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Decision Support

Technology choice and timing with positive network effects

Laura J. Kornish *

The Fuqua School of Business, Duke University, Durham, NC 27708-0120, USA

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Abstract

When two competing and incompatible products coexist in a market, potential users face a choice between the two products and the alternative of deferring the decision. This paper examines the choice between the two substitutes, where each one is subject to a positive network effect. That is, a user of one of the products experiences an increase in the value for each additional person using the same product. We examine this buy or wait problem, either for an individual or a manager making the investment on behalf of a firm, by formulating and analyzing a decision-theoretic model. To model the stochastic evolution of market share, we build on the generalized Polya urn of Arthur et al. [European Journal of Operational Research, 30 (3) (1987) 294], allowing for composition of the market to affect not just the relative market shares but also the absolute growth rate of the market. We show that the optimal strategy is defined by a pair of market penetration thresholds that depend on the market composition. Looking at the effect of the network effects on the optimal strategy, we find that more pronounced network effects can either raise or lower the penetration thresholds.

Keywords: Dynamic programming; Investment decision; Buy or wait; Competing technologies; Stochastic processes

1. Introduction

In this paper, we examine the choice between two competing and incompatible durable products. Each product conforms to a format or standard which creates positive network effects for users of that product. That is, a user experiences an increase in the value from using a product for each additional person using the same one. Because of this network effect, a market in which two incompatible products coexist creates a decision problem for a potential adopter regarding not only which product to choose, but whether or not to wait to see what other potential adopters are doing. For example, a manager thinking about whether

* Tel.: +1 919 660 7844.
E-mail address: kornish@duke.edu

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it is time to invest in an enterprise resource planning system not only asks himself "SAP or Peoplesoft?" but also considers the option of waiting to see if one of those products will come to dominate the market.

With the advances in networked computing and telecommunications of the last decade, issues related to positive network effects now feature prominently in business and personal technology decision making. For example, decisions about which type of cellular phone and service to buy and which computer processor and operating system to choose rely on assessments of the size of the group that has made or will make the same choice. Users value larger networks because they provide both increased opportunities for communication as well as increased availability of complementary products (Katz and Shapiro, 1994). Both the direct communication and indirect complements network benefits are at the heart of positive feedback: the more users in a network, the more attractive it is for any prospective user to join. Recent popular books such as Shapiro and Varian's (1999) *Information Rules* and Gladwell's (2000) *The Tipping Point* have described the pervasiveness of positive feedback in a variety of settings, from high tech markets to eradication of crime in New York subways.

The classic example of the communication benefit is the fax machine. The more users there are to fax, the more value the fax machine has. Examples of the complementary products benefit include computer hardware and software, VCRs and rental tapes, and java and java programmers. David (1985) discusses this issue with respect to the QWERTY format typewriter keyboard. The more popular that format became, the more people decided to learn how to use it and the availability of trained operators made that format more popular still.

Because of these self-reinforcing tendencies, expectations play an important role in the dynamics of network populations. If potential adopters believe that a particular technology will catch on, then they will adopt it, and, in fact, it will catch on. Alternatively, if potential adopters believe that it will not catch on, no one will adopt it and that belief will also be fulfilled. While these self-fulfilling prophecies capture the essential logic of positive feedback systems, this mode of analysis is not particularly helpful to the person facing the choice. It captures neither the inherent uncertainty nor the process by which that uncertainty is resolved. In this paper, we approach this decision problem as a sequential decision under uncertainty.

Using a simple market dynamic that allows for the gradual resolution of uncertainty about adoptions of the products, we derive the structure of the optimal strategy and explore how the market environment affects that rule. Arthur (1989) proposed a model of the evolution of market share in a market with competing technologies. The model formalizes the idea that new consumers are more likely to purchase the product that has the higher proportion of the market. The generalized Polya urn introduced in Arthur et al. (1987) is an intuitively appealing model of a market prone to domination by one standard that allows for an initial period of uncertainty about the outcome: small events early in a process have big consequences later. We further generalize this process to make it more consistent with empirical observations of new product adoption. In particular, we allow for the rate of growth of the market to depend probabilistically on the current market size and composition.

Using a decision-oriented point of view, we use a variation of the generalized Polya urn as our model of stochastic market evolution to analyze the buy or wait decision in the face of competing and incompatible technologies and positive network effects. This framework has a decision-analytic perspective, capturing the fundamental uncertainty about the outcome of the market, the gradual resolution of that uncertainty over time, and the self-reinforcing tendency of the market associated with network effects. We show that the optimal strategy for this problem is a pair of thresholds that depends on the state of the system. The value of waiting comes from two sources in this model. First, waiting allows some of the uncertainty as to which technology will come to dominate the market to be resolved before the decision is made. Second, by waiting, the potential user may join a bigger network. The wait moves the investment closer to a larger stream of benefits, increasing the net value.

In addition, we examine the strength of the network effects (the "tippiness") of the market on the optimal strategy. We offer a technical definition of the strength of the network effects: the more prominent the network effect in the value of product, the more quickly small market leads tend to grow to large market leads, and the less stable is the market initially. In some cases, stronger network effects make waiting less attractive because a small lead by one of the products serves as a strong indication that the market will become dominated by that product. However, stronger network effects can also make waiting more attractive in the case in which the market is moving (probabilistically) towards the product that the decision maker finds less attractive overall.

The contribution of this paper to the operations research/management science literature is to analyze the competing standards decision problem from a decision-oriented perspective, using a stopping problem with sequential resolution of uncertainty. Choice between competing standards has received much attention in the economics literature, but the emphasis has been on the aggregation and efficiency of outcomes. We formulate the decision problem using an intuitively appealing stochastic process which allows for positive feedback in the market, an important characteristic in many technology choice problems.

The paper is organized as follows. Section 2 reviews the related literature. Section 3.1 introduces the formulation of a buy or wait problem and the stochastic process used to describe the market. Sections 3.2 and 3.3 present the two main results about the form of the optimal strategy and the strength of network effects. Section 4 concludes.

2. Related literature

This work draws on decision-theoretic models of technology choice in the fields of operations research/management science and economics. In addition, technology adoption models have strong links to work on the diffusion of innovations from the marketing literature, models of group behavior in sociology, and to work on standards adoption in the economics literature.

As a problem of technology choice with the option to defer purchase, our work is similar in orientation to that of Jensen (1982) who looks at the decision regarding information collection about a new technology and to that of McCardle (1985) who looks at the decision regarding information gathering vs. adoption or dismissal of a new technology. In Jensen (1982), information about a new technology is costlessly observed, and the decision maker can decide to adopt the new technology at any point in time. In McCardle (1985), the decision maker can take costly observations on the new technology, and therefore may choose to reject the technology and maintain the firm's current practice. The decision maker in that model collects information on a single new technology, but the fallback option makes the problem a choice between two technologies, like our formulation. However, in our work, the values of both technologies are uncertain, in the work of McCardle (1985), only the value of the new technology is uncertain. Another, important difference is our consideration of network effects: in our work, the value from adoption depends not only on the current estimate of a technology's stand-alone profitability but also on the number of current and future adopters of the same product.

Other decision-theoretic models of technology choice have explicitly examined multiple technologies, although the competing technologies have been sequential innovations, not simultaneously available choices. Balcer and Lippman (1984), Hopp and Nair (1991), and Nair and Hopp (1992) consider keep or replace problems in the face of improving technology. In Balcer and Lippman (1984), indefinitely many new technologies are forecast and indefinitely many purchases are allowed. Both the timing and the size of the innovations are uncertain. In Hopp and Nair (1991) and Nair and Hopp (1992), there is one technology in use, a better one available, and an even better one not yet available. The appearance time of the "even better" one is uncertain. An important difference between these three papers and our work is that in our model, the stochastic description of the environment focuses on the evolution of market size and share as opposed to the trajectory of technical development.

Studies of technology adoption have natural links to the marketing literature on the diffusion of innovations, spawned by the seminal work of Bass (1969). The basic idea of ''word of mouth'' or social contagion is related to the idea of positive network effects, as both explain how larger groups of adopters attract further adopters. The word of mouth mechanism operates through awareness that influences further adoption, and the positive network effects mechanism operates through increases in value which make adoption without delay a more attractive alternative. In this stream of diffusion research in marketing, the multi-product diffusion model of Peterson and Mahajan (1978) is most relevant to our work in its application to adoption trajectories of competing products. Most of the diffusion work (including Peterson and Mahajan) offers deterministic forecasts. This forecast orientation differs from our work's choice orientation, and the deterministic trajectory differs from our stochastic process describing the evolution of adoptions. However, we incorporate an important insight from these diffusion studies, that adoption rates depend on the size of the installed base, in modeling the stochastic evolution of the market.

A related stream of work about social contagion can be found in the sociology literature. In the ''threshold models'' such as Granovetter (1978) and Granovetter and Soong (1983), used to explain riots, fashions, party departure, and other group behavior, exogenous participation thresholds give rise to deterministic dynamic systems. Unlike those exogenous thresholds, the thresholds in our model are endogenous and dynamic; they change as the state changes.

Issues of new product adoption have also been written about extensively in the economics literature. A group of scholars in economics has written about standards and technology adoption. Katz and Shapiro (1985, 1986), Farrell and Saloner (1985, 1986), and Choi (1994) all consider versions of game-theoretic models of technology choice. All of these papers model and solve simple decision problems, looking at the welfare effects of aggregate behavior. The existence of multiple equilibria (e.g., Katz and Shapiro, 1985) and instantaneous resolution of the standards uncertainty (e.g., Farrell and Saloner, 1986) through rational expectations diminish the usefulness of these models for decision support.

Two papers that explicitly look at the adoption timing decision are Choi (1994) and Farrell and Saloner (1985). Choi (1994) analyzes a two-period model in which the first consumer has the option to wait to buy until the second person arrives in the second period. Choi analyzes the role and magnitude of the externalities associated with the decision. Farrell and Saloner (1985) model an adoption timing game between two heterogenous players who both have the choice of switching to a new technology now or waiting until the second period (and staying with the status quo in the current period). The authors describe the ''bandwagon effect'' in which some types of players will wait until the second period, and then adopt only if a bandwagon has formed, that is, if the other player has adopted. Farrell and Saloner (1985) note that their model is ''timeless'' (p. 82) in that value from adoption is determined by how many adopt, but not when. Farrell and Saloner (1985) followed up in their 1986 paper with a model that features valuation of a stream of benefits over time; however, the results from their ''Model with New Users'' show that adoptions happen either as soon as possible or never. Our emphasis is on the long term evolution of the market and the corresponding series of decisions under uncertainty.

3. Model and analysis

In this section, we first develop a model for a decision maker's choice and then offer the two main results of the paper.

This decision-theoretic model considers the optimal choice given a probabilistic description of the market environment. We build on the generalized Polya urn model (see Arthur et al., 1987) to represent the stochastic market evolution of two competing, incompatible technologies. This urn model captures the idea that the market leader has momentum in gaining even more market share. In the basic Polya urn model (Polya and Eggenberger, 1923; see also Feller, 1966), the urn has an initial endowment of two types of balls;

we call them products X and Y. In each period, a ball is drawn from the urn and replaced along with an additional ball of the same type. One interpretation of the basic Polya urn process is a word of mouth effect, where new customers randomly encounter a person who has made a purchase and buy the same product type as that person.

With the generalization of Arthur et al. (1987), the probability of drawing a type is not necessarily *equal* to the proportion of that type in the urn, but is a function of the proportion. We add a further generalization, allowing the rate of adoptions to vary with the composition of the market. In the Arthur et al. model, one ball (i.e., user) is added each period. In our model, either one user or no users are added each period: the larger the population of each type, the more likely that a user of that type is actually added.

3.1. Model

In our model of the market, each period a potential user either adopts X or Y or does not adopt (i.e., there are zero adoptions or one adoption each period). Each potential user is inclined to choose X or Y with probability equal to a function q of the proportion of people already using that type. Instead of a guaranteed adoption each period, as in the Arthur et al. (1987) generalized Polya urn, the potential user adopts the type he is inclined toward with probability equal to a function g of the users of that type. (He adopts nothing with the complementary probability.) Of course, we assume throughout that q and g are increasing functions. This model captures two types of feedback: a larger user group attracts more potential users *and* makes it more likely that they will actually join the group by adopting.

The decisions of the population at large are not modeled explicitly, nor assumed to be optimal. Instead, in this decision-theoretic view, we analyze a decision maker's alternatives using a stochastic description of the environment he faces. The decision maker can observe the market share evolution of the two incompatible technologies; at any point in time, he can buy one of the products. To capture the positive network externalities, the decision maker's value of having either product in any period is increasing in the number of users who have the same product in that period. The decision maker observes the evolution and can buy only one of the products which he keeps forever. [1]

We define $V(x,y)$ as the value function when there are x X-users and y Y-users ($t \equiv x+y$). The infinite horizon value $V(x,y)$ is the maximum value of three alternatives: buy X now, buy Y now, and wait. The one-period discount factor is δ, the cost of purchasing technology X is K^X, and the cost of purchasing technology Y is K^Y. The probability $q\left(\frac{x}{t}\right)$ that the next potential user is inclined toward X is an increasing function of the proportion x/t of X-users. The probability $g(x)$ that the potential user adopts X given he is inclined toward it is an increasing function of the number of X-users. [2] Hence,

$$V(x,y) = \max \begin{cases} V^X(x,y) - K^X, \\ V^Y(x,y) - K^Y, \\ \delta\left\{\left[q\left(\frac{x}{t}\right)(1-g(x)) + \left(1-q\left(\frac{x}{t}\right)\right)(1-g(y))\right]V(x,y) \right. \\ \quad \left. + q\left(\frac{x}{t}\right)g(x)V(x+1,y) + \left(1-q\left(\frac{x}{t}\right)\right)g(y)V(x,y+1)\right\}. \end{cases} \tag{1}$$

The expression $V^X(x,y)$ is the expected value of owning X when there are x Xs and y Ys,

$$\begin{aligned} V^X(x,y) = f^X(x) + \delta\Big\{&\Big[q\left(\frac{x}{t}\right)(1-g(x)) + \left(1-q\left(\frac{x}{t}\right)\right)(1-g(y))\Big]V^X(x,y) \\ &+ q\left(\frac{x}{t}\right)g(x)V^X(x+1,y) + \left(1-q\left(\frac{x}{t}\right)\right)g(y)V^X(x,y+1)\Big\} \end{aligned}$$

[1] The extension allowing the buyer to switch products after a purchase is briefly discussed in the conclusion.

[2] For simplicity, we use the same function g for the probability that a potential user inclined toward Y will adopt Y. The results we present do not depend on this similarity.

with $f^X(x)$ the per-period benefit of owning product X, as a function of the number x of other people who own product X in the period. The expression for $V^X(x,y)$ is comprised of the immediate reward plus the discounted future reward, and the latter value conditions on the outcome of the potential user in the next period ($V^X(x,y)$—he does not buy, $V^X(x+1,y)$—he buys X, or $V^X(x,y+1)$—he buys Y). The expression for the value of waiting (the third element of $V(x,y)$) is built up in a similar fashion. The function f^X can have both network-size-independent and network-size-dependent parts. In order for f^X to capture the positive network externalities, we assume throughout that f^X is increasing.

Similarly, we define $V^Y(x,y)$ as the expected value of owning Y when there are x Xs and y Ys.

$$
\begin{aligned}
V^Y(x,y) = f^Y(y) + \delta\Big\{ & \Big[q\Big(\frac{x}{t}\Big)(1-g(x)) + \Big(1-q\Big(\frac{x}{t}\Big)\Big)(1-g(y))\Big]V^Y(x,y) \\
& + q\Big(\frac{x}{t}\Big)g(x)V^Y(x+1,y) + \Big(1-q\Big(\frac{x}{t}\Big)\Big)g(y)V^Y_{t+1}(x,y+1)\Big\}.
\end{aligned}
$$

The expressions for $V^X(x,y)$ and $V^Y(x,y)$ hold for $x < t$. [3]

3.2. Optimal strategy

In this section we show the structure of the optimal strategy. For any market size t, there are two thresholds such that for a number of X-users below the lower threshold, the best alternative is to buy Y, for a number of X-users above the higher threshold, the best alternative is to buy X, and in the middle, the best alternative is to wait.

Fig. 1 shows the values of the three strategies for a numerical example.

Proposition 1. *For each t, there exists a pair of numbers $x^U(t)$ and $x^L(t)$ with $x^U(t) \geqslant x^L(t)$ such that if $x < x^L(t)$, it is optimal to buy Y; if $x \geqslant x^U(t)$, it is optimal to buy X; otherwise, it is optimal to wait.*

Proof. See Appendix A. □

The result allows for the possibility that no matter what the market shares, it may not be optimal to buy at a particular stage of the process. For a low total population of the market, it may be better to wait even if nearly one hundred percent of the users have one type. But as the market grows, waiting can become suboptimal for any proportion of X: it is possible that $x^U(t) = x^L(t)$. This case occurs for the following reasons. First, there are enough other users such that the value from purchasing one of the technologies is high enough to justify its cost. Second, when the user population is sufficiently large, an imbalance in favor of one type serves as a strong signal that that type will have the majority of the market in the long run.

This result is similar in structure to the main result in McCardle (1985) in which information gathering on a single new technology proceeds until the estimate of value surpasses an upper threshold or dips below a lower threshold. In our model, the information (which comes in the form of adoptions of others) not only updates the decision maker's valuation of adopting each product but also affects the value through the positive network effect.

3.3. Effects of network strength

Now we look at the effect of an important market characteristic, the forcefulness of the positive feedback, on the optimal strategy. When the self-reinforcing effect in the market is very strong, the market will quickly become dominated by one product, and the other will cease to be adopted further. Strong feedback

[3] This formulation assumes that the decision maker's own purchase does not affect the evolution of the market share. The effect of the decision maker's purchase on the market evolution can be captured with a simple modification to the value function.

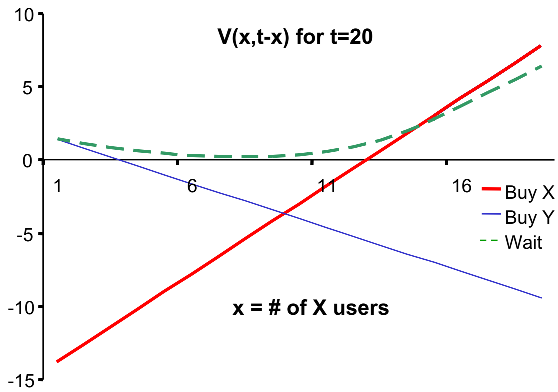

Fig. 1. A simple example of a value function for $t = 20$. For $x > 15$, buy X; do not buy Y for any x. This example has $f^X(x) = 1 + 0.2x$, $f^Y(t - x) = 2 + 0.1\ (t - x)$, $K^X = K^Y = 20$, $\delta = 0.8$, $q(x/t) = x/t$, $g(x) = g(y) = 1$.

makes for a "tippy" market which is highly susceptible to a winner-take-all logic: small leads grow to insurmountable leads quickly.

What makes some markets more tippy than others? This characteristic of the market is related to the relative strength of the network-size-dependent benefits compared to the network-size-independent benefits of the two products. For example, the more valuable are the file-sharing capabilities of handheld computers relative to the stand-alone datebook features, the stronger the network effect. Similarly, the more costly it is for a firm to deviate from its peers in its choice of order handling software because of the decreased availability of technical expertise or support, the stronger the network effect. Strong network effects initially have a destabilizing effect on the market. When a product (e.g., a teleconferencing apparatus) has little or no stand-alone value, but derives its value from the network it is a part of, the market will have a very strong tendency to tip, that is, become dominated by one standard.

Because this is a decision-theoretic model, there is an asymmetry: the decision maker's value and decision are explicitly modeled; the value and behavior of the other market participants are summarized via the urn model. The decision maker's positive network effect is captured by using a value function that is increasing in the number of people who use the same product. The strength of the network effect for the other market participants is encoded in the q and g functions that govern the process. We now explore how changes in the q function affect the decision maker's optimal policy.

Functions that have an s-shape such as the q_1 and q_2 shown in Fig. 2 represent a market with a positive feedback effect, or self-reinforcing tendency: when the proportion of X is low, the next potential user is less likely than the proportion to be inclined toward X; when the proportion of X is high, the next potential user is more likely than the proportion to be inclined toward X. In Fig. 2, for both q_1 and q_2, for proportions of X below the fixed point $\hat{p}$, the market is probabilistically moving toward Y, and for proportions of X above $\hat{p}$, the market is moving toward X. An s-shaped form of q is achieved by potential users randomly encountering a sample of existing users and being inclined toward the product type in the majority of their sample. Any "at least j out of n" rule has a similar shape. Adoption processes that have an s-shaped relationship between proportion and probability will be winner-take-all markets, that is, one of the technologies will eventually become dominant and the other will cease to attract new adopters.

To look at the effect that the prominence of network effects has on the decision rule, we offer a technical definition allowing us to say one q function implies a stronger network effect than another. We limit our comparison to functions that have fixed points at 0 and 1 and share a fixed point $\hat{p}$ in the interval $(0, 1)$. According to the definition below, in Fig. 2, q_2 implies a stronger network effect than q_1; q_2 has a "steeper" s-shape than q_1.

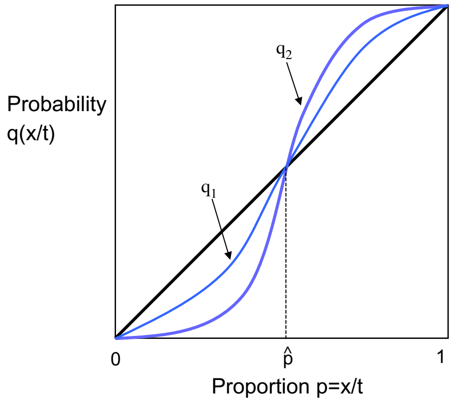

Fig. 2. Two s-shaped functions: q_2 has stronger network effects than q_1.

Definition. Let functions q_1 and q_2 be such that $q_1(0) = q_2(0) = 0$, $q_1(1) = q_2(1) = 1$, and $\hat{p}$ a shared fixed point in $(0, 1)$: $q_2(\hat{p}) = q_1(\hat{p}) = \hat{p}$. If $q_1'(\hat{p}) > 1$ and $q_2'(\hat{p}) > 1$, and if for all $0 < p < \hat{p}$, $q_2(p) < q_1(p)$ and for all $\hat{p} < p < 1$, $q_2(p) > q_1(p)$, then q_2 *implies a stronger network effect than* q_1.

Proposition 2 presents the effect on the optimal thresholds of a change in the strength of the network effects in the market. We use the notation $x^{iU}(t)$ and $x^{iL}(t)$ to denote the upper and lower thresholds, respectively, when the probability that the next user is inclined toward X is $q_i(\frac{x}{t})$.

Proposition 2

(i) *If* q_2 *implies a stronger network effect than* q_1 *and* $\hat{p} > (<, =)\frac{x^{1U}(t)}{t}$, *then* $x^{2U}(t) > (<,=)x^{1U}(t)$.
(ii) *If* q_2 *implies a stronger network effect than* q_1 *and* $\hat{p} > (<, =)\frac{x^{1L}(t)}{t}$, *then* $x^{2L}(t) > (<,=)x^{1L}(t)$.

Proof. See Appendix A. □

Proposition 2 shows that network effects can either serve to increase or decrease the upper and lower thresholds depending on the relative position of the threshold and the fixed point $\hat{p}$ of the q_i functions. The results from Proposition 2 can be summarized in a single statement: Each threshold moves toward the fixed point $\hat{p}$ when the market has stronger network effects.

When the upper threshold is very high, stronger network effects move the threshold down: one needs a smaller proportion of X-users for it to be optimal to buy X. For example, if at the beginning of the process, it is always optimal to wait, a strong (enough) network effect will make buying X the best choice for very high levels of X. In the extreme, for a very tippy market, one is virtually assured of making the right choice, so uncertainty resolution becomes a less compelling reason for delaying purchase.

A special case of this configuration comes in a symmetric formulation with $f^X(x) = f^Y(y)$ and a q function with $\hat{p} = \frac{1}{2}$ (which comes from a "majority of n" rule, for example). Strengthening the network effects always decreases the upper threshold and increases the lower threshold; the "continuation region" shrinks in this special, symmetric case.

When the upper threshold is lower than the fixed point $\hat{p}$ of q, stronger network effects increase that threshold. This relationship will occur when the benefits (stand-alone as well as marginal value of a network member) of owning X are significantly greater than the benefits of owning Y. When the proportion of Xs is below $\hat{p}$, the process is moving (probabilistically) in the direction of becoming a Y-dominated market. In this case, however, because of the relative attractiveness of X, it is optimal to buy X for some market proportions that indicate that Y will win eventually. The short-run benefits of X outweigh the long-term prospects of Y. When the process becomes tippier, Proposition 2 informs us that both thresholds move up (because the urn is tipping more forcefully toward Y) so "buy X" is optimal over a smaller region and "buy Y" is optimal over a larger region.

To examine this result in the context of a current example, consider the recent developments with radio-frequency identification (RFID) tags in applications that track the movement of goods through a supply chain. The technology has been around for a decade, and while there exist pockets of adoption, the market has not yet settled on standards for some of the technology's attributes. In particular, for supply chain applications, two main competing frequency ranges are available: high frequency (HF), 13.56 MHz, which reads tags up to three feet, and ultra-high frequency (UHF), 300 MHz to 1 GHz, which reads tags at 10–20 ft.

The popular press has many examples of firms' "buy or wait" deliberations. In a recent article in *Electronic Business*, Procter and Gamble (P & G) executives discuss the buy or wait question for RFID. So far, waiting still makes sense for them, "We're still in the investigating phase and not ready to make a sizable commitment" (Stackpole, 2003). P & G plans to adopt RFID for their supply chain at some point in time, but they will wait until more firms have adopted, allowing for the possibility that the market will have settled on one of the frequency ranges, or at least have a stronger inclination to be moving in the direction of one. In *Baseline* (Dignan, 2004), the manager at a California shipping facility states the dilemma bluntly. "One of the reasons [he] is less-than-enthusiastic about these systems is the lack of standards. 'I'd have to wait and see what shakes out....Standardization has to happen first." The urn model is an apt metaphor for thinking about the market's evolution and the buy or wait decision.

This RFID example is also relevant to Proposition 2 because of the importance of complementary products that add value to the tracking capabilities of the technology, for example, "intelligent shelves" that know when a product is out of stock or software applications that let supply chain partners share and analyze the data produced by the RFID tag system. The proliferation of complementary products increases the prominence of network-size effects, making the market tippier. Proposition 2 implies that the wider the availability of these complementary, value-adding products, the more the optimal strategy changes. If a firm has no inherent preference for one frequency over the other, but cares solely about matching what other market participants are doing (the special, symmetric case), then the stronger network effects shrink the continuation region. In other words, the complementary products make waiting less attractive. If the firm does have an inherent preference for the UHF product (and of course also cares about what other market participants choose), then stronger network effects should make the firm more cautious in investing in UHF, their preferred choice. The range of market penetration (UHF market share) for which the market is moving toward HF yet it is still optimal for the firm to buy UHF is smaller in the tippier market.

In the different cases, the optimal strategy reacts differently to prominence of the positive feedback effects in the market. The direction of the effect on the optimal strategy depends on the interaction between the range of proportions of the market for which the market is "moving toward X" vs. "moving toward Y" and the relative attractiveness to the decision maker, including network-size-dependent and network-size-independent parts, of each of the products.

4. Conclusion

In this paper we have studied the decision problem facing a consumer or a manager with a choice between two incompatible competing technologies each subject to positive network benefits. While much has been written about the economics of competing standards, most of the work on this subject in the economics literature is motivated by efficiency and policy considerations. We study the problem from the point of view of a decision maker who, uncertain about the future of the market, seeks insight into his problem.

To do this, we built a decision theoretic model for the choice and used a stochastic process that captures the dynamics of market share of two competing technologies. We characterized the optimal strategy to the buy or wait problem as a pair of thresholds and analyzed the effect of the strength of network effects on those thresholds.

Some unaddressed issues bear mention. First, we have assumed that a purchase stops the search process, without allowing switching at a later date. We conjecture that including the possibility of switching will reduce the hurdles for purchase (i.e., will shrink the continuation region). Second, we have taken a decision-theoretic approach to this problem. There are many challenges in a similar game theoretic approach that we have not pursued. For example, can the proposed stochastic process, or some variation of it, be derived from the optimizing behavior of heterogeneous individuals? One challenging aspect of that line of inquiry is handling the multiple coordination equilibria in the dynamic setting.

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Appendix A

A.1. Proof of Proposition 1

To prove this proposition, first we show that the value of owning X is increasing in x and decreasing in y (Lemma 1) and that the *difference* between the optimal value and the value of owning X is decreasing in x and increasing in y (Lemma 2). ("Decreasing" does not imply strictly decreasing; we use it to mean nonincreasing. Likewise, increasing is used in the sense of nondecreasing.) The result follows directly from the second lemma.

Lemma 1. *If f^X, g, and q are increasing, then $V^X(x,y)$ is increasing in x and decreasing in y.*

Proof. Throughout these proofs, we use induction on the number of "decision epochs" remaining. Define $V^{iX}(x,y)$ as the value of owning X when there are i periods to go in the process and there are x Xs and y Ys ($t \equiv x+y$). We define $V^{0X}(x,y) \equiv 0$.

We have two inductive assumptions:

1. $V^{nX}(x+1,y) \geqslant V^{nX}(x,y)$, ($V^{nX}$ increasing in x) and
2. $V^{nX}(x,y) \geqslant V^{nX}(x,y+1)$ (V^{nX} decreasing in y).

The relationship between $V^{n+1X}(x,y)$ and $V^{nX}(x,y)$ is

$$V^{n+1X}(x,y) = f^X(x) + \delta\Big\{\Big[q\Big(\frac{x}{t}\Big)(1-g(x)) + \Big(1-q\Big(\frac{x}{t}\Big)\Big)(1-g(y))\Big]V^{nX}(x,y) + q\Big(\frac{x}{t}\Big)g(x)V^{nX}(x+1,y) + \Big(1-q\Big(\frac{x}{t}\Big)\Big)g(y)V^{nX}(x,y+1)\Big\}.$$

Both parts of the induction hypothesis hold for $n=1$ because $V^{1X}(x,y) = f^X(x)$, which is increasing in x and constant in y.

What remains to be shown is that

1. $V^{n+1X}(x+1,y) \geqslant V^{n+1X}(x,y)$, ($V^{n+1X}$ increasing in x) and
2. $V^{n+1X}(x,y) \geqslant V^{n+1X}(x,y+1)$ (V^{n+1X} decreasing in y).

Both of the induction hypotheses will be approached the same way. The expressions for the RHS and LHS of the inequalities contain convex combinations of V^{nX} terms. We establish the inequalities by showing that the corresponding weighting schemes can be compared via first order stochastic dominance, which in turn implies that the desired weighted value is higher.

First, we address the hypothesis that V^{n+1X} is decreasing in y: $V^{n+1X}(x,y) \geqslant V^{n+1X}(x,y+1)$. The first column of Table 1 contains the V^{nX} terms that appear in the V^{n+1X} expressions and the respective weights. The terms appear in a particular order in the table: ascending value where the ranking is clear from the assumptions, and "worst case" where there is not a clear ranking. Based on the inductive assumptions, some of the terms can be ordered: $V^{nX}(x,y+2) \leqslant V^{nX}(x,y+1) \leqslant V^{nX}(x,y) \leqslant V^{nX}(x+1,y)$. Further, we know that $V^{nX}(x,y+1) \leqslant V^{nX}(x+1,y+1) \leqslant V^{nX}(x+1,y)$. The terms $V^{nX}(x+1,y+1)$ and $V^{nX}(x,y)$ cannot be compared. The order used in the table is the least favorable order for the desired inequality ($V^{n+1X}(x,y) \geqslant V^{n+1X}(x,y+1)$) to hold.

To show stochastic dominance of $V^{n+1X}(x,y)$ over $V^{n+1X}(x,y+1)$, we need to show that the cumulative probability distribution for $V^{n+1X}(x,y)$ lies below the distribution for $V^{n+1X}(x,y+1)$. Both of the distributions have only three points, so it suffices to show (1) that the low, middle, and high points in the $V^{n+1X}(x,y)$ distribution are not lower than the low, middle, and high points respectively in the other distribution, (2) that the weight on the lowest point in the $V^{n+1X}(x,y)$ distribution is lower than the weight on the lowest point in the other distribution, and (3) that the weight on the highest point in the $V^{n+1X}(x,y)$ distribution is higher than the weight on the highest point in the other distribution. The stochastic dominance implies that the weighted sum of the $V^{n+1X}(x,y)$ terms is higher.

The first condition can be seen by inspecting Table 1: the low, middle, and high points in the middle column are higher than the respective points in the right column. The second and third conditions are met because q and g are increasing:

$$\left(1-q\left(\frac{x}{t+1}\right)\right)g(y+1) \geqslant \left(1-q\left(\frac{x}{t}\right)\right)g(y) \tag{2}$$

and

$$q\left(\frac{x}{t}\right)g(x) \geqslant q\left(\frac{x}{t+1}\right)g(x). \tag{3}$$

Finally, we note that $V^{n+1X}(x,y)$ and $V^{n+1X}(x,y+1)$ each contain a $f^X(x)$ term which cancels out in the comparison.

Table 1
$V^{n+1X}(x,y) \geqslant V^{n+1X}(x,y+1)$

Term	Probability (weight) in $V^{n+1X}(x,y)$	Probability (weight) in $V^{n+1X}(x,y+1)$
$V^{nX}(x,y+2)$		$(1-q(\frac{x}{t+1}))g(y+1)$
$V^{nX}(x,y+1)$	$(1-q(\frac{x}{t}))g(y)$	$q(\frac{x}{t+1})(1-g(x))+(1-q(\frac{x}{t+1}))(1-g(y+1))$
$V^{nX}(x,y)$	$q(\frac{x}{t})(1-g(x))+(1-q(\frac{x}{t}))(1-g(y))$	
$V^{nX}(x+1,y+1)$		$q(\frac{x}{t+1})g(x)$
$V^{nX}(x+1,y)$	$q(\frac{x}{t})g(x)$	

Table 2
$V^{n+1X}(x+1,y) \geqslant V^{n+1X}(x,y)$

Term	Probability (weight) in $V^{n+1X}(x+1,y)$	Probability (weight) in $V^{n+1X}(x,y)$
$V^{nX}(x,y+1)$		$(1-q(\frac{x}{t}))g(y)$
$V^{nX}(x+1,y+1)$	$(1-q(\frac{x+1}{t+1}))g(y)$	
$V^{nX}(x,y)$		$q(\frac{x}{t})(1-g(x))+(1-q(\frac{x}{t}))(1-g(y))$
$V^{nX}(x+1,y)$	$q(\frac{x+1}{t+1})(1-g(x+1))+(1-q(\frac{x+1}{t+1}))(1-g(y))$	$q(\frac{x}{t})g(x)$
$V^{nX}(x+2,y)$	$q(\frac{x+1}{t+1})g(x+1)$	

Next we address the hypothesis that $V^{n+1X}(x+1,y) \geqslant V^{n+1X}(x,y)$. The terms and weights are summarized in Table 2.

Based on the inductive assumptions, some of the terms can be ordered: $V^{nX}(x,y+1) \leqslant V^{nX}(x,y) \leqslant V^{nX}(x+1,y) \leqslant V^{nX}(x+2,y)$. Further, we know that $V^{nX}(x,y+1) \leqslant V^{nX}(x+1,y+1) \leqslant V^{nX}(x+1,y)$. The terms $V^{nX}(x+1,y+1)$ and $V^{nX}(x,y)$ cannot be compared. The order used in the table is the least favorable order for the desired inequality $V^{n+1X}(x+1,y) \geqslant V^{n+1X}(x,y)$ to hold.

Once again, all three conditions for stochastic dominance are satisfied: (1) the relative positions of the points, (2) the relative weight on the lowest points:

$$\left(1-q\left(\frac{x}{t}\right)\right)g(y) \geqslant \left(1-q\left(\frac{x+1}{t+1}\right)\right)g(y) \tag{4}$$

and (3) the relative weight on the highest points:

$$q\left(\frac{x+1}{t+1}\right)g(x+1) \geqslant q\left(\frac{x}{t}\right)g(x). \tag{5}$$

Finally, the $f^X(x+1)$ term in $V^{n+1X}(x+1,y)$ is greater than the $f^X(x)$ term in $V^{n+1X}(x,y)$.

Therefore, the inductive hypotheses hold and because for $\delta < 1$, $\lim_{n\to\infty} V^{nX}(x,y)=V^X(x,y), V^X(x,y)$ is increasing in x and decreasing in y. □

Lemma 2. *If f^X, q, and g are increasing, then $A(x,y) = V(x,y) - V^X(x,y)$ is decreasing in x and increasing in y.*

Proof. Again, we use induction on the number of periods left. Define $A^i(x,y) = V^i(x,y) - V^{iX}(x,y)$, with $V^{iX}(x,y)$ the value of owning X when there are i periods to go in the process and there are x Xs and y Ys. We denote $V^i(x,y)$ as the value associated with making optimal choices when there are i periods to go in the process. As above, $V^{0X}(x,y) \equiv 0$ and we define $V^0(x,y) \equiv 0$.

The relationship between A^{n+1} and the V^n and V^{nX} terms is as follows.

$$\begin{aligned} A^{n+1}(x,y) &= V^{n+1}(x,y) - V^{n+1X}(x,y) \\ &= \max\left\{\begin{array}{l} V^{n+1X}(x,y) - K^X, \\ V^{n+1Y}(x,y) - K^Y, \\ \delta\left\{\left[q\left(\frac{x}{t}\right)(1-g(x)) + \left(1-q\left(\frac{x}{t}\right)\right)(1-g(y))\right]V^n(x,y)\right. \\ \quad \left. + q\left(\frac{x}{t}\right)g(x)V^n(x+1,y) + \left(1-q\left(\frac{x}{t}\right)\right)g(y)V^n(x,y+1)\right\} \end{array}\right\} - V^{n+1X}(x,y). \end{aligned}$$

First, show $A^1(x,y) = V^1(x,y) - V^{1X}(x,y)$ decreasing in x and increasing in y.

$$A^1(x,y) = V^1(x,y) - V^{1X}(x,y) = \max\left\{f^X(x) - K^X, f^Y(y) - K^Y, 0\right\} - f^X(x). \tag{6}$$

There are three cases: $f^X(x) - K^X - f^X(x) = -K^X$, $f^Y(y) - K^Y - f^X(x)$, and $-f^X(x)$. They are all decreasing in x and increasing in y.

The inductive assumptions are that $A^n(x,y)$ is decreasing in x and increasing in y. And we will show that $A^{n+1}(x,y)$ decreasing in x and increasing in y.

Using the equation for $A^{n+1}(x,y)$ above, we can see that if $V^{n+1}(x,y)$ comes from the value of buy X or from the value of buy Y, the desired result is immediate based on Lemma 1. The third possibility, that the value of $V^{n+1}(x,y)$ comes from the value of waiting, is examined in more detail below:

$$\begin{aligned}
A^{n+1}(x,y) &= \delta\Big\{\Big[q\Big(\frac{x}{t}\Big)(1-g(x)) + \Big(1-q\Big(\frac{x}{t}\Big)\Big)(1-g(y))\Big]V^n(x,y) \\
&\quad + q\Big(\frac{x}{t}\Big)g(x)V^n(x+1,y) + \Big(1-q\Big(\frac{x}{t}\Big)\Big)g(y)V^n(x,y+1)\Big\} - V^{n+1X}(x,y) \\
&= \delta\Big\{\Big[q\Big(\frac{x}{t}\Big)(1-g(x)) + \Big(1-q\Big(\frac{x}{t}\Big)\Big)(1-g(y))\Big]A^n(x,y) \\
&\quad + q\Big(\frac{x}{t}\Big)g(x)A^n(x+1,y) + \Big(1-q\Big(\frac{x}{t}\Big)\Big)g(y)A^n(x,y+1)\Big\} - f^X(x).
\end{aligned}$$

At this point, the proof is exactly analogous to the proof for Lemma 1. The inequality $A^{n+1}(x,y) \geqslant A^{n+1}(x+1,y)$ holds because the LHS and the RHS are weighted averages of three points and the distribution for $A^{n+1}(x,y)$ stochastically dominates the distribution for $A^{n+1}(x+1,y)$. Table 2 applies, with all the Vs changed to As. And the terms are shown in descending order, or, in cases in which the inductive assumptions don't provide a clear ordering, worst case ordering. The weighted averages have an ordering consistent with $A^{n+1}(x,y) \geqslant A^{n+1}(x+1,y)$ and $-f^X(x+1) < -f^X(x)$. The weights for the inequality $A^{n+1}(x,y+1) \geqslant A^{n+1}(x,y)$ can be found in Table 1 with the same substitution of A for V, and terms in descending not ascending order.

The proposition follows directly from Lemma 2. If $A(x,y) \equiv V(x,y) - V^X(x,y)$ is decreasing in x and increasing in y, then

$$A(x+1,t-x-1) \leqslant A(x,t-x) \leqslant A(x-1,t-x+1).$$

For a given t, if at some point x, $A(x,t-x) + K^X = 0$ (the point at which buy X becomes optimal), then for any $x' > x$ and $t - x' < t - x$, $0 \geqslant V(x',t-x') - V^X(x',t-x') + K^X$. But we know by the definition of $V(x,y)$ that $V(x,y) \geqslant V^X(x,y) - K^X$ for all x. So once buy X becomes optimal, it stays optimal for all greater values of x. The upper cutoff $x^U(t)$ is the lowest x, if any, for which $V(x,t-x) = V^X(x,t-x) - K^X$. The lower cutoff $x^L(t)$ is the highest x, if any, for which $V(x,t-x) = V^Y(x,t-x) - K^Y$. If there is no x for which $V(x,t-x) = V^X(x,t-x) - K^X$ or $V(x,t-x) = V^Y(x,t-x) - K^Y$, then we assign $x^U(t) = \infty$ or $x^L(t) = -\infty$, respectively. □

A.2. Proof of Proposition 2

In the case where $x^{1U}(t) > x^{1L}(t)$, x_t^{1U} is the point of intersection of buy X and wait, i.e., the x such that

$$\begin{aligned}
V^X(x,t-x) - K^X &= \delta\Big\{\Big[q\Big(\frac{x}{t}\Big)(1-g(x)) + \Big(1-q\Big(\frac{x}{t}\Big)\Big)(1-g(t-x))\Big]V(x,t-x) \\
&\quad + q\Big(\frac{x}{t}\Big)g(x)V(x+1,t-x) + \Big(1-q\Big(\frac{x}{t}\Big)\Big)g(t-x)V(x,t-x+1)\Big\}, \\
f^X(x) - K^X &= \delta\Big\{\Big[q\Big(\frac{x}{t}\Big)(1-g(x)) + \Big(1-q\Big(\frac{x}{t}\Big)\Big)(1-g(t-x))\Big]A(x,t-x) \\
&\quad + q\Big(\frac{x}{t}\Big)g(x)A(x+1,t-x) + \Big(1-q\Big(\frac{x}{t}\Big)\Big)g(t-x)A(x,t-x+1)\Big\}. \qquad (7)
\end{aligned}$$

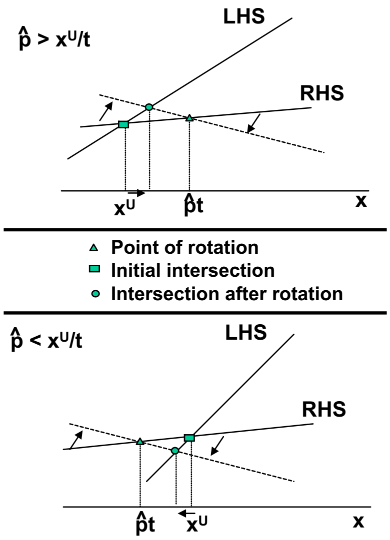

Fig. 3. Figures used in proof of effect of strength of network effects on thresholds for the case of $x^{\mathrm{U}} > x^{\mathrm{L}}$.

The quantity $A(x,y)$ was defined in Lemma 2. From Lemma 2, we know that $A(x,y)$ is decreasing in x and increasing in y: $A(x+1,t-x) \leqslant A(x,t-x) \leqslant A(x,t-x+1)$.

To understand the effect on the point of intersection of buy X and wait, we look at two effects: (a) Making q tippier increases q above $\hat{p}$ and decreases it below $\hat{p}$. (b) An increase in q lowers the RHS of (7) (and a decrease in q raises it).

Effect (a) follows directly from the definition of *stronger network effects*. Effect (b) is derived from the following logic: an increase in q increases the weight $q(\frac{x}{t})g(x)$ on the lowest valued point $A(x+1,t-x)$ and decreases the weight $(1-q(\frac{x}{t}))g(t-x)$ on the highest valued point $A(x,t-x+1)$. A three-point weighted average will unambiguously decrease when the weight on the lowest point is increased and the weight on the highest point is decreased. [4]

Now consider the case in which $\hat{p} > \frac{x^{1\mathrm{U}}(t)}{t}$ (see Fig. 3). Making q tippier rotates the RHS clockwise around $\hat{p}t$ because below $\hat{p}t$, q decreases, increasing the RHS of equation (7), and above $\hat{p}t$, q increases, decreasing the RHS of equation (7). Because the LHS is increasing in x, the RHS rotation increases the point of intersection. Likewise, if $\hat{p} < \frac{x^{1\mathrm{U}}(t)}{t}$, making q tippier rotates the RHS clockwise around $\hat{p}t$. Because the LHS is increasing in x, the RHS rotation decreases the point of intersection. For the special case in which $\hat{p} = \frac{x^{1\mathrm{U}}(t)}{t}$, the RHS rotates around the point of intersection, so that point of intersection does not change.

In the case for which $x_t^{1\mathrm{U}} = x^{1\mathrm{L}}(t) = \hat{x}$, $\hat{x}$ is the point of intersection of buy X and buy Y, i.e., the x such that

$$V^X(x,t-x) - K^X = V^Y(x,t-x) - K^Y,$$

$$\begin{aligned}\frac{K^Y - f^Y(t-x) - K^X + f^X(x)}{\delta} = {} & \left[q\left(\frac{x}{t}\right)(1-g(x)) + \left(1-q\left(\frac{x}{t}\right)\right)(1-g(t-x))\right][V^Y(x,t-x) \\ & - V^X(x,t-x)] + q\left(\frac{x}{t}\right)g(x)[V^Y(x+1,t-x) - V^X(x+1,t-x)] \\ & + \left(1-q\left(\frac{x}{t}\right)\right)g(t-x)[V^Y(x,t-x+1) - V^X(x,t-x+1)].\end{aligned}$$

[4] Consider $a_1x_1 + (1-a_1-a_3)x_2 + a_3x_3$ with $x_1 \leqslant x_2 \leqslant x_3$. If $a_1' = a_1 + \Delta_1$ and $a_3' = a_3 - \Delta_3$, with $\Delta_1, \Delta_3 \geqslant 0$, then $[a_1x_1 + (1-a_1-a_3)x_2 + a_3x_3] - [a_1'x_1 + (1-a_1'-a_3')x_2 + a_3'x_3] = \Delta_1(x_2-x_1) + \Delta_3(x_3-x_2) \geqslant 0$.

From Lemma 1, $V^X(x,y)$ is increasing in x and decreasing in y. By the same logic, $V^Y(x,y)$ is decreasing in x and increasing in y. Therefore, $V^Y(x,y) - V^X(x,y)$ is decreasing in x and increasing in y. If $\hat{p} < \frac{\hat{x}}{t}$, then a tippier q rotates the RHS clockwise around $\hat{p}t$ and reduces the point of intersection.

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