1 - \tau_C)\sigma_f \geq 0$.

When the firm’s TFP is too low ($z_t < \bar{z}$), the firm violates its covenant and looses access to its line of credit. In these circumstances, the mid-year shock must be met with cash: $S_t - (1 - \tau_C)\sigma_f \geq 0$.

Altogether, the liquidity constraint becomes

$$S_t + \bar{L}\mathbf{1}_{(z_t \geq \bar{z})} - (1 - \tau_C)\sigma_f \geq 0,$$

where $\mathbf{1}_{(z_t \geq \bar{z})}$ is an indicator function that takes a value of one when $z_t \geq \bar{z}$.\(^8\)

\(^8\)We note that the credit line is contingent on the TFP shock only. If it were contingent on the mid-year shock, the firm would never use its credit line as the covenant would block the firm’s access to the line of credit when the
The firm’s intertemporal problem can be described by the following dynamic programming problem. The firm’s mid-year problem is to choose cash and a credit line loan to solve

\[ W(K_{t+1}, B_{t+1}, S_t; z_t, f_t) = \max_{\{M_{t+1}, L_{t+1}\}} \beta E_{t+1} \left[ V(K_{t+1}, B_{t+1}, L_{t+1}, M_{t+1}; z_{t+1}, f_{t+1}) \right] \]

subject to equation (11), the non-negativity constraints \( M_{t+1} \geq 0 \) and \( L_{t+1} \geq 0 \), and the upper limit on the loan \( L_{t+1} \leq \bar{L} \). Note that the conditional expectation \( E_{t+1} \) is taken on an information set \( \Phi_{t+1} \) that includes all beginning-of-the-year choices of capital \( K_{t+1} \), debt \( B_{t+1} \), and cash saving \( S_t \), plus the realization of TFP \( z_t \) and the mid-year shock \( f_t \).

When the firm violates its covenant, the problem simplifies to holding cash \( M_{t+1} = S_t - (1 - \tau_C)\sigma_f \epsilon f_t \) and no credit line \( L_{t+1} = 0 \).

The firm’s beginning-of-the-year problem is to choose the payout, capital stock, debt level, and cash savings to solve

\[ V(K_t, B_t, L_t, M_t; z_{t-1}) = \max_{\{D_t, K_{t+1}, B_{t+1}, S_t\}} U(D_t) + E_t \left[ W(K_{t+1}, B_{t+1}, S_t; z_t, f_t) \right] \]

subject to equations (1) to (5), (8) to (10), (12) and (13). Here, the conditional expectation \( E_t \) is taken on an information set \( \Phi_t \) that includes the beginning of the year values for the capital stock \( K_t \), debt level \( B_t \), line of credit \( L_t \), cash holdings \( M_t \), and TFP \( z_t \), but not the mid-year realization of \( f_t \). The appendix presents the optimality conditions for this problem. In what follows, we summarize the salient results.

The necessary optimality conditions include the complementary-slackness conditions associated with the liquidity constraint (13):

\[ \lambda_t \geq 0, \quad S_t + \bar{L} 1_{\{z_t \geq \bar{z}\}} - (1 - \tau_C)\sigma_f \geq 0, \quad \text{and} \quad \lambda_t \left[ S_t + \bar{L} 1_{\{z_t \geq \bar{z}\}} - (1 - \tau_C)\sigma_f \right] = 0. \]

When the multiplier is positive \( \lambda_t > 0 \), the firm saves just enough cash to satisfy the liquidity constraint with equality. That is, all cash holdings are driven by the liquidity motive. When \( \lambda_t = 0 \), the firm may prudently save more than required to meet immediate liquidity needs. We summarize these findings in the following proposition.

**Proposition 1.** When \( \lambda_t > 0 \), \( S_t = (1 - \tau_C)\sigma_f - \bar{L} 1_{\{z_t \geq \bar{z}\}} \) so that the firm saves cash only as a safeguard against the mid-year shock realization over the portion covered by the credit line. When \( \lambda_t = 0 \), \( S_t \geq (1 - \tau_C)\sigma_f - \bar{L} 1_{\{z_t \geq \bar{z}\}} \) so that the firm may prudently hold more cash than required.

The mid-year shock turns out to be too low. The covenant would effectively constrain the firm to accumulate enough cash at the beginning of the year to cover all possible mid-year shock realizations. As a result, the firm would rely on cash only.
The optimality conditions also include Euler equations for cash and the credit line. The cash saving decision $S_t$ is characterized by

$$(17) \quad E_t [m_{t+1}] R^M + \left\{ E_t [\gamma^M_{t+1}] + \lambda_t \right\} / U'(D_t) = 1,$$

where $m_{t+1} = \beta U'(D_{t+1})/U'(D_t) > 0$ is the pricing kernel, $R^M = 1 + (1 - \tau_C)\xi$ is the return to cash, and $\gamma^M_{t+1}$ is the multiplier on the non-negativity constraint $M_{t+1} \geq 0$ in the mid-year problem. The credit line decision is characterized by

$$(18) \quad E_t [m_{t+1}] R^L - \left\{ E_t [\gamma^L_{t+1} - \gamma^U_{t+1}] + \lambda_t \right\} / U'(D_t) = 1,$$

where $R^L = 1 + (1 - \tau_C)\xi$ is the return on the credit line loan, $\gamma^L_{t+1}$ is the multiplier on the non-negative constraint $L_{t+1} \geq 0$, and $\gamma^U_{t+1}$ is the multiplier on the upper limit $L_{t+1} \leq \bar{L}$.

Recall that there is no cost to adjust cash holdings or credit line usage. The relation between the returns $R^M < R^L$, implied by $\iota < \xi$, determines the substitutability between the two sources of liquidity. Proposition 2 highlights the substitution.

**Proposition 2.** The firm will never hold cash $M_{t+1} > 0$ and use its credit line $L_{t+1} > 0$. Either $M_{t+1} \geq 0$ and $L_{t+1} = 0$, or $M_{t+1} = 0$ and $L_{t+1} > 0$.

**Proof.** See Appendix 6.2.

Without observed data on credit lines, we cannot corroborate the substitutability between cash holdings and credit lines. In the data, there are many reasons why cash holdings and credit lines could have only a limited range of substitutability. First, the model abstracts away from the realistic feature of a minimum balance requirement. With such a requirement, the cash balance would rarely fall to zero, implying that a credit line could be used with a positive cash balance. Second, credit lines are contingent on good performance. As a result, the substitution can only go from credit lines to cash holdings when the firm’s performance is poor. Indeed, Viral Acharya et al. (2014) exploit a quasi-experiment around the downgrade of General Motors and Ford in 2005, and confirmed that firms moved out of credit lines and into cash holdings in the aftermath of the downgrade. Finally, credit line loans have a preset upper limit, so there is a restricted range in which the firm has access to credit lines. We have modeled these last two restrictions on credit lines. Proposition 2 is meant to highlight the possible substitution at the margin between cash holdings (above the minimum required for working capital management) and credit lines (within their restrictions on usage).
Whether or not the firm holds cash at the end of a year depends on its beginning-of-the-year cash savings and the size of the mid-year shock. Proposition 3 below states that the firm will hold cash at the end of the year \( M_{t+1} \) when it has enough cash savings \( S_t \) or when the mid-year shock \( f_t \) is low enough.

**Proposition 3.** The firm will hold cash \( M_{t+1} > 0 \) when the mid-year shock is low, \( f_t < \sigma_f - \bar{L} \mathbf{1}(z_t \geq \bar{z})/(1 - \tau_C) \), or when beginning cash savings are high, \( S_t > (1 - \tau_C)\sigma_f \).

**Proof.** See Appendix 6.2.

Finally, Proposition 4 states that the firm will save enough at the beginning of the year so that it accumulates cash holdings by the end of the year only if two requirements are met. First, cash must not be dominated by the firm’s other instruments and, second, the firm must be sufficiently prudent. The first requirement is that cash must not be dominated in return by either debt or capital. To see this, note that debt and capital decisions are characterized by

\[
E_t \left[ m_{t+1} \right] R^B_t = 1
\]

and

\[
E_t \left[ m_{t+1} \right] E_t \left[ R^K_{t+1} \right] + \text{Cov}_t \left[ m_{t+1}, R^K_{t+1} \right] = 1,
\]

where \( R^B_t = \frac{[1 + (1 - \tau_C)\omega_B]}{1 - \omega_B (B_{t+1} - \bar{B})} \) is the return to debt and

\[
R^K_{t+1} = \left( 1 + (1 - \tau_C) \left[ \alpha \exp(z_{t+1}) R^K_{t+1}^{(\alpha - 1)} - \delta \right] + \frac{\omega_K}{2} \left[ \left( \frac{K_{t+1}}{K_t} \right)^2 - 1 \right] \right) / \left[ 1 + \omega_K \left( \frac{K_{t+1}}{K_t} - 1 \right) \right]
\]

is the return to capital.

Cash is not dominated in return whenever \( R^M = R^B = E_t \left[ R^K_{t+1} \right] + \text{Cov}_t \left[ m_{t+1}, R^K_{t+1} \right] / E_t \left[ m_{t+1} \right] \). In this circumstance, a comparison of equation (17) with equations (19) and (20) indicates that \( E_t \left[ \gamma^M_{t+1} \right] + \lambda_t = 0 \). This requires that \( \gamma^M_{t+1} = 0 \) for all realizations of the mid-year shock and that \( \lambda_t = 0 \). The former implies that the non-negativity constraint on cash holdings never binds \( (M_{t+1} \geq 0) \) and that the firm will never use its credit line \( (L_{t+1} = 0) \). To ensure this outcome, the firm must save at least \( S_t \geq (1 - \tau_C)\sigma_f \) (which then implies that the liquidity constraint (13) does not bind, \( \lambda_t = 0 \)).

Otherwise, if cash is dominated, the firm saves less at the beginning of the year. Cash is dominated when \( R^M < R^B = E_t \left[ R^K_{t+1} \right] + \text{Cov}_t \left[ m_{t+1}, R^K_{t+1} \right] / E_t \left[ m_{t+1} \right] \), which implies that \( E_t \left[ \gamma^M_{t+1} \right] + \lambda_t > 0 \). For this, either \( \gamma^M_{t+1} > 0 \) for some realizations of the mid-year shock or \( \lambda_t > 0 \) or both. According to Proposition 1, the latter implies that the firm saves less than above. This
ensures that there are realizations of the mid-year shock for which the firm uses its credit line (which then implies that the non-negativity constraint on cash holdings binds for some realizations, $\gamma_{t+1}^M > 0$). In the model, instances when cash is dominated in return by debt and capital occur whenever the debt is at its target $B_{t+1} = \bar{B}$ and whenever capital either yields a high expected return or when the covariance term is positive and large. A positive covariance implies that the return to capital net of taxes and adjustment costs $R_{t+1}^K$ is high when payouts $D_{t+1}$ are low, such that the firm may be able to self-insure against future adverse shocks with physical capital.

The second requirement is that the firm be sufficiently prudent to enact the precautionary motive. This prudence is conveyed by a convex marginal payout function. Here, $U'(\cdot)$ must be sufficiently convex to allow $R^M E_t[m_{t+1}] = 1$ or, equivalently, to allow $U'(D_t) = \beta R^M E_t[U'(D_{t+1})]$ when $\beta R^M < 1$.

Proposition 4 summarizes these requirements.

**Proposition 4.** The firm will select high cash savings, $S_t > (1 - \tau_C)\sigma_f$, only if $R^M = R^B_t = E_t[R_{t+1}^K] + \text{Cov}_t[m_{t+1}, R_{t+1}^K]/E_t[m_{t+1}]$ and $U'(D_t)$ is sufficiently convex.

**Proof.** See Appendix 6.2.

### 3 Data

The data comes from the North American COMPUSTAT file and covers the period from 1971 to 2006 excluding the crisis period. To explain the large change in cash holdings, the data is split in two extreme time periods: the first third of the sample period from 1971 to 1982 and the last third from 1995 to 2006. The COMPUSTAT sample includes firm-year observations with positive values for total assets (COMPUESTAT Mnemonic AT), property, plant, and equipment (PPENT), and sales (SALE). The measure of cash holdings (CHE) is composed of cash (CH) and short-term investments (IVST). The sample includes firms with at least five years of consecutive data from all industries, excluding utilities and financials. The data are winsorized to limit the influence of outliers at the 1 percent and 99 percent tails. The final sample contains 2,093 firms for the 1971-82 period and 4,526 firms for the 1995-06 period.

We seek to explain the large increase in cash holdings using model-simulated data. The model does not possess an analytical solution and is solved numerically. Once the model is solved, we simulate series for all variables from random outcomes of the innovations $\epsilon_{zt}$ and $\epsilon_{ft}$. These simulated series are used to construct operating income $OI_t = Y_t - F_t$, net income $NI_t = (1 - \tau_C)(Y_t - F_t - \delta K_t -$
For each of the two time periods, we construct five panels that have the same number of firms as observed in the COMPUSTAT panel.

The numerical method requires values for all parameters. To this end, a number of parameters are estimated directly from the data, and others are calibrated in a moment matching exercise. The appendix provides an extensive description of the numerical and calibration methods, as well as a discussion of the parameter estimates. The resulting parameter values appear in Tables 2 to 4. Two important qualifications are in order. First, we examine the cash behavior of COMPUSTAT (listed) firms only. Second, our structural model is based on a representative firm, e.g., as in Hennessy and Whited (2005). In this context, changes in the parameter estimates over the two periods could reflect changes at the firm level or changes in the composition of firms. In particular, the representative firm that aggregates firms operating in the 1971-82 period may be different from the representative firm that aggregates firms operating in the later period. A next step for further research could explore compositional changes.

Table 2 presents parameters whose values are estimated directly from the data. The parameters $\alpha$, $\rho_z$, and $\sigma_z$ are estimated directly from Equation (3) and the autoregressive process (4). The corporate tax rates are calibrated to the top marginal tax rate, while the personal tax rates are calibrated to the average marginal tax rates reported in NBER’s TAXSIM. Finally, the interest rates $r$, $\iota$, and $\xi$ are also measured from the data. In calibrating the interest rate on cash $\iota$, we recognize that, by 1995, sweep money market accounts became increasingly available to firms (see Anderson, 2003). For a fee, firms could have their cash savings in excess (or in shortage) of a minimum balance automatically transferred to (or from) money market accounts. During the 1971-82 period, cash not held in short-term investments would simply lose value at the inflation rate. Regarding the calibration of $\xi$, we apply a premium above the real interest rate $r$, based on the premium reported in Ham and Melnik (1987) for the 1971-82 period and in Sufi (2009) for the 1995-06 period. Finally, we allow the preset limit on the credit line to include all mid-year shock realizations: $\bar{L} = (1 - \tau_C)\sigma_f$, but specify that the firm must be experiencing average or better TFP ($\bar{z} = 0$) to use its line of credit.

**TABLE 2 HERE**

The remaining parameters are set to values chosen to ensure that simulated series from the model replicate important features of the data. Table 3 shows the parameter values and target moments for the 1971-82 period, while Table 4 does so for the 1995-06 period. The target moments
are based on the average and the standard deviation of the investment-to-total assets ratio, the average and standard deviation of the leverage ratio, the average operating income-to-total assets ratio, and the standard deviation of the net income-to-total assets ratio. Of special interest, the coefficient of absolute prudence $\phi$, which dictates the strength of the firm’s precautionary motive, is calibrated in the 1971-82 period so that the average of cash holdings-to-total assets matches the data. This choice ensures that we have the right starting point from which to investigate the increase in cash holdings. We are interested in predicting average cash holdings during the 1995-06 period without changing the coefficient of prudence from its 1971-82 parameter value. The average of COMPSTAT firms’ cash holdings-to-total assets was 7.9 percent during the 1971-82 years. Matching this moment requires a convexity parameter of $\phi = 0.0045$.

TABLE 3 HERE
TABLE 4 HERE

We acknowledge that our dynamic capital structure model may not be well suited to describe the behavior of all firms. For example, our model may not be a good representation of firms that do not benefit from a tax deduction (very low tax firms) or firms whose production involves little physical capital (low tangibility firms). To this end, we have examined the effects of excluding firms on the target moments.\footnote{The details of the data restrictions appear in an appendix available from the authors.} We have examined the effects of excluding firms with “permanently” low tax rates, i.e., firms with tax rates below 5 percent for every year within a given sub-sample. As it turns out, the exclusion of those permanently low tax rate firms does not lead to significantly different moments calculated from the data. We have also examined excluding low tangibility firms. We identify low tangibility firms as those with a ratio of Net Property, Plant and Equipment to Total Assets less than 5 percent for all years within a sub-period. The exclusion of those “permanently” low tangibility firms does not make a difference for the early period (1971-82). For the later period (1995-06), however, the data indicate that firms with more tangible assets experience slightly less volatility in their debt policy (20.4 percent compared to 23 percent for all firms). For the other target moments including cash holdings, the exclusion of those low tangibility firms does not lead to significantly different moments calculated from the data.

As is current practice, we standardize moments by Total Assets. Bates et al. (2009) standardize cash holdings by Total Assets, precisely to show that the increase in cash outweighs possible confounding factors in other asset categories. In considering the closest measure of Total Assets in the model, we have chosen to compare moments standardized by accounting values for Total
Liabilities and Owner’s Equity (i.e., Total Assets) in COMPUSTAT to moments standardized by the sum of debt and equity values generated from the model. For robustness, we have verified that our results obtain with a standardization that more closely relates observed and simulated data. For this, we standardize by year-end values for Property, Plant, and Equipment (PPENT) and Cash and Marketable Securities (CHE) in the data, and standardize by end-of-year capital ($K_{t+1}$) and cash holdings ($M_{t+1}$) in the model. The resulting moments matched to the data are different but our qualitative results remain the same.\footnote{The results appear in an appendix available from the authors.}

4 Results

4.1 Do Simulated Financial Policies Behave as in the Data?

Before studying cash holdings in detail, we verify that the model provides a reasonable description of firms’ observed behavior. Admittedly, the moment matching exercise ensures that specific simulated moments match the targeted moments in the data. In what follows, the analysis moves on to other moments related to dividend smoothness and debt countercyclicality. These results appear at the bottom of Tables 3 and 4.

It has long been recognized that firms smooth dividends (see Lintner, 1956). In our COMPUSTAT data, payout policies are smooth in the first period. The average standard deviation of payouts is only 4 percent during the 1971-82 period, and rises to 12.6 percent during the 1995-06 period. The model replicates the smooth payout policies in the first period and the greater volatility in the last period. The simulated standard deviation is only 2 percent for the first period, and rises to 11.6 percent for the last period. We note, however, that the model underestimates the volatility of payouts, especially in the first period.

In the model, firms smooth payouts to avoid large taxes on payouts and large equity issuing costs. Specifically, the net payout function $U(D)$ recognizes the convexity of taxes and issuing costs and therefore describes firms as if they are risk averse. Firms smooth payouts well in the first period. Unavoidably, the observed volatility increase in TFP and mid-year shocks in the last period affects the payout volatility.

It has also been recognized that debt issues are countercyclical (see Choe, Masulis, and Nanda, 1993; Covas and Den Haan, 2010; and Korajczyk and Levy, 2003). In our COMPUSTAT data, the correlation between debt issues and revenues is $-0.267$ for the 1971-82 period and $-0.191$ for the
1995-06 period. In the model, the correlations are $-0.291$ and $-0.049$, respectively. Both in the data and in the model, debt issues are countercyclical, and the negative correlation attenuates in the recent time period.

The countercyclicality of debt issues in the model is surprising because standard dynamic capital structure models with a tax benefit of debt generate procyclical debt. In these models, firms take on more debt in persistent good times to benefit from the tax advantage because of their stronger abilities to repay the debt. In our model, the “risk averse” firm chooses to smooth the effect of adverse TFP realizations on payouts by filling the gap with debt issues.

4.2 Do Simulated Liquidity Policies Behave as in the Data?

The model provides a reasonable overall description of average cash holdings. In COMPUSTAT data, cash (including short-term investments) represents 7.9 percent of total assets in the 1971-82 period. That number dramatically rises to 20.6 percent in the 1995-06 period. In the model, the average ratio of cash holdings-to-total assets is specifically targeted by our calibration using the prudence parameter $\phi$ for the first period. Holding $\phi$ constant, the model predicts cash holdings of 21.8 percent of total assets in the last period. As dramatic as the observed cash increase has been in the data, the model slightly overshoots the observed cash increase. Because the 1995-06 cash holdings simulated with the 1971-82 prudence parameter are larger than observed, the model suggests that the large increase in cash holdings is not associated with an increase in prudence. To match observed cash holdings, simulated firms must have become slightly less prudent over time.

Another quantity of interest is the used line of credit. Conditional on using the line of credit, our model predicts an average loan of 0.9 percent of total assets during the 1971-82 period. The later period usage rises to 5 percent, slightly overshooting Sufi’s (2009) average of 4.7 percent among firms with a credit rating.

Firms in the model do not reflect the rich heterogeneity of firm characteristics displayed in COMPUSTAT. In the model, the only source of heterogeneity arises from different realizations of TFP and mid-year shocks. Nevertheless, the model can inform on some of the cross-sectional variations in liquidity policies. For example, the model predicts a negative propensity to save, as documented in Riddick and Whited (2009). Firms in the model that experience low TFP realizations and thus low cash flows become financially constrained as they violate their credit line covenant. As a result, these firms must save more as a proportion of their revenues to meet the liquidity constraint. In addition, firms experiencing low TFP realizations invest less and become
small. Sorting on TFP realizations, firms who experience the worst TFP realizations hold more cash than average.

The model can also inform on the relation between cash holdings and debt. First, most firms simultaneously have positive debt and cash holdings. The COMPUSTAT data indicate that 96.2 percent of firm-year observations in the 1971-82 period and 81.0 percent of firm-year observations in the 1995-06 period have both positive debt and cash holdings. In accord, our model predicts that 94.0 percent of simulated firm-year observations for the 1971-82 period and 80.5 percent of simulated firm-year observations for the 1995-06 period have both positive debt and cash holdings. Second, the number of firms that have more cash than debt has risen substantially. The COMPUSTAT data indicate that 17.8 percent of firms had on average more cash than debt in the 1971-82 period. This percentage has increased to 45.0 percent during the 1995-06 period. Our model also predicts a large increase in the number of firms with positive cash net of debt, but it replicates only crudely this increase. No simulated firm has more cash than debt during the 1971-82 period but 51.3 percent of simulated firms have on average more cash than debt outstanding in the 1995-06 period.

Finally, we also calibrate the model on COMPUSTAT data for the middle 1983-94 period. This middle period characterizes the progressive change in firm policies well, as the average moments calculated for the 1983-94 period lie within the moments for the 1971-82 period reported in Table 3 and the moments from the 1995-06 period reported in Table 4. Importantly, cash holdings observed in the data represent 13.9 percent of total assets for the 1983-94 period. Holding the prudence parameter to its 1971-82 value, the model predicts very similar cash holdings (13.8 percent) for the 1983-94 period.

### 4.3 What Drives Liquidity?

Given that the model replicates the large increase in cash holdings and produces significant credit line usage, the analysis now turns to identifying which economic forces are responsible for the increase in firm liquidities. In what follows, we discuss the most important forces. First, Table 2 documents a substantial reduction in the opportunity cost of holding cash: the extent to which cash is dominated in return has decreased. Second, Tables 3 and 4 document that the later period

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11 We note that these percentages are measured differently in Armenter and Hnatkovska (2011). Using Flow of funds data (rather than COMPUSTAT) during different time periods, Armenter and Hnatkovska (2011) measure 26.9 percent net financial assets to capital in the 1970s, with the percentage increasing to 43.6 percent during 2000-07 period.

12 The details of this exercise appear in an appendix available from the authors.
is characterized by more risk.

To gauge the importance of these forces, Table 5 presents the results of a sensitivity analysis, which proceeds on the basis of the first period parametrization. In turn each parameter is reset from its 1971-82 value to its 1995-06 value, leaving all other parameters to their 1971-82 values. The three most important parameters are the interest rate on cash holding \( \iota \), the TFP volatility \( \sigma_z \), and the mid-year shock volatility \( \sigma_f \).

**TABLE 5 HERE**

Table 5 documents that the increase in the interest rate \( \iota \) has a large effect on cash holdings. The economy-wide return to holding cash has risen dramatically from a real rate of \(-5.32\) percent during the 1971-82 period to a real rate of \(0.35\) percent during the 1995-06 period. This increase is due to two fundamental changes: the reduction in inflation rate from an average of \(7.9\) percent in the 1971-82 period to \(2.6\) percent in the 1995-06 period, and the introduction of sweep accounts in the mid-1990s. All else equal, the increase in \( \iota \) alone would spur an otherwise similar firm to hold \(25.1\) percent of its assets in cash in the 1995-06 period, overshooting the observed average cash holdings of \(20.6\) percent. With such large cash liquidities, the simulated firm would counterfactually substitute entirely away from its credit line.

Table 5 also documents that the large volatility increases for TFP \( \sigma_z \) and the mid-year shock (scaled by mean assets) \( \sigma_f / \bar{A} \) generate large increases in cash holdings. Over the two periods, the increase in the standard deviation of the innovation to TFP, \( \sigma_z \), makes cash flow much more volatile and this triggers a large response in cash saving. The volatility rose from \(0.250\) in the first period calibration to \(0.441\) in the last period calibration. Table 5 shows that this increase promotes an increase in cash holdings from \(7.9\) percent of total assets to \(22\) percent. With such large liquidities, the simulated firm again counterfactually substitutes away from the credit line.

Over the two periods, the mid-year shock volatility (scaled by mean total assets) grows from \(0.081\) to \(0.273\). This forces the firm to save more cash to meet its current liquidity needs. Accordingly, Table 5 shows that, by changing only the mid-year shock volatility \( \sigma_f / \bar{A} \) to its last period value, the calibration otherwise based on the first period predicts a rise in cash holdings from \(7.9\) percent of total assets to \(17.7\) percent. This change also produces a large rise in credit line usage from \(0.9\) percent to \(6.9\) percent. Thus, the change in mid-year volatility is key in explaining both the rise in cash holdings and the widespread use of credit lines.
4.4 Extensions

4.4.1 The Prudence Parameter

One area of concern is whether our results are robust to an alternative calibration of the prudence parameter $\phi$. In particular, our analysis assumes that the prudence parameter (set in the 1971-82 period to replicate cash holdings) remains unchanged in the 1995-06 period.

Table 6 presents the predicted cash holdings and credit line usage for two experiments to gauge the robustness of our results. In the first experiment, we set the prudence parameter $\phi$ to match cash holdings in the 1995-06 period, and re-calibrate the other parameters for both periods given this value. Consistent with previous results, matching the observed cash holdings in the last period requires a slightly lower prudence parameter of 0.0040, compared to the prudence parameter of 0.0045 set to match cash holdings during the 1971-82 period. Applying the lower prudence parameter to the 1971-82 period, we find that the model generates the same credit line usage as before, but slightly lower simulated cash holdings, at 7.4 percent of total assets. In other words, to match the cash holdings, the model requires firms to be more prudent in the 1971-82 period than in the 1995-06 period. As before, this suggests that the increase in cash holdings cannot be explained by heightened prudence over time.

TABLE 6 HERE

In a second experiment, we set the prudence parameter $\phi$ to match the volatility of payouts, and re-calibrate all other parameters. The standard deviation of payouts to assets is 0.040 in the 1971-82 period and 0.126 in the 1995-06 period. The resulting estimates of $\phi$ are 0.0005 for the 1971-82 period and 0.0039 for the 1995-06 period. For the 1971-82 period, the model predicts cash holdings of only 4.3 percent of total assets and a small 0.6 percent credit line usage. For the 1995-06 period, the model predicts realistic cash holdings of 20.5 percent of total assets and 5.1 percent of credit line usage. This calibration suggests a rise in prudence, but a rise in prudence does not square well with US experience. A rise in $\phi$ would be coherent with a more progressive tax system and more convex issuing costs. However, Piketty and Saez (2007) document that changes to the US tax system have made the federal tax system somewhat less progressive, while Chen and Ritter (2000) show that initial public offerings have become less costly over time.

4.4.2 Flexible Payout Policy

Another important area of concern is that firms in the model can only react to the mid-year shock using accumulated cash and credit lines. The choice was motivated by the greater flexibility of cash
and credit lines given the predetermined choices in payout and other policies. In line with Lintner (1956), the model recognizes that firms view cash savings as a by-product of predetermined payout policies: “savings in a given period generally are largely a by-product of dividend action taken in terms of pretty well established practices and policies; dividends are rather seldom a by-product of current decisions regarding the desired magnitude of savings as such.” More recent evidence presented in Brav, Graham, Harvey, and Michaely (2005) confirms that Lintner’s findings largely hold in the more recent period.

Nevertheless, we construct a version of the model where firms would have a more flexible payout policy. In particular, we give the firm the flexibility to adjust payouts in response to the mid-year shock. In this version, the firm chooses $K_{t+1}$, $B_{t+1}$, $S_t$ knowing an information set $\Phi_t$ that includes all state variables ($K_t$, $B_t$, $L_t$, and $M_t$) and the current realization of $z_t$, but not $f_t$. Then, the firm chooses the allocation of saving $S_t$ between payout $D_t$, cash $M_{t+1}$ and credit line loan $L_{t+1}$ knowing an information set $\Phi_{t+}$ that includes the information in $\Phi_t$, plus the new states ($K_{t+1}$, $B_{t+1}$, and $S_t$) and the realization of $f_t$.\(^{13}\)

Table 6 also reports the results of this experiment. A more flexible payout policy relaxes the firm’s need for liquidity. As a result, firms hold less cash and choose more volatile payouts. Applying the prudence parameter that matches cash holdings of the 1971-82 period, we find that the model generates lower simulated cash holdings in the 1995-06 period (18.8 percent of total assets) and no credit line usage. In other words, firms react to the mid-year shock by changing payouts. Payouts are therefore more volatile. For example, the model predicts a standard deviation for payouts to total assets of 0.179 for the 1995-06 period, compared to 0.126 in the data.

5 Conclusion

The cash hoarding behavior of US firms since the 1970s provides an interesting setting in which to evaluate the different motives for corporate liquidities. Our results document that the increase in cash holdings is mostly attributable to three economic forces. First, the return on cash holdings has increased tremendously because of lower inflation in the later period and the innovation of sweep money market accounts. Second, firms’ revenues have become more volatile, as estimated by the TFP process. Both these changes, however, cannot explain the widespread use of credit lines. It is the increased volatility in the firms’ bottom lines that is unrelated to the scale of operations, as

\(^{13}\)The details of this version of the model and the resulting parameter estimates are presented in an appendix available from the authors.
captured by the mid-year operating leverage shock, that magnifies both cash holdings and credit lines.

The results shed light on two widely acknowledged motives for holding cash. The precautionary motive (Leland, 1968) plays the same role in translating the exposure to risk into cash holdings. Our results however do not suggest that firms have become more prudent over time: firms facing various taxes and issuing costs were likely more prudent in the 1970s in terms of their payout policies than firms in the late 1990s and early 2000s. The liquidity motive (Miller and Orr, 1966) plays an increasingly important role. In particular, our results suggest that it is the higher liquidity needs that has forced firms to hold more cash and use more credit lines.

A next step for future research would be to investigate the sources of the time-varying operating leverage. It is possible that firms have endogenously chosen more risk over time. For example, firms’ decisions may have evolved to reflect more volatile advertising budgets, executive compensation, and research and development expenses. It would be helpful to investigate the possible mechanisms by which firms may have endogenously incorporated more volatility in their production and financing.

Another step would be to investigate the changes in the composition of firms over time, rather than relying on a representative firm. Further research should investigate the mechanisms that may have spurred a change in the composition of firms and how these mechanisms may be linked to cash holdings and credit line usage.

6 Appendix

6.1 The Intertemporal Problem

At the beginning of the year, the firm chooses \( D_t, K_{t+1}, B_{t+1}, \) and \( S_t \) knowing an information set \( \Phi_t \) that includes all state variables \( (K_t, B_t, L_t, \) and \( M_t) \) and the current realization of \( z_t \), but not \( f_t \). During the year, the firm chooses the allocation of \( S_t \) between \( M_{t+1} \) and \( L_{t+1} \) knowing an information set \( \Phi_{t+} \) that includes all the information in \( \Phi_t \) plus all the new relevant states \( (K_{t+1}, B_{t+1}, \) and \( S_t) \) as well as the realization of \( f_t \).

Given the firm’s choice of cash savings \( S_t \), the firm’s problem during the year consists of choosing between cash holdings and credit line. Of course, if \( z_t < \bar{z} \), the firm does not have access to the credit line. In this circumstance, the solution is \( L_{t+1} = 0 \) and \( M_{t+1} = S_t + (1 - \tau_c) f_t \). If \( z_t \geq \bar{z} \),

\[ \text{We thank an anonymous referee for suggesting this possibility.} \]
the firm solves

(21) \[ W(K_{t+1}, B_{t+1}, S_t; z_t, f_t) = \max_{\{M_{t+1}, L_{t+1}\}} \beta E_{t+1} [V(K_{t+1}, B_{t+1}, L_{t+1}, M_{t+1}; z_{t+1}, f_t)] \]

subject to

(22) \[ M_{t+1} = S_t + L_{t+1} - (1 - \tau_C) f_t \]

as well as \( M_{t+1} \geq 0, 0 \leq L_{t+1} \leq \bar{L} \), where the \( E_{t+1} \) denotes that the expectation is conditional on
the information set \( \Phi_{t+1} \).

The solution must satisfy the following first-order conditions:

(23) \[ \zeta_{t+} - \gamma^M_{t+} = \beta E_{t+} [V_M(K_{t+1}, B_{t+1}, L_{t+1}, M_{t+1}; z_{t+1}, f_t)] \]

(24) \[ \zeta_{t+} + \gamma^L_{t+} - \gamma^U_{t+} = -\beta E_{t+} [V_L(K_{t+1}, B_{t+1}, L_{t+1}, M_{t+1}; z_{t+1}, f_t)] \]

(25) \[ \gamma^M_{t+} \geq 0, \quad M_{t+1} \geq 0, \quad \gamma^M_{t+} M_{t+1} = 0 \]

(26) \[ \gamma^L_{t+} \geq 0, \quad L_{t+1} \geq 0, \quad \gamma^L_{t+} L_{t+1} = 0 \]

(27) \[ \gamma^U_{t+} \geq 0, \quad \bar{L} - L_{t+1} \geq 0, \quad \gamma^U_{t+} (\bar{L} - L_{t+1}) = 0, \]

where \( \zeta_{t+} \) is the multiplier associated with constraint (22).

At the optimum, we also have

(28) \[ W_K(K_{t+1}, B_{t+1}, S_t; z_t, f_t) = \beta E_{t+} [V_K(K_{t+1}, B_{t+1}, L_{t+1}, M_{t+1}; z_{t+1}, f_t)] \]

(29) \[ W_B(K_{t+1}, B_{t+1}, S_t; z_t, f_t) = \beta E_{t+} [V_B(K_{t+1}, B_{t+1}, L_{t+1}, M_{t+1}; z_{t+1}, f_t)] \]

(30) \[ W_S(K_{t+1}, B_{t+1}, S_t; z_t, f_t) = \zeta_{t+}. \]

At the beginning of the year, the firm’s problem is

(31) \[ V(K_t, B_t, M_t; z_t, f_{t-1}) = \max_{\{D_t, K_{t+1}, B_{t+1}, S_t\}} U(D_t) + E_t [W(K_{t+1}, B_{t+1}, S_t; z_{t+1}, f_t)] \]

subject to

(32) \[ S_t = (1 - \tau_C) (Y_t + F - \delta K_t - r B_t - \xi L_t + \iota M_t) - \Delta K_{t+1} + \Delta B_{t+1} - L_t + M_t - \Omega^K_t - \Omega^B_t - D_t \]
(33) \[ Y_t = \exp(z_t)K_t^\alpha \]

(34) \[ S_t + \bar{L}1(z_t \geq \bar{z}) - (1 - \tau_C)\sigma_f \geq 0, \]

where the \( E_t \) denotes that the expectation is conditional on the information set \( \Phi_t \), and \( \Omega^K_t \) and \( \Omega^B_t \) are as defined in equations (9) and (10).

The first-order conditions of this problem are

(35) \[ \eta_t = U'(D_t) \]

(36) \[ \eta_t - \lambda_t = E_t [W_S(K_{t+1}, B_{t+1}, S_t; z_t, f_t)] \]

(37) \[ \eta_t \left[ 1 + \omega_K \left( \frac{K_{t+1}}{K_t} - 1 \right) \right] = E_t [W_K(K_{t+1}, B_{t+1}, S_t; z_t, f_t)] \]

(38) \[ \eta_t \left[ 1 - \omega_B (B_{t+1} - \bar{B}) \right] = -E_t [W_B(K_{t+1}, B_{t+1}, S_t; z_t, f_t)] \]

(39) \[ \lambda_t \geq 0, \quad S_t + \bar{L}1(z_t \geq \bar{z}) - (1 - \tau_C)\sigma_f \geq 0, \quad \lambda_t \left[ S_t + \bar{L}1(z_t \geq \bar{z}) - (1 - \tau_C)\sigma_f \right] = 0, \]

where \( \eta_t \) is the multiplier associated with constraint (32), \( \lambda_t \) is associated with (34), and

(40) \[ V_K(K_t, B_t, L_t, M_t; z_t, f_{t-1}) = \eta_t \left[ 1 + (1 - \tau_C)(\alpha \exp(z_t)K_t^{(\alpha-1)} - \delta) + \frac{\omega_K}{2} \left[ \left( \frac{K_{t+1}}{K_t} \right)^2 - 1 \right] \right] \]

(41) \[ V_B(K_t, B_t, L_t, M_t; z_t, f_{t-1}) = -\eta_t (1 + (1 - \tau_C)r) \]

(42) \[ V_L(K_t, B_t, L_t, M_t; z_t, f_{t-1}) = -\eta_t (1 + (1 - \tau_C)\xi) \]

(43) \[ V_M(K_t, B_t, L_t, M_t; z_t, f_{t-1}) = \eta_t (1 + (1 - \tau_C)t). \]

We can write the Euler equations that describe the different decisions as

(44) \[ U'(D_t) - \lambda_t = \beta R^M E_t [U'(D_{t+1})] + E_t \left[ \gamma^M_{t+1} \right] \]

(45) \[ U'(D_t) - \lambda_t = \beta R^L E_t [U'(D_{t+1})] - E_t \left[ \gamma^L_{t+1} - \gamma^U_{t+1} \right] \]

(46) \[ U'(D_t) = \beta R^B_t E_t [U'(D_{t+1})] \]

and

(47) \[ U'(D_t) = \beta E_t \left[ R^K_{t+1} U'(D_{t+1}) \right], \]

where \( R^M = 1 + (1 - \tau_C)t \), \( R^L = 1 + (1 - \tau_C)\xi \), \( R^B = [1 + (1 - \tau_C)r] / [1 - \omega_B (B_{t+1} - \bar{B})] \) and

\[ R^K_{t+1} = \left( 1 + (1 - \tau_C) \left[ \alpha \exp(z_{t+1})K_{t+1}^{(\alpha-1)} - \delta \right] + \frac{\omega_K}{2} \left[ \left( \frac{K_{t+2}}{K_{t+1}} \right)^2 - 1 \right] \right) / \left[ 1 + \omega_K \left( \frac{K_{t+1}}{K_t} - 1 \right) \right]. \]
6.2 Proofs

Proof of Proposition 2

We first rewrite the first-order conditions of the mid-year problem as follows.

If \( z_t < \bar{z} \), the solution is simply that \( L_{t+1} = 0 \) and \( M_{t+1} = S_t - (1 - \tau_C) f_t \geq 0 \).

If \( z_t \geq \bar{z} \), the solution must satisfy the following first-order conditions:

\[
\begin{align*}
(48) & \quad \zeta_t^+ - \gamma_t^M = \beta R^M E_t^+ [\eta_{t+1}] \\
(49) & \quad \zeta_t^+ + \gamma_t^L - \gamma_t^U = \beta R^L E_t^+ [\eta_{t+1}] \\
(50) & \quad \gamma_t^M \geq 0, \quad M_{t+1} \geq 0, \quad \gamma_t^M M_{t+1} = 0 \\
(51) & \quad \gamma_t^L \geq 0, \quad L_{t+1} \geq 0, \quad \gamma_t^L L_{t+1} = 0 \\
(52) & \quad \gamma_t^U \geq 0, \quad \bar{L} - L_{t+1} \geq 0, \quad \gamma_t^U (\bar{L} - L_{t+1}) = 0.
\end{align*}
\]

The first two conditions impose that

\[
\begin{align*}
(53) & \quad \gamma_t^L - \gamma_t^U + \gamma_t^M = \beta (R^L - R^M) E_t^+ [\eta_{t+1}] > 0
\end{align*}
\]

because \( R^L > R^M \) and \( \eta_t = U'(D_t) > 0 \).

We note that the constraints on the credit line are such that

If \( \gamma_t^L > 0 \), then \( L_{t+1} = 0 \) and \( \gamma_t^U = 0 \)

If \( \gamma_t^U > 0 \), then \( L_{t+1} = \bar{L} \) and \( \gamma_t^L = 0 \)

If \( \gamma_t^L = \gamma_t^U = 0 \), then \( 0 \leq L_{t+1} \leq \bar{L} \).

Then, the constraints and equation (53) imply that

If \( \gamma_t^M > 0 \) and \( \gamma_t^L > 0 \) and \( \gamma_t^U = 0 \), then \( M_{t+1} = L_{t+1} = 0 \)

If \( \gamma_t^L > 0 \), \( \gamma_t^L = 0 \), and \( \gamma_t^U = 0 \), then \( M_{t+1} = 0 \) and \( 0 \leq L_{t+1} \leq \bar{L} \)

If \( \gamma_t^M > 0 \), \( \gamma_t^L = 0 \), and \( \gamma_t^U > 0 \), then \( M_{t+1} = 0 \) and \( L_{t+1} = \bar{L} \)

If \( \gamma_t^M = 0 \) then \( \gamma_t^L > 0 \), \( \gamma_t^U = 0 \), then \( M_{t+1} \geq 0 \), and \( L_{t+1} = 0 \).
These resulting cases suggest that we can only have $M_{t+1} \geq 0$ and $L_{t+1} = 0$, or $M_{t+1} = 0$ and $0 < L_{t+1} \leq \bar{L}$.

**Proof of Proposition 3**

The relevant set of equations here is

(54) \[ M_{t+1} = L_{t+1} + S_t - (1 - \tau_C) f_t \]

(55) \[ S_t \geq -\bar{L}1(z_t \geq \tilde{z}) + (1 - \tau_C) \sigma_f. \]

To obtain $M_{t+1} > 0$:

If $z_t < \tilde{z}$, then $M_{t+1} = S_t - (1 - \tau_C) f_t \geq (1 - \tau_C) (\sigma_f - f_t) \geq 0$

1. $M_{t+1} > 0$ ($L_{t+1} = 0$) when $f_t < \sigma_f$ for all values of $S_t \geq (1 - \tau_C) \sigma_f$
2. $M_{t+1} > 0$ ($L_{t+1} = 0$) when $S_t > (1 - \tau_C) \sigma_f$ for all values of $f_t$.

If $z_t \geq \tilde{z}$, then $M_{t+1} = S_t + L_{t+1} - (1 - \tau_C) f_t \geq L_{t+1} - \bar{L} + (1 - \tau_C) (\sigma_f - f_t)$

1. $M_{t+1} > 0$ ($L_{t+1} = 0$) when $f_t < \sigma_f - \bar{L}/(1 - \tau_C)$ for all values of $S_t \geq -\bar{L} + (1 - \tau_C) \sigma_f$
2. $M_{t+1} > 0$ ($L_{t+1} = 0$) when $S_t > (1 - \tau_C) \sigma_f$ for all values of $f_t$.

**Proof of Proposition 4**

Proposition 3 shows that high cash savings, $S_t > (1 - \tau_C) \sigma_f$, ensure $M_{t+1} > 0$ and $L_{t+1} = 0$. This requires that $\lambda_t = \gamma_{t+1}^M = 0$. The relevant Euler equations (in terms of returns are):

(56) \[ E_t [m_{t+1}] R^M_t = 1 - \{E_t \left[ \gamma_{t+1}^M \right] + \lambda_t \} / U'(D_t) \]

(57) \[ E_t [m_{t+1}] R^B_t = 1 \]

and

(58) \[ E_t [m_{t+1} R^K_{t+1}] = E_t [m_{t+1}] E_t [R^K_{t+1}] + \text{Cov}_t [m_{t+1}, R^K_{t+1}] = 1. \]

A comparison of these equations yields

(59) \[ \frac{E_t \left[ \gamma_{t+1}^M \right] + \lambda_t}{U'(D_t) E_t [m_{t+1}]} = R^B_t - R^M = E_t [R^K_{t+1}] + \frac{\text{Cov}_t [m_{t+1}, R^K_{t+1}]}{E_t [m_{t+1}]} - R^M. \]
When $R^M = R^B_t = E_t \left[ R^K_{t+1} \right] + \text{Cov}_t \left[ m_{t+1}, R^K_{t+1} \right] / E_t \left[ m_{t+1} \right]$, the firm is indifferent between holding cash, debt, and capital. As a result, $E_t \left[ \gamma^M_t \right] + \lambda_t = 0$ which requires both $\gamma^M_t = 0$ for all values of $f_t$ and $\lambda_t = 0$. According to Proposition 2, $\gamma^M_t = 0$ implies $\gamma^L_t > 0$ and the firm does not use its credit line $L_{t+1} = 0$. To ensure this, the firm must choose a high level of liquidity $S_t \geq (1 - \tau_C)\sigma_f$ to ensure that $L_{t+1} = 0$ for all values of $f_t$.

If the first requirement holds, the Euler equation for cash can be rewritten as

$$(60) \quad U'(D_t) = \beta R^M E_t \left[ U'(D_{t+1}) \right].$$

Note that $U'(D_t) = \beta R^M E_t \left[ U'(D_{t+1}) \right] < E_t \left[ U'(D_{t+1}) \right]$ because $\beta R^M < 1$. The last inequality requires that $U'(\cdot)$ be convex. If it is sufficiently convex, then the firm can choose $S_t > (1 - \tau_C)\sigma_f$ and ensure that $M_{t+1} > 0$. In our model, the condition $E_t \left[ U'(D_{t+1}) \right] > U'(D_t)$ is possible because our assumptions on the schedule of taxes and equity issuing costs $T(D_t)$ imply that $U'(D_t)$ is convex.

### 6.3 Numerical Method

The model is solved numerically using a finite element method as in Coleman’s (1990) algorithm. The policy functions $K_{t+1}$, $M_{t+1}$, $B_{t+1}$, $L_{t+1}$, and co-states $\lambda_t$, $V_t$ are approximated by piecewise linear interpolants of the state variables $K_t$, $M_t$, $B_t$, $L_t$, as well as $z_t$ and $f_t$. The numerical integration involved in computing expectations is approximated with a Gauss-Hermite quadrature rule with two quadrature nodes.

This state space grid consists of 3125 uniformly spaced points for the beginning-of-the-year state variables. The lowest and highest grid points for the endogenous state variables $K_t$, $M_t$, $B_t$, and $L_t$ are specified outside the endogenous choices of the firm. The lowest and highest grid points for the income shock $z_t$ are specified three standard deviations away, at $\exp(-3\sigma_\epsilon)$ and $\exp(3\sigma_\epsilon)$.

The approximation coefficients of the piecewise linear interpolants are chosen by collocation, i.e., to satisfy the relevant system of equations at all grid points. The approximated policy interpolants are substituted in the equations, and the coefficients are chosen so that the residuals are set to zero at all grid points. The time-stepping algorithm is used to find these root coefficients. Given initial coefficient values for all grid points, the time-stepping algorithm finds the optimal coefficients that minimize the residuals at one grid point, taking coefficients at other grid points as given. In turn, optimal coefficients for all grid points are determined. The iteration over coefficients stops when the maximum deviation of optimal coefficients from their previous values is lower than a specified tolerance level, e.g., 0.0001.
6.4 Calibration by Simulation

We calibrate the parameters using a procedure similar to Ingram and Lee (1991). We compute moments in the data and in the simulation as

\[
\bar{H}(x) = \frac{1}{F} \sum_{f=1}^{F} \left[ \frac{1}{T} \sum_{t=1}^{T} h(x_{f,t}) \right]
\]

and

\[
\bar{H}_s(\theta) = \frac{1}{F} \sum_{f=1}^{F} \left[ \frac{1}{T} \sum_{t=1}^{T} h(x_{s,f,t}(\theta)) \right],
\]

where \(\bar{H}(x)\) is an \(\tilde{m}\)-vector of statistics computed on the actual data matrix \(x\) and \(\bar{H}_s(\theta)\) is an \(\tilde{m}\)-vector of statistics computed on the simulated data for panel \(s\). The simulated statistics depend on the \(k\)-vector of parameters \(\theta\). We use these statistics to construct the \(m < \tilde{m}\) moments \(H(x)\) and \(H_s(\theta)\) on which the estimation is based. The estimator \(\hat{\theta}\) of \(\theta\) is the solution to

\[
\min_{\theta} \left( H(x) - \frac{1}{S} \sum_{s=1}^{S} H_s(\theta) \right)^{\top} W \left( H(x) - \frac{1}{S} \sum_{s=1}^{S} H_s(\theta) \right)
\]

where \(W\) is a positive definite weighting matrix.

For the first period, we compute \(\tilde{m} = 9\) statistics to form the \(m = 7\) targeted moments and identify the \(k = 7\) parameters of the first period (see Table 3). For the last period, we compute \(\tilde{m} = 8\) statistics to form the \(m = 6\) targeted moments to identify the \(k = 6\) parameters of the last period (see Table 4). We construct simulated samples that have the same number of firms as the data. The actual data sample contains 2,093 firms for the first period and 4,526 firms for the second sample. To replicate the data, the simulated samples contain \(F = 2,093\) firms for the first period and \(F = 4,526\) firms for the last period. In practice, we simulate 50 years, but keep only the last \(T = 10\) years. In both periods, we construct \(S = 5\) simulated panels. We use an identity weighting matrix as in Cochrane (1996) and Carlson et al. (2004).

6.5 Parameters Estimated from the Data

Table 2 presents the first set of parameter estimates for both periods. The capital intensity \(\alpha\), the persistence of the TFP innovations \(\rho_z\), and their volatility \(\sigma_z\) are estimated from equation (3) and the autoregressive process (4). For each time periods, we use a panel regression specification that includes time fixed effects to capture productivity variations in the aggregate economy, as in Gourio and Miao (2010). In addition, we include firm fixed effects to capture productivity variation between firms. Revenues \(Y_t\) are measured as sales, and the beginning-of-the-year capital stock \(K_t\) is measured as lagged property, plant, and equipment.\(^{15}\)

\(^{15}\)As an alternative, the capital stock could be reconstructed from the accumulation equation (8) using capital expenditures (CAPX) assuming an initial value for the capital stock and a value for the depreciation rate. We do
For the 1971-82 period, the estimates are \( \alpha = 0.695 \), \( \rho_z = 0.490 \), and \( \sigma_z = 0.250 \). For the 1995-06 period, the estimates are \( \alpha = 0.521 \), \( \rho_z = 0.461 \) and \( \sigma_z = 0.441 \). The values for \( \alpha \) are in line with the values used in Moyen (2004), Hennessy and Whited (2005, 2007), and Gamba and Triantis (2008). The values for \( \rho_z \) are smaller than those in Hennessy and Whited (2007) and Gourio and Miao (2010) likely due to the firm fixed effects we include in the estimation. Over a short sample time period, the firm fixed effect might soak up some firm dynamics explaining our lower persistence. The low persistence is well within the range of other studies who include firm fixed effects, including Cooper and Ejarque (2003) with \( \rho = 0.11 \). Overall, the estimates of the stochastic process indicate a threefold increase in the unconditional variance of TFP, \( \sigma^2_z/(1 - \rho^2_z) \), from 8.22 percent during the 1971-82 period to 24.70 percent during the 1995-06 period.

The corporate tax rate \( \tau_C \) is set to the top marginal rate. The top marginal tax rate was 48 percent from 1971 to 1978 and 46 percent from 1979 to 1982. The top corporate marginal tax rate has been constant at 35 percent from 1993 until the end of our sample in 2006. As a result, the corporate tax rate is set to its twelve year average of \( \tau_C = 0.473 \) for the first period and to \( \tau_C = 0.35 \) for the last period. The personal tax rates are set to the average marginal tax rates reported in NBER’s TAXSIM. Over the 1971-82 period, the marginal interest income tax rate averaged \( \tau_r = 0.276 \) while the marginal dividend tax rate averaged \( \tau_D = 0.395 \). Over the 1995-06 period, the marginal interest income tax rate averaged \( \tau_r = 0.244 \) while the marginal dividend tax rate averaged \( \tau_D = 0.233 \).

The real interest rate \( r \) is set to the average of the monthly annualized t-bill rate deflated by the consumer price index. High inflation characterized much of the 1971-82 period. As a result, the real interest rate was quite low, at 0.585 percent. As for the 1995-06 period, the real interest rate was higher, at 1.609 percent. For the interest rate earned on cash holdings \( \iota \), we disentangle the two components of cash (CHE): short-term investments (IVST) and cash (CH). In 1971-82, firms held 30.7 percent of their cash in short-term investments earning a rate of return \( r \) and 69.3 percent in cash earning a zero nominal interest rate. Given an average inflation rate of 7.9 percent, the interest rate on cash holdings is set to \( \iota = 0.307r + 0.693(0.079) = -0.053 \). By 1995, sweep money market accounts became available to firms. Therefore we calibrate the interest rate on cash holdings as the risk free rate minus 100 basis points for the expense ratio and FDIC rate, except for a minimum balance of eight percent, inspired by the Basel requirement, that depreciates at the rate of account balances.

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not pursue this alternative approach for two reasons. First, it requires a value for the depreciation rate, which would prevent our estimation of the depreciation rate. Second, our results obtained by measuring the capital stock as lagged property, plant, and equipment yield parameter estimates similar to those obtained elsewhere in the literature.

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inflation rate of 2.6 percent: \( \iota = 0.92(r - 0.01) + 0.08(0 - 0.026) = 0.00352 \). Finally, for the interest rate on the credit line, we apply a premium above the real interest rate. For the 1971-82 period, we use Ham and Melnik (1987) and set \( \xi = r + 0.008 \). For the 1995-06 period, we use Sufi (2009) and set \( \xi = r + 0.015 \).

6.6 Parameters Calibrated by Matching Moments

The last set of parameters is calibrated through the simulated method of moments. In spirit, the strategy targets a particular moment for each parameter. In practice, a change to one parameter affects all simulated moments.

Two moments of the capital policy are targeted to calibrate the depreciation rate \( \delta \) and the adjustment cost parameter \( \omega_K \). The depreciation rate \( \delta \) ensures that the average investment-to-total assets simulated from the model matches the average investment found in the data. In the COMPUSTAT data, the ratio is computed as capital expenditures (CAPX) divided by total assets (AT). In the model simulated data, the ratio is computed as investment \( I_t \) divided by total assets \( A_t \). The adjustment cost parameter \( \omega_K \) ensures that the simulated standard deviation of investment \( I_t/A_t \) normalized by the standard deviation of revenues \( Y_t/A_t \) matches that of the data. We normalize by the standard deviation of revenues so that the capital adjustment cost \( \omega_K \) can target the volatility of investment in reference to the volatility of the TFP innovations \( \sigma_z \).

Two moments of debt policy are targeted to calibrate the long-run debt level \( \bar{B} \) and the cost parameter \( \omega_B \). The value of \( \bar{B} \) ensures that the simulated average leverage \( B_t/A_t \) matches that of COMPUSTAT firms. Leverage is measured by the sum of long-term debt (DLTT) and debt in current liabilities (DLC) divided by total assets. The value of \( \omega_B \) ensures that the simulated standard deviation of debt relative to the standard deviation of revenues matches that of the observed data. Because costs are more relevant to long-term debt than to short-term debt, we focus on the standard deviation of long-term debt-to-capital stock. This standard deviation is then normalized by the standard deviation of revenues-to-capital stock.

The average mid-year shock level \( \bar{F} \) ensures that the average of operating income-to-total assets ratio \( OI_t/A_t \) matches the data, where operating income \( OI_t \) is measured before depreciation (OIBDP). The value of \( \sigma_f \) ensures that the standard deviation of net income-to-total assets \( NI_t/A_t \) matches the data. We target net income because we want to allow for special items: expenses that may not be part of the regular operations of the firm but that can affect the firm’s financial health.

Finally, we calibrate the coefficient of absolute prudence \( \phi \) to ensure that the average of cash
holdings-to-total assets $M_{t+1}/A_t$ matches the data in the 1971-82 period, where cash holdings are measured by cash and short-term investments (CHE).

Tables 3 and 4 present the results of the moment matching exercise. Table 3 shows the parameter values and the target moments for the 1971-82 period, while Table 4 does so for the 1995-06 period.

In the data, the average investment-to-total assets is 9.6 percent in the first time period and 6.5 percent in the last time period. To hit these moments, the depreciation rate $\delta$ is set to 14.9 percent in the first time period and to 10 percent in the last time period. In COMPUSTAT data, investment has an average standard deviation of 20.2 percent of the average standard deviation of revenues during the 1971-82 period and a relative average standard deviation of 14.5 percent during the 1995-06 period. To replicate these moments, the capital adjustment cost $\omega_K$ is set to 1.864 in the first time period and to 1.844 in the last time period. These values are of magnitudes similar to those obtained by Cooper and Haltiwanger (2006).

The average leverage of COMPUSTAT firms has decreased over time: 28.6 percent during the 1971-82 period and 21.9 percent during the 1995-06 period. To replicate these moments, the long-run debt level (standardized by mean total assets) $\bar{B}/\bar{A}$ is set to 0.287 in the first time period and to 0.220 in the last time period. The long-term debt-to-capital stock of COMPUSTAT firms has an average standard deviation of 17.7 percent of the average standard deviation of revenues-to-capital stock during the 1971-82 years and a relative average standard deviation of 23.0 percent during the 1995-06 years. To replicate these moments, the debt cost $\omega_B$ values are 0.007 in both periods.

In COMPUSTAT data, operating income has declined from an average of 15.5 percent of total assets during the 1971-82 period to an average of 0.6 percent of total assets during the 1995-06 period. This represents a large reduction in operating income for the average firm in our sample. A much higher average mid-year shock is required to explain the reduction in the average operating income over time. The average mid-year shock level (standardized by mean total assets) $\bar{F}/\bar{A}$ is set to $-0.012$ (a mean inflow) in the first period and to 0.149 (a mean outflow) in the last period. In the data, the standard deviation of net income-to-total assets has greatly increased over time from an average of 0.046 during the 1971-82 years to an average of 0.180 during the 1995-06 years. The volatility parameter (standardized by mean total assets) $\sigma_f/\bar{A}$ is set to 0.081 in the first period and to 0.273 in the last period.

$^16$The relative standard deviation computed in the data differs considerably from the relative standard deviation commonly shown using macroeconomic data. This simply results from different measurements. For example, we compute the standard deviation of the ratio of investment-to-total assets, while macroeconomists might compute the standard deviation of the logarithm of investment detrended using the Hodrick-Prescott filter.
7 References


### Table 1

**Standard Deviations of Net Income Components**

**Panel A: Average Standard Deviations for Components of Net Income**

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1971-82</td>
<td>0.046</td>
<td>0.073</td>
<td>0.012</td>
<td>0.011</td>
<td>0.028</td>
<td>0.000</td>
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<tr>
<td>1995-06</td>
<td>0.180</td>
<td>0.134</td>
<td>0.016</td>
<td>0.048</td>
<td>0.012</td>
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**Panel B: Average Standard Deviations for Components of Operating Income**

<table>
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<th>Sales</th>
<th>Operating Expenses</th>
<th>Depreciation</th>
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<tbody>
<tr>
<td>1971-82</td>
<td>0.239</td>
<td>0.224</td>
<td>0.008</td>
</tr>
<tr>
<td>1995-06</td>
<td>0.271</td>
<td>0.304</td>
<td>0.017</td>
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**Panel C: Average Standard Deviations for Components of Operating Expenses**

<table>
<thead>
<tr>
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<th>COGS</th>
<th>XSGA</th>
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<tr>
<td>1971-82</td>
<td>0.184</td>
<td>0.055</td>
</tr>
<tr>
<td>1995-06</td>
<td>0.193</td>
<td>0.114</td>
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</table>

Note: The data comes from the North American COMPUSTAT file and covers the period from 1971 to 2006 excluding the crisis period, where we focus on the first third of the sample period from 1971 to 1982 and the last third from 1995 to 2006. The COMPUSTAT sample includes firm-year observations with positive values for total assets (COMPUSTAT Mnemonic AT), property, plant, and equipment (PPENT), and sales (SALE) from all industries except utilities and financials, with at least five years of consecutive data. The data is winsorized to limit the influence of outliers at the 1 percent and 99 percent tails. COGS refers to Cost of Goods Sold and XSGA refers to Selling, General and Administrative Expenses. We compute the standard deviation of a firm’s time series (standardized by the firm’s total assets), then average over all firms for each sub-sample.
### Table 2
Parameter Estimates of the Calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>1971-82 Period</th>
<th>1995-06 Period</th>
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<tbody>
<tr>
<td>Revenues</td>
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<tr>
<td>$\alpha$</td>
<td>0.695</td>
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<td>$\rho_z$</td>
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<td>$\sigma_z$</td>
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<td>Tax Rates</td>
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<tr>
<td>$\tau_C$</td>
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<td>$\tau_r$</td>
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<td>$\tau_D$</td>
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<td>0.233</td>
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<td>$r$</td>
<td>0.585</td>
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<td>$\iota$</td>
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<tr>
<td>$\xi$</td>
<td>0.593</td>
<td>1.624</td>
</tr>
</tbody>
</table>

Note: The parameter estimates are based on North American data from COMPUSTAT for the sample periods 1971 to 1982 and 1995 to 2006. The COMPUSTAT samples include firm-year observations with positive values for total assets (COMPUSTAT Mnemonic AT), property, plant, and equipment (PPENT), and sales (SALE). The sample includes firms from all industries except for utilities and financials, with at least five years of consecutive data. The data are winsorized to limit the influence of outliers at the 1% and 99% tails. For each of the two time periods, we estimate the parameters of the revenue function using a panel of firm-year observations with fixed firm and year effects. The corporate tax rates are calibrated to the top marginal rate, while the personal tax rates are calibrated to the average marginal tax rates reported in NBER’s TAXSIM. The real interest rates are calibrated to the average of the monthly annualized t-bill rate deflated by the consumer price index. For the 1971-82 period, the interest rate earned on cash holdings is calibrated as the proportion of cash held in short-term investments (CHE) which earns the real interest rate, plus the proportion held in cash (CH) which earns a zero nominal interest rate deflated by the consumer price index. For the 1995-06 period, the interest rate earned on cash holdings is calibrated as the real interest rate minus 100 basis points of the expense ratio and FDIC rate, except for a minimum balance that earns a zero nominal interest rate deflated by the consumer price index. The interest rates paid on credit line debt are calibrated as the real interest rate plus a credit line premium.
Table 3
Matching Moments for the 1971-1982 Period

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Targeted Moments</th>
<th>Calibrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.149</td>
<td>Mean($I/A$)</td>
<td>0.096</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_K$</td>
<td>1.864</td>
<td>SD($I/A$)/SD($Y/A$)</td>
<td>0.202</td>
</tr>
<tr>
<td></td>
<td>(1.016)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{B}/\bar{A}$</td>
<td>0.287</td>
<td>Mean($B'/A$)</td>
<td>0.287</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_B$</td>
<td>0.007</td>
<td>SD($B'/K'/Y/K')$</td>
<td>0.177</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{F}/\bar{A}$</td>
<td>-0.012</td>
<td>Mean($OI/A$)</td>
<td>0.155</td>
</tr>
<tr>
<td></td>
<td>(0.100)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_f/\bar{A}$</td>
<td>0.081</td>
<td>SD($NI/A$)</td>
<td>0.046</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.0045</td>
<td>Mean($M'/A$)</td>
<td>0.079</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other Moments</th>
<th>Simulated</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean($L^+/A$)</td>
<td>0.009</td>
<td>n.a.</td>
</tr>
<tr>
<td>SD($D/A$)</td>
<td>0.020</td>
<td>0.040</td>
</tr>
<tr>
<td>Corr($\Delta B'/A, Y/A$)</td>
<td>-0.291</td>
<td>-0.267</td>
</tr>
</tbody>
</table>

Note: The observed moments are computed using a sample of North American data from COMPU-STAT for the sample period 1971 to 1982. The simulated moments are computed using 5 simulated panels of 2,093 firms over 10 years. $I$ denotes investment, $A$ total assets, $Y$ revenues, $B$ debt level, $K$ capital stock, $OI$ operating income, $NI$ net income, $M$ cash holdings, $L^+$ credit line used, $D$ dividends, $\Delta B'$ debt issues, and primed variables refer to time $t+1$ values rather than time $t$ values. The model is solved using a finite-element method. The parameters are estimated using a just identified system of moment matching. The numbers in parenthesis are the standard deviations of the estimated parameters.
Table 4
Matching Moments for the 1995-2006 Period

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Targeted Moments</th>
<th>Calibrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.100</td>
<td>Mean($I/A$)</td>
<td>0.065</td>
</tr>
<tr>
<td></td>
<td>(0.452)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_K$</td>
<td>1.844</td>
<td>SD($I/A$)/SD($Y/A$)</td>
<td>0.145</td>
</tr>
<tr>
<td></td>
<td>(1.724)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{B}/\tilde{A}$</td>
<td>0.220</td>
<td>Mean($B'/A$)</td>
<td>0.220</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_B$</td>
<td>0.007</td>
<td>SD($B'/K'$)/SD($Y/K'$)</td>
<td>0.230</td>
</tr>
<tr>
<td></td>
<td>(0.059)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{F}/\tilde{A}$</td>
<td>0.149</td>
<td>Mean($OI/A$)</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(0.519)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_f/\tilde{A}$</td>
<td>0.273</td>
<td>SD($NI/A$)</td>
<td>0.180</td>
</tr>
<tr>
<td></td>
<td>(0.134)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other Moments</th>
<th>Simulated</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean($M'/A$)</td>
<td>0.218</td>
<td>0.206</td>
</tr>
<tr>
<td>Mean($L^+/A$)</td>
<td>0.050</td>
<td>0.047</td>
</tr>
<tr>
<td>SD($D/A$)</td>
<td>0.116</td>
<td>0.126</td>
</tr>
<tr>
<td>Corr($\Delta B'/A, Y/A$)</td>
<td>$-0.049$</td>
<td>$-0.191$</td>
</tr>
</tbody>
</table>

Note: The observed moments are computed using a sample of North American data from COMPUSTAT for the sample period 1995 to 2006, except for the used line of credit which is taken from Sufi (2009). The simulated moments are computed using 5 simulated panels of 4,526 firms over 10 years. $I$ denotes investment, $A$ total assets, $Y$ revenues, $B$ debt level, $K$ capital stock, $OI$ operating income, $NI$ net income, $M$ cash holdings, $L^+$ credit line used, $D$ dividends, $\Delta B'$ debt issues, and primed variables refer to time $t+1$ values rather than time $t$ values. The model is solved using a finite-element method. The parameters are estimated using a just identified system of moment matching. The numbers in parenthesis are the standard deviations of the estimated parameters.
### Table 5
Sensitivity Analysis

<table>
<thead>
<tr>
<th>Predicted Moments</th>
<th>Mean($M'/A$)</th>
<th>Mean($L^+/A$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark Calibration</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1971-82</td>
<td>0.079</td>
<td>0.009</td>
</tr>
<tr>
<td>1995-06</td>
<td>0.218</td>
<td>0.050</td>
</tr>
<tr>
<td><strong>Liquidity Policy Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>interest rate on cash (%)</td>
<td>$\iota$</td>
<td>0.251</td>
</tr>
<tr>
<td>interest rate on debt (%)</td>
<td>$r$</td>
<td>0.083</td>
</tr>
<tr>
<td>interest rate on credit line (%)</td>
<td>$\xi$</td>
<td>0.096</td>
</tr>
<tr>
<td>corporate tax rate</td>
<td>$\tau_C$</td>
<td>0.085</td>
</tr>
<tr>
<td>interest income tax rate</td>
<td>$\tau_r$</td>
<td>0.078</td>
</tr>
<tr>
<td>dividend tax rate</td>
<td>$\tau_D$</td>
<td>0.053</td>
</tr>
<tr>
<td>mid-year shock average</td>
<td>$\bar{F}/\bar{A}$</td>
<td>0.081</td>
</tr>
<tr>
<td>mid-year shock volatility</td>
<td>$\sigma_f/\bar{A}$</td>
<td>0.177</td>
</tr>
<tr>
<td><strong>Debt Policy Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>debt adjustment cost</td>
<td>$\omega_B$</td>
<td>0.078</td>
</tr>
<tr>
<td>debt target</td>
<td>$\bar{B}/\bar{A}$</td>
<td>0.079</td>
</tr>
<tr>
<td><strong>Capital Policy Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>depreciation rate</td>
<td>$\delta$</td>
<td>0.095</td>
</tr>
<tr>
<td>capital adjustment cost</td>
<td>$\omega_K$</td>
<td>0.078</td>
</tr>
<tr>
<td>capital intensity</td>
<td>$\alpha$</td>
<td>0.032</td>
</tr>
<tr>
<td>TFP persistence</td>
<td>$\rho_z$</td>
<td>0.079</td>
</tr>
<tr>
<td>TFP volatility</td>
<td>$\sigma_z$</td>
<td>0.220</td>
</tr>
</tbody>
</table>

Note: The simulated moments are computed using 5 simulated panels of 4,526 firms over 10 years.
For each parameter, we report the cash holdings and credit line usage obtained from changing the first period parameter value to its second period value, holding all other parameters constant.
Table 6
Extensions

<table>
<thead>
<tr>
<th></th>
<th>Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean($M'/A$)</td>
</tr>
<tr>
<td><strong>Observed Moments</strong></td>
<td></td>
</tr>
<tr>
<td>1971-82</td>
<td>0.079</td>
</tr>
<tr>
<td>1995-06</td>
<td>0.206</td>
</tr>
<tr>
<td><strong>Predicted Moments</strong></td>
<td></td>
</tr>
<tr>
<td>Setting $\phi$ to Match 1995-06 Cash Holdings</td>
<td></td>
</tr>
<tr>
<td>1971-82</td>
<td>0.074</td>
</tr>
<tr>
<td>1995-06</td>
<td>0.206</td>
</tr>
<tr>
<td>Setting $\phi$ to Match Payout Volatility</td>
<td></td>
</tr>
<tr>
<td>1971-82</td>
<td>0.043</td>
</tr>
<tr>
<td>1995-06</td>
<td>0.205</td>
</tr>
<tr>
<td>Flexible Payout Policy</td>
<td></td>
</tr>
<tr>
<td>1971-82</td>
<td>0.079</td>
</tr>
<tr>
<td>1995-06</td>
<td>0.188</td>
</tr>
</tbody>
</table>

Note: The observed moments are the same as those reported in Tables 3 and 4. For predicted moments, the experiment under “Setting $\phi$ to Match 1995-06 Cash Holdings” entails setting the prudence parameter $\phi$ to match the 1995-06 cash holdings, and we use this value to predict cash holdings and credit line usage for 1971-82. The experiment under “Setting $\phi$ to Match Payout Volatility” entails setting $\phi$ in each period to match the observed payout volatility. Finally, the experiment under “Flexible Payout Policy” uses the alternative timing model where dividends are chosen after observing the mid-year shock. For all experiments, the other parameters are estimated using a just identified system of moment matching.