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Abstract: The Unit Commitment Problem --- one that is of great interest to energy providers --- consists of finding an energy production plan in order to meet the forecasted demand. Unlike most of the existing solution methods, we adopt a "robust" approach that takes into account the uncertainty around the forecasted demand. Performance is measured by estimating the savings achieved by the robust method over the typical approach that ignores uncertainty and solves a deterministic optimization model with expected values. Our experiments verify the benefits of modeling uncertainty in key data by considering ranges instead of point estimates and by capturing risk with a minimax and a min-max regret function.

***Research Highlights**

Highlight 1

We investigate the benefits of robust approaches for the Unit Commitment Problem.

Highlight 2

We test the minimax and the minimax regret criteria with Demand and Price ranges.

Highlight 3

A Bender Decomposition algorithm is proposed and computationally tested

Highlight 3

We test different levels of forecast quality and robustness

Highlight 5

Our results suggest our method leads to significant savings versus current approaches

Minimax Robust Unit Commitment Problem with Demand and Market Price uncertainty

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Abstract

The Unit Commitment Problem — one that is of great interest to energy providers — consists of finding an energy production plan in order to meet the forecasted demand. Unlike most of the existing solution methods, we adopt a ”robust” approach that takes into account the uncertainty around the forecasted demand. Performance is measured by estimating the savings achieved by the robust method over the typical approach that ignores uncertainty and solves a deterministic optimization model with expected values. Our experiments verify the benefits of modeling uncertainty in key data by considering ranges instead of point estimates and by capturing risk with a minimax and a min-max regret function.

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1. Introduction

Given a demand for energy and a set of power plants, the Unit Commitment (UC) problem consists of finding the optimal energy production plan that meets the demand while minimizing the production cost. Typically, a solution of the UC provides a production plan for the next day and involves two sets of managerial decisions. The first is the selection of power plants to be used and the level of power production; the second is the amount of energy to buy and sell in the market. While planning the activity of the power units must be done in advance, buying needed energy or selling surplus energy is done in real time, once the demand for energy is known. However, when the deterministic version of the UC problem is solved, both aspects are taken into consideration at the same time by assuming the forecast of the energy demand and the sell/purchase prices as certain. That is, the deterministic UC problem does not consider forecast errors. One way of taking into account the uncertainty associated with demand and prices is to assume that the unknown value is contained within a range. We approach the UC problem in the framework of robust optimization by modeling the uncertainty of demand and prices with ranges.

Scenario-based robust optimization has received a fair amount of attention in recent years. For the basic concepts we refer the reader to [1] and [2]. Uncertainty in key data has been typically modeled in two ways: with the use of scenarios and with ranges [3]. In the first case, uncertainty is represented by a set of scenarios, with each one consisting of a combination of values for the uncertain parameters. In the second case, each uncertain value is replaced by a range of possible values. In either case, decisions can be made according to several criteria, including minimax and minimax regret ([4], [5]).

The goal of the minimax criterion, which is associated with the most conservative approach, is to obtain a solution that will result in the best performance under the worst possible scenario. The goal of the minimax regret criterion, on the other hand, is to obtain a solution that minimizes the maximum difference between the values of the solution that will be adopted and that of the optimal solution that could be obtained with perfect information. Both of these criteria assume that the value of the objective function must be minimized.

The mathematical definition of these problems is as follows. Let the set of feasible solutions be $x \in X$ and the set of scenarios (i.e., realizations of the unknown parameters within the given data intervals) be $k \in \Gamma$, then the minimax criterion is:

$$\tau_{MM} := min_{x \in X} \; max_{k \in \Gamma} \left(F_k(x) \right) \tag{1}$$

where $F_k(x)$ is the objective function value (total cost in the UC problem) of the solution computed under scenario k. Let F_k^* be the optimal solution value under scenario k. Then we can define the minimax regret criterion as:

$$\tau_{MMR} := min_{x \in X} \; max_{k \in \Gamma} \left(F_k(x) - F_k^* \right) \tag{2}$$

Further details on the minimax and minimax regret models may be found in [5]. There are other relevant types of robust optimization models, whose details can be found in [6] and [7].

The literature on the Unit Commitment Problem problem is equally vast. Existing mathematical models share a common structure but have a number of differences in the functional form of the objective function and the set of constraints. We refer the reader to [8] for a complete survey of this literature. Production cost has been modeled as both a quadratic function [9] and a linear function [10].

The linear model speeds up the execution of the solution method and facilitates the use of the robustness strategies. Models have been enriched with an increasing number of realistic constraints, such as ”ramping constraints” [9], maximum production constraints, and others [11]. Solution methods include both exact and heuristic approaches ([12], [11] and [13]).

To the best of our knowledge, the first attempt to address the robust version of this problem was described in [14]. The authors introduce a Stochastic UC model that includes fuel constraints and electricity prices, and they solve it across a set of given scenarios. Likewise, [15], which describes a Stochastic Integer Programming Model for the Hydro-Thermal UC, assumes that the complete set of possible scenarios is known. In [16], the authors present a robust Two-Stage UC Problem that minimizes the total power generation cost under the worst case scenario. In [10], a Robust UC Problem with demand response and wind energy is described, and [17] presents a Two-Stage Robust Power Grid Optimization Problem. In [18] the authors develop a Bender Decomposition for the Two-Stage Security Constrained Robust UC Problem and in [19] the authors propose a two-stage adaptive robust optimization model for the same variant of the UC problem. Finally, a strategic planning model for an integrated oil supply chain is developed in [20], comparing different kinds of robust formulations and taking into account three uncertainty typologies: the crude oil production, the demand for refined products and the market prices.

As far as we know, our current work is the first one that deals with uncertainty in the UC problem by creating a risk-averse formulation based on data ranges. Our main contribution consists of developing a solution method, based on Bender Decomposition, capable of producing high-quality robust solutions to the UC problem with uncertainty. Furthermore, we perform a detailed computational analysis to provde answers to the following research questions:

- Is it beneficial to use a robust approach for the UC problem?
- In the presence of uncertainty, is it better to overproduce or underproduce?
- What is the best trade off between robustness and performance?
- Does performance (measured as cost savings) improve by including uncertainty on market prices?

From a technical point of view, our approach is novel for two reasons. First, uncertainty is modeled using ranges around uncertain values not only in the objective function but also in the constraints coefficients. Second, we include scenario-dependent variables to model the fact that the trading decisions are made after the production decisions.

2. Unit Commitment Base Model

We now describe a mathematical model for the UC problem with known demand and market power prices. The goal is to find the minimum cost production plan that satisfies the demand for power, which may also be met by buying or selling energy in the market. We have attempted to include all the production requirements that have been introduced in the literature so far.

We consider n power production units $i \in I$ and a set of time periods $t \in T$. Each production unit i is characterized by a lower (upper) bound l_i (u_i) on the power production, a minimum up (down) time t^+ (t^-), and a maximum ramp up (down) power production Δ_i^+ (Δ_i^-). The power cost is determined by a fixed production cost r_i, paid for each time period where the production unit i is on, a setup cost a_i, paid whenever unit i is activated, and a linear power production cost function, with a linear term coefficient c_i. In case of quadratic functions, the quadratic part of the power production cost can be linearized with a linear function passing through the origin and the cost of producing maximum power. Finally, in each time period t there is a demand d_t and a price for purchasing (selling) in the regulatory market q_t (e_t). The input data for the model are described in Table 1.

Table 1: UC Notation

$i \in I$	: set of n units
$t \in T$	: set of time periods
l_i (u_i)	: lower (upper) bound on the power production of unit i
d_t	: power demand at time t
t^+ (t^-)	: minimum up (down) time (in terms of time intervals)
Δ_i^+ (Δ_i^-)	: maximum ramp up (down)
c_i	: variable power production cost
r_i (a_i)	: fixed (setup) cost of production of unit i
e_t (q_t)	: buy (sell) price at time t

We model the UC problem with a set of binary variables y_{it}, which indicate if a power unit i is available at time t, a set of binary variables z_{it}, which indicate if a power unit has been activated at time t, a set of continuous variables x_{it}, which represent the amount of energy produced by unit i at time t, and two sets of continuous variables b_t and s_t, which represent the amount of energy bought and sold, respectively, at time t. For notational convenience, we also define p_t as the total amount of energy produced in time period t, i.e., $p_t = \sum_{i \in I} x_{it}$. We assume that all excess production will be sold in the period when it occurs and, likewise, power will be bought to cover production shortages. In other words, for each period t, $s_t = max(p_t - d_t, 0)$ and $b_t = max(d_t - p_t, 0)$. The mathematical programming model for the UC problem is:

$$
\begin{aligned}
\textbf{(UC)} \quad min \sum_{i\in I}\sum_{t\in T}(c_i x_{it} + r_i y_{it} + a_i z_{it}) & & \\
+ \sum_{t\in T}(e_t b_t - q_t s_t) & & (3)\\
\sum_{i\in I} x_{it} + b_t - s_t = d_t & \quad t \in T & (4)\\
l_i y_{it} \leq x_{it} \leq u_i y_{it} & \quad i \in I,\ t \in T & (5)\\
-y_{it-1} + y_{it} - z_{it} \leq 0 & \quad i \in I,\ t \in T & (6)\\
-y_{it-1} + y_{it} - y_{ih} \leq 0 & \quad i \in I, 1 \leq t \leq h \leq t^+ & (7)\\
y_{it-1} - y_{it} + y_{ih} \leq 1 & \quad i \in I, 1 \leq t \leq h \leq t^- & (8)\\
x_{it+1} \leq x_{it} + y_{it}\Delta_i^+ + (1 - y_{it})u_i & \quad i \in I,\ t \in T & (9)\\
x_{it} \leq x_{it+1} + y_{it}\Delta_i^- + (1 - y_{it})u_i & \quad i \in I,\ t \in T & (10)\\
x_{it},\ b_t,\ s_t \geq 0 & \quad i \in I,\ t \in T & (11)\\
y_{it},\ z_{it} \in \{0, 1\} & \quad i \in I,\ t \in T & (12)
\end{aligned}
$$

The objective function (3) minimizes the total cost. Constraints (4) force the demand in each time period to be met, constraints (5) impose bounds on the power production for each power unit and each time period, constraints (6) charge the cost of switching a unit on, constraints (7) and (8) model the minimum a maximum up (down) time, and constraints (9) and (10) limit the maximum ramp-up (down) power production.

3. Minimax Unit Commitment Problem

In order to formulate the UC problem as minimax model, we consider a set Γ of scenarios that represent possible combinations of demand values (d). The exact values for the demand are unknown, but, for each scenario $k \in \Gamma$ and time period $t \in T$ they are assumed to fall within a range: $d_t^k \in [d_t^-, d_t^+]$.

Let x be a power production plan that is developed prior to knowing the energy demand. In terms of a mathematical model, the production plan is a commitment that is made before the realization of a scenario that instantiates the values of d. In a robust optimization formulation of the UC problem, the x variables (as well as the ancillary y and z variables) do not depend on the scenarios, but the purchase and selling decisions do. Therefore, for a given production plan x, $b_{tk} = max(d_t^k - p_t, 0)$ represents the energy purchased in period t under scenario k. Likewise, $s_{tk} = max(p_t - d_t^k, 0)$ represents the energy sold in period t under scenario k.

Using a continuous variable ρ, the Minimax Unit Commitment Problem (MM-UC) may be formulated as follows:

$$\textbf{(MM-UC)} \quad min \quad \rho \tag{13}$$

$$\sum_{i \in I} x_{it} + b_{tk} - s_{tk} = d_t^k \quad k \in \Gamma, t \in T \tag{14}$$

$$\sum_{i \in I} \sum_{t \in T} (c_i x_{it} + r_i y_{it} + a_i z_{it}) + \sum_{i \in I} (e_t b_{tk} - q_t s_{tk}) \leq \rho \qquad k \in \Gamma \tag{15}$$

$$(5) - (12) \tag{16}$$

For convenience of notation, we define F_k as the objective function value associated with scenario k for production plan x:

$$F_k = \sum_{i \in I} \sum_{t \in T} (c_i x_{it} + r_i y_{it} + a_i z_{it}) + \sum_{i \in I} (e_t b_{tk} - q_t s_{tk}) \qquad (17)$$

Therefore, constraint set (15) may be simplified as $F_k \leq \rho$. The following theorem shows that the MM-UC problem is solved by considering only the "worst-case" scenario, where demand values are high.

Theorem 1. *The scenario k that maximizes the cost F_k of a solution x is such that, for all $t \in T$, $d_t^k = d_t^+$.*

Proof:

For any scenario j such that $d_t^- < d_t^j < d_t^+$, there exists a scenario k with $d_t^k = d_t^+$ for which $F_k \geq F_j$. Let $p_t = \sum_i x_{it}$ be the total energy produced in period t. If $d_t^- < d_t^j \leq p_t$, then by making $d_t^k = d_t^+$ the cost increases, because less energy is sold and more needs to be purchased. If, on the other hand, $p_t \leq d_t^j < d_t^+$, then by making $d_t^k = d_t^+$ the cost increases, because more energy needs to be purchased.

□

Because MM-UC is solved for $d_t = d_t^+$, solutions to the MM-UC problem lead to overproduction, whose magnitude is correlated to that of the d_t^+ values.

4. Minimax Regret Unit Commitment Problem

By keeping the same notation used for the MM-UC model, and redefining the variable ρ to represent the regret, the Minimax Regret Unit Commitment Problem (MMR-UC) may be formulated as follows:

$$(\textbf{MMR-UC}) \quad min \quad \rho \qquad (18)$$

$$\sum_{i\in I} x_{it} + b_{tk} - s_{tk} = d_t^k \quad k\in\Gamma, t\in T \tag{19}$$

$$\sum_{i\in I} F_k - F_k^* \leq \rho \qquad k\in\Gamma \tag{20}$$

$$(5) - (12) \tag{21}$$

where F_k^* is the objective function value associated with the optimal solution (x^k) of the UC problem under scenario k. The following theorem shows that when formulating the UC problem as a minimax regret problem with ranges, it is not necessary to explore solutions for which the uncertain data take on values inside the given interval. That is, a solution procedure may focus on solutions for which the parameter values are equal to one of the bounds. A similar theorem has been proven in [5] (Proposition 8) for $0-1$ problems with a linear objective function and intervals only in the objective function coefficients.

Theorem 2. *The scenario k that maximizes the regret of a solution x is such that, for all t, $d_t^k = d_t^-$ or d_t^+ .*

Proof:

For any scenario j such that $d_t^- < d_t^j < d_t^+$, there exists a scenario k with $d_t^k = d_t^-$ or d_t^+ for which $F_k - F_k^* \geq F_j - F_j^*$. Let $p_t^k = \sum_i x_{it}^k$ be the total energy produced in period t associated with the optimal solution of the UC problem under scenario k. Similarly, let b_t^k and s_t^k be the optimal purchasing and selling decisions associated with the production plan x^k, which is optimal for the solution of the UC problem under scenario k.

1. If $d_t^- < d_t^j < d_t^+$ and $p_t > d_t^-$ then by making $d_t^k = d_t^-$ the regret associated with solution x either remains the same if $p_t^k = p_t^j$ or it increases if $p_t^k < p_t^j$. Because $d_t^k < d_t^j$, at optimality, p_t^k cannot be greater than p_t^j.

2. If $d_t^- < d_t^j < d_t^+$ and $p_t \leq d_t^-$ then by making $d_t^k = d_t^+$ the regret associated with solution x either remains the same if $p_t^k = p_t^j$ or it increases if $p_t^k > p_t^j$. Because $d_t^k > d_t^j$, at optimality, p_t^k cannot be less than p_t^j.

□

The number of scenarios to consider is therefore limited to 2^T. However, the explicit consideration of the whole set is not practical. Thus, we develop a specialized procedure to find a smaller scenario set $\tilde{\Gamma}$. Given a solution $\tilde{x}$ to the master problem MMR-UC obtained by considering only a subset of scenarios $\tilde{\Gamma}$, we aim at finding a "target" scenario that leads to the maximum regret with respect to $\tilde{x}$. This scenario, as explained later, is added to the set $\tilde{\Gamma}$, so that a new master solution can be obtained by solving MMR-UC with the larger set of scenarios. The target scenario can be found by solving the following problem, called Scenario Generation Model (SG):

$$\begin{aligned}
\textbf{(SG)}\ \ max \sum_{t \in T} (e_t b_{tm} - q_t s_{tm}) - \sum_{i \in I} \sum_{t \in T} c_i x_{it} & \\
-r_i y_{it} - a_i z_{it} - \sum_{t \in T} (e_t b_t - q_t s_t) & \qquad\qquad (22)\\
\sum_{i \in I} \tilde{x_{it}} + b_{tm} - s_{tm} = d_t \quad t \in T & \qquad\qquad (23)\\
\sum_{i \in I} x_{it} + b_t - s_t = d_t \quad t \in T & \qquad\qquad (24)\\
\sum_{i \in I} \tilde{x_{it}} \geq d_t - \mu_t M_t^1 \quad t \in T & \qquad\qquad (25)\\
\sum_{i \in I} \tilde{x_{it}} \leq d_t + (1 - \mu_t) M_t^2 \quad t \in T & \qquad\qquad (26)\\
b_{tm} \leq \mu_t M_t^1 \quad t \in T & \qquad\qquad (27)\\
s_{tm} \leq (1 - \mu_t) M_t^2 \quad t \in T & \qquad\qquad (28)
\end{aligned}$$

$$d_t^- \leq d_t \leq d_t^+ \quad t \in T \tag{29}$$

$$b_t, s_t, b_{tm}, s_{tm} \geq 0 \quad t \in T \tag{30}$$

$$\mu_t \in \{0, 1\} \quad t \in T \tag{31}$$

$$(5) - (12) \tag{32}$$

The objective is to maximize $\tilde{F}_m - F_m^*$, i.e., the difference between the cost paid by the master solution $\tilde{x}$ under the target scenario m and the cost paid by the target scenario's optimal solution, which in the SG model is given by the optimal values of the d variables. The objective function does not include the fixed and setup costs of the master solution because they are constants. Constraint set (23) determines the energy bought (b_{tm}) and sold (s_{tm}) in the master solution in each time period t. Constraint set (24) ensures that the demand is met by solution x. Constraints (25-28) avoid that the problem be unbounded: without these constraints, an arbitrarily large objective value is achieved by letting b_{tm} and s_{tm} grow while keeping their difference constant and positive. Constraints (25-26) sets the binary variable μ_t to 0 (1) if and only if the production in the master solution is greater (smaller) than or equal to the demand. Practically, μ_t indicates if energy is purchased in the master solution under scenario m. Constraints (27-28) allow b_{tm} (s_{tm}) to be positive only if $\mu_t = 1$ ($\mu_t = 0$). To obtain the tightest linear relaxation, the "Big-M" constants are set to $M_t^1 = max(0, d_t^+ - \sum_{i \in I} \tilde{x_{it}})$ and $M_t^2 = max(0, \sum_{i \in I} \tilde{x_{it}} - d_t^-)$. The set $\tilde{\Gamma}$ of scenario is initialized with the scenario obtained with the average values (step 1, Figure 1). The model is then solved (step 3) and a solution is found by solving MMR-UC (equations (18)-(21)), which minimizes the maximum regret considering the scenarios in $\tilde{\Gamma}$. A scenario k is found that maximizes the regret in the current MMR-UC solution (step 4), and is added to $\tilde{\Gamma}$ (step 5). The optimal UC solution under this scenario is then found (step 6). Finally, if the regret of this

UC solution is larger than the regret currently obtained by the MMR-UC solution (step 7), then a new cut is inserted by updating $\tilde{\Gamma}$ and the MMR-UC (constraints (19) and (20)). Otherwise, the procedure ends and the current MMR-UC solution is returned.

begin

1. Initialize MMR-UC with $\tilde{\Gamma}$
2. **repeat**
3. Solve MMR-UC
4. Solve (SG) and obtain the vector d^k;
5. Add scenario k to $\tilde{\Gamma}$;
6. Solve the UC with d^k obtaining the SG' solution (x, b, s);
7. **while** scenario k cuts the current MMR-UC solution .

end.

Figure 1: Bender Decomposition with demand data ranges

Note that, because the scenarios are generated using an exact procedure, the Bender Decomposition method described in Figure 1 is an exact method. However, it may be impractical to execute to completion for hard problem instances, for which the algorithm may instead be terminated after a given time limit.

5. Generation of Problem Instances

In order to investigate the merit of our approach, each instance comprises the characteristics of the production units and two pairs of data: the real data and the forecasted data. The real data are composed of the vectors of actual demand

and price values; the forecasted data are composed of the vectors of forecasted demand and price values. The performance of a robust solution method on a problem instance is evaluated by 1) obtaining a production plan by considering only the forecasted data, and 2) evaluating the production plan on the real data.

5.1. Real Data

Artificial (but realistic) characteristics of n thermal systems, with $n \in \{10, 20\}$, are configured by the procedure presented in [13]. This procedure also generates an artificial demand load d_t^a ($t = 1...24$) which is appropriate to the set of thermal units, but follows the same profile for all instances. To make our experiments more realistic and to consider a variety of demand profiles, we combine real energy demand data recorded in 2006 in the State of Colorado (USA) with the characteristics of the thermal system. However, the real demand d_t^r may be inappropriate for the artificial characteristics of the thermal systems. To ensure that the load d_t is appropriate, the values of d_t are set as follows:

$$d_t := d_t^r \times max_{i=1...24}\{d_i^a\}/HY$$

where HY is the highest demand recorded throughout the entire 2006 in Colorado. In this way, even if d_t^r is the largest demand load of 2006 (i.e., $d_t^r = HY$), the d_t can be satisfied with the thermal systems.

The selling and purchase prices are set so that:

1. The purchase price is always larger than an upper bound of the production price
2. The selling price is always lower than a lower bound of the production price
3. They fluctuate in agreement with the demand (they are high when the demand is high)

Requirements 1 and 2 ensure that energy trading is discouraged. Otherwise, it may be profitable to produce nothing and purchase enough energy to meet demand; or, it may be profitable to produce as much as possible and sell the surplus. Requirements 1 and 2 also avoid arbitrage. The lower bound of the production unit price corresponds to the case where the unit is active throughout all the time periods and produces the maximum power u_i. In this case, the cost of producing a unit of energy is the summation of the variable cost c_i and the fixed cost r_i spread over the u_i energy units produced. Let us define w^- as the lower bound computed across all units.

$w^- = min_{i=1...n}[c_i + \frac{r_i}{u_i}]$

The upper bound corresponds to the case where the unit produces the minimum power l_i for the minimum "up time" t^+. Then, it is off for the minimum "down time" t^-. In this case, the cost of producing a unit of energy is the summation of the variable cost c_i and the fixed cost r_i spread over the l_i energy units produced. Furthermore, every time the unit is switched on, a cost a_i is incurred, which needs to be spread over all t^+l_i energy units produced while the unit is on. Thus, the upper bound is computed as:

$w^+ = max_{i=1...n}[c_i + \frac{r_i}{l_i} + \frac{a_i}{t^+l_i}]$

In order to satisfy the three requirements above, the demand profile is used to build a "mid-price profile", which has the same "shape" as the demand and represents the mid-point between the selling price and the purchase price. While the demand is included between the minimum demand value d^- and the maximum demand value d^+, the mid-price is included in $[w^-, w^+]$. The purchase and the selling price are obtained by respectively adding and subtracting $w^+ - w^-$ from the mid-price values. The equations used to compute e_t and q_t are:

$midPrice_t = \frac{d_t - d^-}{d^+ - d^-}(w^+ - w^-) + w^-$

$e_t = midPrice_t + (w^+ - w^-)$

$q_t = midPrice_t - (w^+ - w^-)$

The problem set from [13] includes 5 instances for each value of n. Therefore, a problem instance is uniquely identified by a value of $n \in \{10, 20, 50, 75, 100, 150, 200\}$, $i = 1...5$, and a day $d = 1...365$. Let n_i be the set of instances corresponding to given values of n and i. All the benchmark instances can be found in the online supplement. However, in the remainder of the paper we consider only $n \in \{10, 20\}$ because the UC formulation (3)-(12) cannot be solved by commercial solvers for larger values of n. Note that our focus is not on the solution method for the UC problem, which is therefore treated as a black box. Should a more efficient solution method be available, our formulation could be easily replaced, and the computational times required by our approaches would decrease accordingly.

5.2. Forecasted Data

The forecasted data are generated by modifying the real data. While this could be achieved by perturbing the real hourly demand and prices by a randomly generated amount, this procedure would fail to capture two important aspects of forecast generation: the correlation among forecast errors made in consecutive time periods (correlation 1) and the correlation between the magnitude of the forecast errors and the sudden changes in temperature (correlation 2). Because demand and prices are correlated, for brevity we will simply refer to "demand".

Correlation 1 implies, for example, that if at time period t the forecasted value is much larger than the real one, at time period $t+1$ the forecasted value should be similarly larger (and not, for example, much smaller). The the use of weather forecasts to predict demand causes Correlation 1 to be larger in very cold or very hot days than in mild days. Because weather forecasts for consecutive time periods tend to be similar, so do consecutive demand forecasts.

Correlation 2 states that the magnitude of the forecasting error is proportional to the forecasted change in temperature. If, for example, the temperature is forecasted to drop by 40 degrees during a period of 24 hours, the error of such prediction is larger than if the forecasted drop of temperature were of 5 degrees. Errors in the forecasted demand behave similarly. This is an important aspect for places with a continental climate, such as Colorado.

We account for both of these aspects by allowing the next-day forecasts to belong to different groups, which are found by mining the 2006 demand data set of Colorado. In particular, we use the data mining algorithm called "clustering" (see [21] for further details), which consists of assigning objects to groups (clusters) in order to maximize the similarity among objects belonging to the same cluster while minimizing that of objects belonging to different clusters. In our case, the objects are *next-day variations* of demand, which are 24-element vectors, whose t-th element is the daily percentage change in demand between two consecutive days at time period t. Given a *training set* of known next-day variations, this procedure finds clusters made of similar next-day variations. A cluster can include, for example, all those next-day variations that correspond to a sudden drop in temperature, or all those next-day variations corresponding to the increase in demand of the day after a holiday.

In order to predict the demand of a day t that does not belong to the training set, we consider the real demand vectors of day $t-1$ and of day t to compute the real next-day variation. Then, we identify the cluster whose centroid is closest to the real next-day variation. We use the centroid of this cluster as the forecasted next-day variation. By adding this variation to the real demand of $t-1$, t's forecasted demand is obtained. The forecast prices are a function of the forecasted demand, and are computed as explained above. Each cluster may be interpreted as a different "way" in which the demand may change from day to day. It is important to note that this procedure is only used to build the vector of forecasted demand and price values, and is not part of the methodology used to solve the problem.

6. Computational Results

We now assess the benefits of finding unit commitment plans by applying a robust optimization approach when compared to plans that are found by solving the deterministic version of the unit commitment problem using expected values. Two different types of robustness criteria are tested: minimax regret (MMR-UC) and minimax (MM-UC). MM-UC and MMR-UC are the models presented in sections 3 and 4, respectively. We measure the performance of these methods in terms of savings that can be obtained compared to solving the determinisitc UC, as follows:

1. The UC problem is solved using the real data in order to provide a lower bound for the cost. Let us call this cost λ—the cost in the absence of uncertainty.
2. The UC problem is solved using the forecasted data. The cost of the production plan obtained is then evaluated on the real data. Let us call this cost β—the cost if no ranges are considered.
3. Ranges of $\pm robLevel\%$ are built around the forecasted demand and prices.

4. The robust method is executed with these ranges and evaluated on the real data. Let us call the cost obtained γ.
5. The savings (%) given by the robust method is computed as $100(1-\frac{\gamma-\lambda}{\beta-\lambda})$. In this way, positive savings indicate that the robust method obtains a lower-cost solution than the basic UC model; conversely, negative savings indicate that the robust method obtains a higher-cost solution than the basic UC model.

Note that when a method is tested on a set of instances, λ, β, and γ are the average costs obtained across the instances.

Each problem instance is denoted by n_i_X, where n is the size, i is the instance number, and $X = A$ for training and $X = B$ for test. The training set, which comprises 1/8 of the data, represents past (known) data; the test set, which also comprises 1/8 of the data, represents a set of future days in which we want to assess the performance of our method. The instances are divided in such a way that each set includes an equal number of each day of the week (as many Mondays as Tuesdays, Wednesdays, etc...), and an equal number of days from each month of the year. Hence, both sets include days from every period of the year. By considering the training set only, we build clusters of next-day variations as explained above. Clusters are built using two different techniques: Expectation-Maximization ([22]) and K-Means ([23]). The EM procedure results in 12 clusters; K-Means, which, unlike EM, requires the number of clusters to be specified, is run by setting the number of clusters to 3. This low number results in a much less accurate prediction than that obtained by EM. Therefore, in our experiments, EM represents the case where the prediction accuracy is high, while K-Means the case where the prediction accuracy is low.

In order to test the robust optimization methods, a robustness level $robLevel$ needs to be chosen. Thus, we assess the savings given by MM-UC and MMR-UC via the procedure above by using different levels of robustness. Our experiments were run on a cluster of 4 computers: two equipped with a 4-core 2.93 GHz CPU and 4 GB or RAM, one equipped with an 8-core 2.27 GHz CPU and 4 GB of RAM, and one equipped with a 3.40 GHz 8-core CPU and 16 GB or RAM. All problems are solved using ILOG CPlex 12.3 and only one processor. Because these experiments are computationally intensive, they are run with a time limit of 20 minutes. Results are shown in Table 2. The first 2 columns report the instance name (n_i_A) (where A indicates that it belongs to the training set) and the $robLevel$. The following columns report the performance of MM-UC and MMR-UC. For both methods, we report the the savings (%) and the average computing time in seconds. For MMR-UC, we also report the total number of "time limit" occurrences and the average number of cuts generated during the procedure.

On one hand, it is evident that solving the MMR-UC takes much longer than solving the MM-UC. This is explained by the need of MMR-UC to solve at least one instance of the UC for each cut made during the Bender decomposition procedure, whereas solving the MM-UC consists of solving the UC problem under the worst-case scenario. On the other hand, the savings obtained by MMR-UC are generally larger than those obtained by MM-UC. This is particularly clear for large values of $robLevel$, where MMR-UC performs much better than MM-UC. In this case, the MM-UC tends to overproduce by a large amount, which leads to significant losses (negative savings). Interestingly, for small values of $robLevel$, the MM-UC leads to significant savings. From a managerial standpoint, this means that it is cheaper to slightly overproduce than to slightly under produce.

Figure 2 analyzes the effect of the clustering method chosen on the best value of $robLevel$ and on the savings generated by MM-UC and MMR-UC. The tuning phase suggests $robLevel = 0.01$ for MM-UC irrespective of the clustering technique used to generate the forecasted instances, and $robLevel = 0.02$ ($robLevel = 0.03$) for MMR-UC if the clustering technique used is EM (KMeans). A larger tolerance level for KMeans is unsurprising, given the inferior forecast accuracy of this method compared to EM. For the same reason, the optimal value of $robLevel$ leads to a better performance for both methods when the forecast is generated by EM (12.62% for MM-UC and 22.43% for MMR-UC) than when it is generated by KMeans (7.19% for MM-UC and 13.49% for MMR-UC). Table 3 (columns 2–4) reports the performance on the test set obtained by executing MM-UC and MMR-UC with the values of $robLevel$ found above and a 1-hour time limit. Although smaller, the savings obtained on the test set are in line with those obtained on the training set. In particular, the savings yielded by MMR-UC (15.11%) are much higher than those yielded by MM-UC (7.40%).

We executed the same experiment (tuning on the training set and evaluating performance on the test set) on larger instances, those with $n = 20$. Noticeably, the average time taken to solve the UC increases from 1.01 seconds for $n = 10$ to 27.63 seconds for $n = 20$. Therefore, the occurrences of ”time limit” events incurred by MMR-UC is much higher than in the previous case. Nevertheless, the main pattern observed in Figure 2 does not change: the performance of MM-UC is influenced by the value of $robLevel$ much more heavily than that of MMR-UC, which achieves larger savings. Interestingly, the best values of $robLevel$ obtained in the training phase are the same as those found for $n = 10$. The details of the training phase for $n = 20$ are not reported because of their similarity to those of the case $n = 10$. However, Table 3 (columns 5–7) reports the results on the test set, obtained by solving the instances in the test set with the best values of $robLevel$ found in the training phase. Again, MMR-UC outperforms MM-UC for all instances with $n = 20$. Note that these experiments have been run with a 2-hour time limit. However, since typically the grid operator solves the Unit Commitment problem once a day, a longer time can be allocated to its solution. Table 4 reports the test set performance, limited to the EM clustering case, obtained with a 12-hour time limit. This larger value results in an average savings increase of just 1.93% (from 11.75% to 13.68%), indicating that MMR-UC is capable of obtaining good quality solutions early in the search.

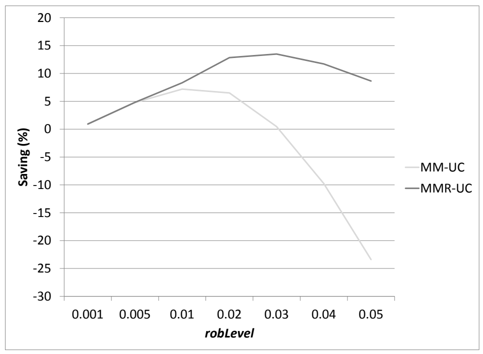
Although this seems to suggest that larger instances can be successfully solved, this is not the case. In fact, the simple UC problem cannot be solved to optimality for all the next-size instances ($n = 50$) within a reasonable time (we allowed 5 hours). Therefore, the MMR-UC may not generate any cuts, terminating without finding a feasible solution. This experiment shows how strongly the performance of MMR-UC depends on that of UC. Should a new solution method for UC be developed, our MMR-UC procedure can be applied as is, leading to higher quality solutions than those obtained by UC and MM-UC.

7. Uncertainty on the demand and energy prices

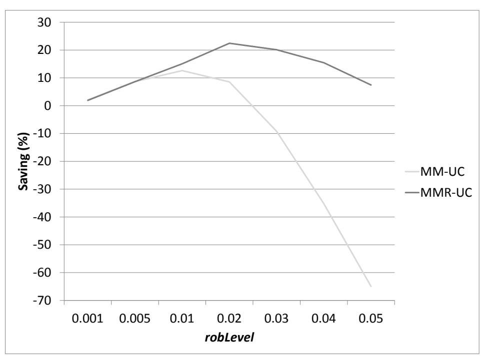
Here, we study the case where the uncertainty considered is not only on the demand, but also on the selling and purchase prices. Our goal is to investigate if this extension results in further savings. In addition to demand ranges, now we consider that for each scenario $k \in \Gamma$ and time period $t \in T$ prices fall within

		MM-UC		MMR-UC			
Instance	$robLevel$	sav%	time	sav%	time	n. TL	n. cuts
10_1_A		-6.62	2.42	9.48	1058.51	495	35.8
	0.001	1.33	2.36	1.10	523.10	6	30.5
	0.005	5.23	2.54	6.54	974.61	49	40.9
	0.01	8.75	2.51	11.32	1115.26	74	43.9
	0.02	5.36	2.42	16.46	1197.73	91	40.4
	0.03	-5.58	2.41	16.69	1198.80	91	35.2
	0.04	-19.82	2.28	11.02	1200.04	92	31.4
	0.05	-41.63	2.44	3.24	1200.05	92	28.1
10_2_A		-5.22	0.84	11.89	731.61	289	45.0
	0.001	1.63	0.72	1.40	164.58	1	28.5
	0.005	6.72	0.71	6.48	337.53	5	36.9
	0.01	10.18	0.80	11.11	523.50	17	45.0
	0.02	8.51	0.81	16.80	840.38	46	55.0
	0.03	-2.15	0.90	17.79	996.20	63	53.8
	0.04	-19.86	0.94	17.29	1084.26	73	50.9
	0.05	-41.55	0.99	12.37	1174.81	84	44.6
10_3_A		-2.18	0.34	10.97	758.37	312	53.4
	0.001	1.57	0.28	1.35	102.46	1	29.5
	0.005	7.21	0.28	6.20	276.92	4	43.2
	0.01	11.00	0.28	10.82	513.96	10	57.5
	0.02	10.07	0.33	16.54	943.48	47	70.9
	0.03	1.48	0.36	15.38	1095.92	73	64.2
	0.04	-13.81	0.43	16.43	1179.29	87	57.7
	0.05	-32.80	0.39	10.11	1196.58	90	50.8
10_4_A		-8.29	0.82	10.35	810.11	349	40.2
	0.001	1.14	0.75	1.32	182.48	1	28.0
	0.005	6.17	0.76	5.98	386.39	11	36.1
	0.01	7.89	0.77	10.28	603.74	22	44.1
	0.02	5.01	0.87	16.35	1002.30	58	50.7
	0.03	-8.19	0.87	17.10	1123.80	78	45.8
	0.04	-25.48	0.92	11.80	1172.05	87	40.0
	0.05	-44.56	0.81	9.64	1200.03	92	36.7
10_5_A		-3.95	0.82	9.57	900.86	396	40.5
	0.001	1.18	0.83	1.42	232.22	0	30.1
	0.005	6.38	0.85	6.33	572.82	13	43.5
	0.01	9.20	0.90	11.15	878.35	46	49.4
	0.02	8.61	0.89	16.65	1068.93	71	45.9
	0.03	-0.15	0.79	13.37	1154.86	83	41.7
	0.04	-17.51	0.75	10.92	1198.78	91	37.9
	0.05	-35.36	0.73	7.14	1200.05	92	34.8
tot/avg		-5.25	1.05	10.45	851.89	1841	43.0

Table 2: Training set results (10 units).



(a) *KMEANS Clustering (10 units)*



(b) *EM Clustering (10 units)*

Figure 2: Effects of robustness level on savings

		Saving (%)			Saving (%)	
Cluster	Instance	MM-UC	MMR-UC	Instance	MM-UC	MMR-UC
KMeans		6.47	14.73		5.76	8.84
	10_1_B	6.58	14.26	20_1_B	6.86	8.92
	10_2_B	4.72	15.97	20_2_B	5.97	6.93
	10_3_B	6.76	15.37	20_3_B	5.83	9.24
	10_4_B	7.04	15.06	20_4_B	6.34	9.78
	10_5_B	7.22	13.01	20_5_B	3.77	9.34
EM		8.68	15.80		9.05	11.75
	10_1_B	8.61	15.67	20_1_B	9.72	12.66
	10_2_B	8.02	15.24	20_2_B	8.91	9.00
	10_3_B	11.27	17.07	20_3_B	9.30	12.07
	10_4_B	6.60	14.16	20_4_B	10.89	14.41
	10_5_B	8.87	16.84	20_5_B	6.41	10.64
tot/avg		7.57	15.26		7.40	10.30

Table 3: Performance on Test Set.

		Saving (%)	
Cluster	Instance	MM-UC	MMR-UC
EM		9.05	13.68
	20_1_B	9.72	14.47
	20_2_B	8.91	11.07
	20_3_B	9.30	14.61
	20_4_B	10.89	17.77
	20_5_B	6.41	10.50
tot/avg		9.05	13.68

Table 4: Performance with 12 hours time limit.

the ranges $e_t^k \in [e_t^-, e_t^+]$ and $q_t^k \in [q_t^-, q_t^+]$. To avoid arbitrage, purchase prices are considered to be larger than or equal to selling prices (i.e., $e_t^- \geq q_t^+$ for all t). To solve the MM-UC we need to extend Theorem 1 in order to take into account price ranges.

Theorem 3. *The scenario k that maximizes the cost F_k of a solution x is such that, for all $t \in T$, $d_t^k = d_t^+$, $e_t^k = e_t^+$ and $q_t^k = q_t^-$.* Proof:

In addition to the considerations in Theorem 1, $e_t^k = e_t^+$ and $q_t^k = q_t^- \forall t \in T$ because, as shown by [5], they appear only in the objective function with positive and negative sign, respectively. □

Theorem 3 proves that the optimal solution to the MM-UC is obtained by solving the UC in the scenario that has the largest demand, the highest purchase prices, and the lowest selling prices. Similarly, to solve the MMR-UC we need to extend Theorem 2 as follows:

Theorem 4. *The scenario k that maximizes the regret of a solution x is such that, for all t, $d_t^k = d_t^-$ or d_t^+, $e_t^k = e_t^-$ or e_t^+ and $e_t^k = e_t^-$ or e_t^+.*

Proof:

For any scenario j such that either $d_t^- < d_t^j < d_t^+$ or $e_t^- < e_t^j < e_t^+$ or $q_t^- < q_t^j < q_t^+$, there exists a scenario k with $d_t^k = d_t^-$ or d_t^+ and $e_t^k = e_t^-$ or e_t^+ and $e_t^k = e_t^-$ or e_t^+ for which $F_k - F_k^* \geq F_j - F_j^*$. Let $p_t^k = \sum_i x_{it}^k$ be the total energy produced in period t associated with the optimal solution of the UC problem under scenario k. Similarly, let b_t^k and s_t^k be the optimal purchasing and selling decisions associated with the production plan x^k, which is optimal for the solution of the UC problem under scenario k.

1. If $d_t^- < d_t^j < d_t^+$ and $p_t > d_t^-$ then by making $d_t^k = d_t^-$ the regret associated with solution x either remains the same if $p_t^k = p_t^j$ or it increases if $p_t^k < p_t^j$.

Because $d_t^k < d_t^j$, at optimality, p_t^k cannot be greater than p_t^j.

2. If $d_t^- < d_t^j < d_t^+$ and $p_t \leq d_t^-$ then by making d_t^k = d_t^+ the regret associated with solution x either remains the same if p_t^k = p_t^j or it increases if $p_t^k > p_t^j$. Because $d_t^k > d_t^j$, at optimality, p_t^k cannot be less than p_t^j.
3. If $e_t^- < e_t^j < e_t^+$ and $b_{tj} > b_t^j$ then by making e_t^k = e_t^+ the regret associated with solution x increases by exactly (b_{tj} - b_t^k)(e_t^+ - e_t^j) when p_t^k=p_t^j (and therefore b_t^k = b_t^j) and by more when $p_t^k > p_t^j$.
4. If $e_t^- < e_t^j < e_t^+$ and $b_{tj} \leq b_t^j$ then by making e_t^k = e_t^- the regret associated with solution x increases by exactly (b_t^k - b_{tj})(e_t^j - e_t^-) when p_t^k = p_t^j (and therefore b_t^k = b_t^j) and by more when $p_t^k < p_t^j$.
5. The cases related to $q_t^- < q_t^j < q_t^+$ are similar to 3 and 4.

□

Theorem 4 proves that we need to consider only the scenarios where demand and prices assume boundary values. To generate the regret-maximizing scenario, we cannot use the SG model. If the constants e_t and q_t in SG become variables, the SG model becomes nonlinear. Therefore, we tackle the scenario-generation subproblem (step 4, Figure 1) heuristically. The heuristic procedure, reported in the Appendix, is such that the regret increases at each step.

We performed the same test as in Table (3) and compared the performance of the MMR-UC with price ranges to that obtained without the price ranges. On the 10-unit instances, the overall savings are 1.43% lower; on the 20-unit instances, they are 10.81% lower. On one hand, considering price ranges models the problem realistically but its increased complexity requires either a nonlinear or a heuristic solution method; on the other hand, MMR-UC models the problem less realistically, but the solution method is exact. Clearly, our results suggest that the benefit of using the exact method is greater than the benefit obtained by introducing price ranges. Similarly, we performed the same comparison for the MM-UC approach, but the performance obtained with price ranges is not statistically different from that without price ranges.

8. Concluding remarks

In this paper, we investigated the benefits of robust approaches for the UC problem. In particular, we tested the minimax and the minimax regret criteria with and without price ranges, which required novel solution approaches. Our results suggest that our method leads to significant savings compared to the current approaches. We also tested different levels of forecast quality and found that the savings yielded by our method are substantial both in the case of accurate and in the case of inaccurate forecast. This has positive implications to smart grids, where the demand can be somewhat adjusted in case of mismatch with supply. Mathematically, the ability of "changing" the demand is equivalent to a better quality demand forecast. Therefore, smart grids operators can potentially benefit from our work too. Our computational results allow us to answer the research questions formulated in the introduction.

Using a robust approach for the UC problem is beneficial. Although both MM-UC and MMR-UC lead to significant savings, MMR-UC outperforms MM-UC. The reason is that MM-UC always leads to overproduction (because it considers only the case where the demand is highest) and, therefore, it incurs in unnecessary energy trading. On the other hand, MMR-UC takes into account all possible demand realizations, avoiding unprofitable energy trading. However, overproducing is preferable to underproducing, as confirmed by the MM-UC outperforming UC.

We attribute this to the fact that the purchase price is higher than the selling price.

Figure 2 graphically shows the impact of robustness on performance. If the robustness range is too narrow, there is no advantage in adopting robustness; if the robustness range is too wide, robustness leads to overproduction and, in the case of the MM-UC, to monetary losses. If, on the other hand, the robustness range is set to an intermediate level, both robust approaches (MM-UC and MMR-UC) outperform the non-robust approach (UC).

One limitation of our approach is the generation of the prices, which are calculated from the lower and upper bounds of the production cost. An analysis of the generated instances suggest that while the lower bound is close to the production price, the upper bound tends to be much farther. This asymmetry affects the structure of the problem, by making it even more attractive to overproduce than to underproduce. While our problem instances are available online, a promising future development is designing other data generation procedures. Also, we hope that utility companies apply our ideas to their real-world data.

Interestingly, including uncertainty on market prices is detrimental to the solution quality. The reason, as discussed in Section 7, lies in the increased complexity of the solution method needed to generate scenarios. Our scenario generator procedure (see Appendix for further details) is heuristic; however, since it iteratively solves a UC problem, it is time consuming, limiting in this ways the number of scenarios generated within the time limit. A different approach to generating scenarios may consist of using alternative nonlinear approaches; however, we leave this extension as a future development.

Appendix A. Scenario Generation in the case of price and demand ranges

We consider the problem of generating the scenario that maximizes the regret with respect to the current set of scenarios. Unlike the case where the uncertainty is only on the demand, here we cannot solve the SG problem efficiently, as it becomes nonlinear. Instead, we tackle this problem with a heuristic method. The procedure starts by constructing an initial scenario $k = 0$ that results in a positive regret. Considering the absolute difference between the production $\tilde{p}$ and the extreme values of the demand range (d_t^- and d_t^+), the initial scenario is built as follows:

$$d_t^0 = \begin{cases} d_t^+ & \text{if } \tilde{p}_t \text{ is "closer" to } d_t^- \text{ than to } d_t^+ \\ d_t^- & \text{otherwise} \end{cases}$$

$$e_t^0 = \begin{cases} e_t^+ & \text{if } \tilde{p}_t \text{ is "closer" to } d_t^- \text{ than to } d_t^+ \\ e_t^- & \text{otherwise} \end{cases}$$

$$q_t^0 = \begin{cases} q_t^+ & \text{if } \tilde{p}_t \text{ is "closer" to } d_t^- \text{ than to } d_t^+ \\ q_t^- & \text{otherwise} \end{cases}$$

If a small amount of energy in time period t is produced (i.e., $\tilde{p}_t$ is "closer" to d_t^- than to d_t^+), a scenario with high demand and high trade costs is generated. In this way, the regret increases because the master solution is forced to buy energy at a high price. If, on the other hand, a large amount of energy in time period t is produced (i.e., $\tilde{p}_t$ is "closer" to d_t^+ than to d_t^-), a scenario with low demand and low trade costs is generated. In this way, the master solution is forced to sell energy at a low price, while the target scenario has the advantageous option of purchasing at a low price.

The initial scenario (e^0, q^0, d^0) does not necessarily maximize the regret. Therefore, the local search procedure outlined in Figure A.3 is executed to improve upon the initial regret value. At steps 3–6, the current scenario is modified in accordance to the Regret-Increasing Rules presented below, in order to increase the regret. At steps 7–8, the optimal solution under the modified scenario is updated by solving the UC. The local search terminates when the regret value stops increasing. Our procedure is an approximation of the optimal solution to SG, where the demand and prices are determined heuristically. We refer to it as the SG' procedure.

begin

1. Compute initial values of e^0, q^0, and d^0
2. **repeat**
3. Compute values $\tilde{p}_t$ for each time period t
4. **for each** t
5. Modify e_t, q_t, and d_t according to the Regret-Increasing Rules;
6. **end for**
7. Solve the UC and compute the production values p_t for each time period t;
8. Compute the current regret
9. **until** the regret stops increasing

end.

Figure A.3: SG' Local Search

The solution method is the same as that in Figure 1, except that the scenario is built using the algorithm described in Figure A.3. Note that, because the scenarios are generated using a heuristic procedure, the Bender Decomposition for MMR-

UC is a heuristic method. We compared the solution quality it obtains by to the one obtained by generating all scenarios via complete enumeration in step 1 (that is, by setting $\tilde{\Gamma} = \Gamma$). Our tests show that the Heuristic Bender Decomposition method finds near-optimal solutions.

Regret-Increasing Rules: Let p_t be the production of the SG' solution at time t and $\tilde{p}_t$ the production of the master solution at time t. For each of the following cases, we must find prices and demand values that increase (do not decrease) the regret value. The goal is finding a combination for which the objective function of the current SG' solution (without re-optimizing it) increases by less (or, equivalently, decreases by more) than that of the master solution.

1. $p_t \leq d_t^- \leq \tilde{p}_t \leq d_t^+$.
   - If $d_t = d_t^+$, set $d_t = d_t^-$. The SG' objective decreases by $e_t(d_t^+ - d_t^-)$, while the master objective decreases by a smaller quantity, $e_t(d_t^+ - \tilde{p}_t) + q_t(\tilde{p}_t - d_t^-)$.
   - If $e_t = e_t^+$, set $e_t = e_t^-$. The SG' objective decreases by $(d_t - p_t)(e_t^+ - e_t^-)$, while the master objective decreases by a smaller quantity λ, where
   $$\lambda = \begin{cases} (d_t^+ - \tilde{p}_t)(e_t^+ - e_t^-) & \text{if } d_t = d_t^+ \\ 0 & \text{if } d_t = d_t^- \end{cases}$$
   - If $q_t = q_t^+$, set $q_t = q_t^-$. The SG' objective does not change, while the master objective increases by a non-negative quantity λ, where
   $$\lambda = \begin{cases} 0 & \text{if } d_t = d_t^+ \\ (\tilde{p}_t - d_t^-)(q_t^+ - q_t^-) & \text{if } d_t = d_t^- \end{cases}$$

2. $\tilde{p}_t \le d_t^- \le p_t \le d_t^+$

- If $d_t = d_t^-$, set $d_t = d_t^+$. The master objective increases by $e_t(d_t^+ - d_t^-)$, while the SG' objective increases by a smaller quantity, $e_t(d_t^+ - p_t) + q_t(p_t - d_t^-)$.
- If $e_t = e_t^-$, set $e_t = e_t^+$. The master objective increases by $(d_t - \tilde{p}_t)(e_t^+ - e_t^-)$, while the SG' objective increases by a smaller quantity λ, where
$$\lambda = \begin{cases} (d_t^+ - p_t)(e_t^+ - e_t^-) & \text{if } d_t = d_t^+ \\ 0 & \text{if } d_t = d_t^- \end{cases}$$
- If $q_t = q_t^-$, set $q_t = q_t^+$. The master objective does not change, while the SG' objective decreases by a non-negative quantity λ, where
$$\lambda = \begin{cases} 0 & \text{if } d_t = d_t^+ \\ (p_t - d_t^-)(q_t^+ - q_t^-) & \text{if } d_t = d_t^- \end{cases}$$

3. $d_t^- \le p_t \le d_t^+ \le \tilde{p}_t$.

- If $d_t = d_t^+$, set $d_t = d_t^-$. The SG' objective decreases by $q_t(d_t^+ - p_t) + e_t(p_t - d_t^-)$, while the master objective decreases by a smaller quantity, $q_t(d_t^+ - d_t^-)$.
- If $e_t = e_t^+$, set $e_t = e_t^-$. The master objective does not change, while the SG' objective decreases by a non-negative quantity λ, where
$$\lambda = \begin{cases} (d_t^+ - p_t)(e_t^+ - e_t^-) & \text{if } d_t = d_t^+ \\ 0 & \text{if } d_t = d_t^- \end{cases}$$
- If $q_t = q_t^+$, set $q_t = q_t^-$. The master objective increases by $(\tilde{p}_t - d_t)(q_t^+ - q_t^-)$, while the SG' objective increases by a smaller quantity

λ, where

$$\lambda = \begin{cases} 0 & \text{if } d_t = d_t^+ \\ (p_t - d_t^-)(q_t^+ - q_t^-) & \text{if } d_t = d_t^- \end{cases}$$

4. $d_t^- \leq \tilde{p}_t \leq d_t^+ \leq p_t$

   - If $d_t = d_t^-$, set $d_t = d_t^+$. The SG' objective increases by $q_t(d_t^+ - d_t^-)$, while the master objective increases by a larger quantity, $e_t(d_t^+ - \tilde{p}_t) + q_t(\tilde{p}_t - d_t^-)$.
   - If $e_t = e_t^-$, set $e_t = e_t^+$. The SG' objective does not change, while the master objective increases by a non-negative quantity λ, where

     $$\lambda = \begin{cases} 0 & \text{if } d_t = d_t^- \\ (d_t^+ - \tilde{p}_t)(e_t^+ - e_t^-) & \text{if } d_t = d_t^+ \end{cases}$$

   - If $q_t = q_t^-$, set $q_t = q_t^+$. In this way, the SG' objective decreases by $(p_t - d_t)(q_t^+ - q_t^-)$, while the master objective decreases by a smaller quantity λ, where

     $$\lambda = \begin{cases} (\tilde{p}_t - d_t^-)(q_t^+ - q_t^-) & \text{if } d_t = d_t^- \\ 0 & \text{if } d_t = d_t^+ \end{cases}$$

5. $p_t \leq d_t^- \leq d_t^+ \leq \tilde{p}_t$.

   - If $d_t = d_t^+$, set $d_t = d_t^-$. The master objective decreases by $q_t(d_t^+ - d_t^-)$, while the SG' objective decreases by a larger quantity, $e_t(d_t^+ - d_t^-)$.
   - If $e_t = e_t^+$, set $e_t = e_t^-$. The master objective does not change, while the SG' objective decreases by a non-negative quantity $(d_t - p_t)(e_t^+ - e_t^-)$.

- If $q_t = q_t^+$, set $q_t = q_t^-$. The SG' objective does not change, while the master objective increases by a non-negative quantity $(\tilde{p}_t - d_t)(q_t^+ - q_t^-)$.

6. $\tilde{p}_t \leq d_t^- \leq d_t^+ \leq p_t$

   - If $d_t = d_t^-$, set $d_t = d_t^+$. The master objective increases by $e_t(d_t^+ - d_t^-)$, while the SG' objective increases by a smaller quantity, $q_t(d_t^+ - d_t^-)$.
   - If $e_t = e_t^-$, set $e_t = e_t^+$. The SG' objective does not change, while the master objective increases by a non-negative quantity $(d_t - \tilde{p}_t)(e_t^+ - e_t^-)$.
   - If $q_t = q_t^-$, set $q_t = q_t^+$. The master objective does not change, while the SG' objective decreases by a non-negative quantity $(p_t - d_t)(q_t^+ - q_t^-)$.

7. $d_t^- \leq p_t \leq \tilde{p}_t \leq d_t^+$

   - If $d_t = d_t^+$, set $d_t = d_t^-$. The master objective decreases by $q_t(\tilde{p}_t - d_t^-) + e_t(d_t^+ - \tilde{p}_t)$, while the SG' objective decreases by a larger quantity, $q_t(p_t - d_t^-) + e_t(d_t^+ - p_t)$.
   - If $e_t = e_t^+$, set $e_t = e_t^-$. If $d_t = d_t^+$, the master objective decreases by $(d_t^+ - \tilde{p}_t)(e_t^+ - e_t^-)$, while the SG' objective decreases by a larger quantity, $(d_t^+ - p_t)(e_t^+ - e_t^-)$. If, on the other hand, $d_t = d_t^-$, neither the master nor the SG' objective changes.
   - If $q_t = q_t^+$, set $q_t = q_t^-$. If $d_t = d_t^-$, the master objective increases by $(\tilde{p}_t - d_t^-)(q_t^+ - q_t^-)$, while the SG' objective increases by a smaller quantity, $(p_t - d_t^-)(q_t^+ - q_t^-)$. If, on the other hand, $d_t = d_t^+$, neither the master nor the SG' objective changes.

8. $d_t^- \leq \tilde{p}_t \leq p_t \leq d_t^+$

   - If $d_t = d_t^-$, set $d_t = d_t^+$. The SG' objective increases by $q_t(p_t - d_t^-) + e_t(d_t^+ - p_t)$, while the master objective increases by a larger quantity, $q_t(\tilde{p}_t - d_t^-) + e_t(d_t^+ - \tilde{p}_t)$.
   - If $e_t = e_t^-$, set $e_t = e_t^+$. If $d_t = d_t^+$, the SG' objective increases by $(d_t^+ - p_t)(e_t^+ - e_t^-)$, while the master objective increases by a larger quantity, $(d_t^+ - \tilde{p}_t)(e_t^+ - e_t^-)$. If, on the other hand, $d_t = d_t^-$, neither the master nor the SG' objective changes.
   - If $q_t = q_t^-$, set $q_t = q_t^+$. If $d_t = d_t^-$, the SG' objective decreases by $(p_t - d_t^-)(q_t^+ - q_t^-)$, while the master objective decreases by a smaller quantity, $(\tilde{p}_t - d_t^-)(q_t^+ - q_t^-)$. If, on the other hand, $d_t = d_t^+$, neither the master nor the SG' objective changes.

9. $p_t \leq \tilde{p}_t \leq d_t^- \leq d_t^+$

   - If $e_t = e_t^+$, set $e_t = e_t^-$. The SG' objective decreases by $(d_t - p_t)(e_t^+ - e_t^-)$, while the master objective decreases by a smaller quantity $(d_t - \tilde{p}_t)(e_t^+ - e_t^-)$.

10. $\tilde{p}_t \leq p_t \leq d_t^- \leq d_t^+$

    - If $e_t = e_t^-$, set $e_t = e_t^+$. The master objective increases by $(d_t - \tilde{p}_t)(e_t^+ - e_t^-)$, while the SG' objective increases by a smaller quantity $(d_t - p_t)(e_t^+ - e_t^-)$.

11. $d_t^- \leq d_t^+ \leq p_t \leq \tilde{p}_t$

    - If $q_t = q_t^+$, set $q_t = q_t^-$. The SG' objective increases by $(p_t - d_t)(q_t^+ - q_t^-)$, while the master objective increases by a larger quantity $(\tilde{p}_t - d_t)(q_t^+ - q_t^-)$.

12. $d_t^- \leq d_t^+ \leq \tilde{p}_t \leq p_t$

- If $q_t = q_t^-$, set $q_t = q_t^+$. The master objective decreases by $(\tilde{p}_t - d_t)(q_t^+ - q_t^-)$, while the SG' objective decreases by a larger quantity $(p_t - d_t)(q_t^+ - q_t^-)$.

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