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Optimal Referral Bonuses with Asymmetric Information: Firm-Offered and Interpersonal Incentives

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Referral bonuses, in which an existing customer gets an in-kind or cash reward for referring a new customer, are a popular way to stimulate word of mouth. In this paper, we examine key firm decisions about such bonuses. Others have studied referral bonus programs; a key difference is that we study the role of recommendations not just in spreading awareness (as they do) but also in providing assessments. We start with the idea that people have a variety of reasons for making product recommendations, including placing a value on a friend's outcome with a product they recommend. We apply that idea in a context of asymmetric information: A customer combines his knowledge about the product and his familiarity with friends' tastes, making him more informed than the friends. Thus, the recommendation is a signal about the value of the product to the friend. In this setting, we consistently find that the greater the concern for others' outcomes, the higher the referral bonus should be, as long as the firm cannot more efficiently motivate recommendations with a lower price. Moreover, if price is the more efficient lever, the optimal bonus is zero, and the optimal price is low. We also show that greater concern tends to reduce firm profit and, in some cases, actually reduces consumer welfare as well.

Key words: word of mouth; reward programs; pricing
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1. Introduction

It is well accepted in the marketing and consumer behavior literatures that word of mouth can often have a more persuasive impact than other nonpersonal forms of influence such as advertising (e.g., Kiel and Layton 1981, Price and Feick 1984, Feick and Price 1987). Moreover, there is rationale rooted in economic theory that this difference in impact is even more pronounced for experience goods, i.e., those goods that must be tried to assess quality. Nelson (1974) and Milgrom and Roberts (1986) have argued that for experience goods, "ads...cannot credibly convey much direct information about the product" (Milgrom and Roberts 1986, p. 797). Their conclusion follows from a firm's incentive to exaggerate claims of quality and consumers' tendency to discount those claims. However, the incentives of the parties to word of mouth are different, in part, because personal relationships tend to have one of the key components of "source credibility" (Tellis 1998): trustworthiness. Between the advantage of word of mouth in conveying credible information and the difficulty of doing that with advertising, it is not surprising to see firms trying different techniques to promote word of mouth.

In this paper, we study one such technique, referral bonus programs. A referral bonus is essentially a finder's fee to an existing customer. The customer receives a reward, usually cash or a period of free service, for bringing in a new customer. This type of program is currently quite popular in many settings. For example, four of the five largest U.S. diet programs offer some referral bonus program, as do three of the five largest cable and satellite television operators. We also estimate that over 60% of local apartment complexes offer referral bonus programs as do 75% of national Internet service providers.[1] The prevalence of these programs suggests that many firms deem them effective, and yet, if a customer thinks a friend will like a product, we wondered why that would not be sufficient incentive to prompt a recommendation.

There has been some experimental work on the use of referral bonuses (e.g., Ryu and Feick 2007, Wirtz and Chew 2002) and, to our knowledge, one published analytical model (Biyalogorsky et al. 2001) on the topic. The following research questions guide us: (1) Can customers still make credible recommendations to friends, even with referral bonuses? (2) If we account for the concern customers have for their

[1] These findings are based on data collected in spring 2008 for diet programs, local apartments, and Internet service providers, and fall 2007 for cable and satellite television operators.

friends' outcomes, how do such interpersonal incentives affect the optimal design of bonus programs and pricing schemes? (3) Given the interpersonal incentives, what are the implications of bonus programs for firm profit and consumer welfare?

Like Biyalogorsky et al. (2001), we take the firm's perspective and study the bonus decision together with a pricing decision. Unlike Biyalogorsky et al. (2001), we focus on the case in which customers explicitly care about their friends' satisfaction with their recommendations rather than their own delight with the product. In their work, the referral serves to spread awareness about a new product. In our work, the referral also helps resolve some uncertainty on the part of the potential new customer about the value of the new product: the referral both spreads awareness and provides an assessment. Thus, we explicitly study the asymmetry of information between the provider and the recipient of the referral and the credibility of the referral.

These two key elements—concern for others and asymmetric information—drive the answers to our research questions. On the first question, we find that customers *can* make credible recommendations to friends, even with referral bonuses. Customers balance the value of the bonus to themselves with the net value of the product to their friends. The result is that some recommendations may, in fact, be exaggerated (i.e., have negative expected value), but the information asymmetry permits their credibility.

On the second question, we find systematic relationships between concern for others and the optimal bonus and price levels. The referral bonus should be higher the greater the concern, as long as the firm cannot more efficiently motivate recommendations with a lower price. Moreover, if price is the more efficient lever, the optimal bonus is zero. The results for price are more nuanced. When a bonus is used, greater concern for the marginal group of friends (i.e., those who are on the border between buying and not buying) raises the optimal price, but greater concern for inframarginal friends (at least weakly) lowers the optimal price. An important intuition underlying these findings is that the more a customer cares about a friend, the more selective the customer will be in making recommendations. More selectivity means a smaller quantity of recommendations but with a recommendation implying a higher value. The firm can use a higher bonus to counteract that selectivity and a higher price to leverage that selectivity.

On the third question, we see that greater concern tends to hurt profitability. When a firm uses a bonus, an increase in price and bonus in response to greater concern leaves the same margin on a smaller quantity, hurting the bottom line. However, the welfare consequences are not zero-sum, with greater concern hurting the firm but helping the consumers. Instead, consumers can also suffer in aggregate from greater concern when increases in price erode their surplus.

In interpreting our answers to the research questions, we argue that the intensity with which someone cares about friends' outcomes will vary by product category. Some categories, such as those related to child care or health conditions, are more sensitive with regard to recommendations. In other, more mundane categories such as Internet, cable, telephone, or movie service, people will not be as sensitive to their friends' outcomes based on their recommendations. Sensitivity can arise from social factors influenced by the visibility or importance of the category, or it can arise from financial or physical risk related to the probability of a bad outcome. Furthermore, sensitivity can be rooted explicitly in altruism or it can be more indirect, based on concern for one's own reputation. Interpreting our results in this light, we conclude that the referral bonuses should be higher in the more sensitive categories, unless customers care so much about their friends' outcomes in the category that it is more efficient for the firm to simply lower the price and avoid a bonus program altogether.

The rest of this paper is organized as follows. In §2, we describe the related literature broadly against the backdrop of "social motives" and, more specifically, in the context of incentives for word of mouth. Section 3 explains the signaling game we use to model recommendations. Sections 4 and 5 build on that model of interaction between individuals to examine the firm's problem in setting the optimal bonus and price in "recommendation risk-neutral" and "recommendation risk-averse" settings, respectively. In §6, we discuss the implications of our results and the limitations of the current study.

2. Literature Review

We review the place of our work in both a larger conceptual theory of social motives as well as in the more narrowly defined topic of word-of-mouth incentives.

2.1. Conceptual Theory: Social Motives

Behavorial economists and psychologists have accumulated much evidence to flesh out the idea that "people care about the outcomes of others" (the opening line of Loewenstein et al. 1989). To give some structure to the notion of caring, Scott (1972) and MacCrimmon and Messick (1976) offer lists of social motives including self-interest, altruism, and cooperation. This "social motives" framework has been studied in the context of helping behavior (e.g., saving lives and property, donating time to worthy causes) by Lynch and Cohen (1978), but it has primarily been applied in negotiation, bargaining, and dispute resolution contexts (including the ultimatum and dictator games widely used in behavioral economics; e.g., Güth et al. 1982, Hoffman et al. 1996, and work by Loewenstein et al. 1989 and Corfman and Lehmann 1993).

Our work applies the idea of social motives in a new domain: consumer word-of-mouth behavior for new products. In particular, we use a variation of what MacCrimmon and Messick (1976) call cooperation, which combines altruism and self-interest, to analyze referral behavior and the effect of financial incentives. In our application, we look at the implications for the consumers as well as firms.

2.2. Word-of-Mouth Incentives

As mentioned in §1, the work of Biyalogorsky et al. (2001) is most similar in perspective to our paper. They study customer referral programs and consider the effects of pricing as we do here. They model a customer's delight with a product as the motivating force behind a referral. In contrast, we use a compensatory model in which the customer considers a possible referral bonus together with a factor related to the expected results for the friend who receives the recommendation. We discuss why the friend's uncertainty about the product's value is an important part of recommendation behavior and, therefore, firm decisions.

Since Biyalogorsky et al. (2001) was published, there has been a surge of interest in managing word of mouth without explicit reward programs. The work of Mayzlin and Godes (2009) examines mechanisms for encouraging word of mouth absent reward payments from the firm. In Godes and Mayzlin (2004), the authors describe important aspects of online word of mouth. Mayzlin (2002) investigates how the connectedness of a social network's structure influences the effectiveness of buzz as a marketing instrument and how the network effect moderates the payoff from a firm's investment to promote buzz. In Godes et al. (2005), the authors discuss a firm's active role in managing the social interactions. They suggest that the firm go beyond simply gathering information about conversations and take steps to foster the conversations, such as establishing a customer recommendation program. In our paper, we analyze such active roles by investigating the relative value of manipulating price and bonus.

Chen and Shi (2001) investigate the reward programs and types of rewards in both monopoly and competitive settings. They find that monopolists prefer to give out cash rewards, whereas duopolistic firms may be better off offering future discounts to the customers. In our paper, we are concerned not only with the tangible reward the customer may receive from the firm but also the intangible rewards from making a good recommendation. However, we do not analyze the firm's competitive environment.

Experimental work by Wirtz and Chew (2002) and Ryu and Feick (2007) investigates the effectiveness of referral bonuses. Wirtz and Chew (2002) study the role of deal proneness, satisfaction, and tie strength; and Ryu and Feick (2007) study the role of tie strength, brand strength, and recipient of the reward. Our analytical conclusions are consistent with the experimental results that bonuses can be effective at changing behavior. Our additional contribution is analysis from the firm's perspective about the optimality of bonuses.

There is other recent work in the marketing science literature about the role of and credibility of information sources, for example, the work on referral infomediaries by Chen et al. (2002). In that setting, industry websites such as Autobytel.com point consumers to retail establishments. A key difference from our work is that in the framework of Chen et al. (2002), no personal relationship is involved in the referral. Likewise, Chen and Xie (2005) study third-party product reviews, which also deal with issues of information credibility but not in a personal sense.

3. Signaling Model

In this section, we describe a game of incomplete information, a signaling game, between a customer (the sender of the signal) and a friend (the receiver of the signal). We use the signaling framework to capture the customer's decision about whether to recommend the product and the friend's decision about whether to accept the recommendation. In the two subsequent sections, we build on this model of interaction between individuals to analyze the firm's price and bonus-setting decisions in a heterogeneous population. (Table 1 contains a notation summary.)

3.1. Action Spaces

A customer's action, which we denote as a_S (subscript S stands for "sender" of the signal), is his recommendation: he either recommends ($a_S = 1$) or he does not ($a_S = 0$). The $a_S = 0$ choice can be interpreted in two ways, either as a "not recommended" signal or as "no signal." This action is contingent on the customer's private information about his assessment of the friend's expected value from the product.

Based on the signal received from the customer, the friend either buys or does not. We denote the friend's action as a_R and define $a_R = 1$ as buying and $a_R = 0$ as not buying. We use the subscript R, for "receiver" of the signal, to denote the friend.

3.2. Information

Product recommendations from a customer to a friend are most useful when two conditions are met: First, the value of a new product is difficult to assess with inspection. Second, the experienced party knows

Table 1 Notation Summary

a_S	The action that the customer (sender of signal) takes; 1 (recommend) or 0 (not).
a_R	The action that the friend (receiver of signal) takes; 1 (buy) or 0 (not).
x	The value a friend will gain from using the product; this is an uncertain quantity.
m	The mean of x; we assume m is uniformly distributed over [0, 1] in the population.
p	The price the firm sets for the product.
B	The referral bonus the firm sets.
u_R	The friend's utility.
u_S	The customer's utility.
j	The function that describes how u_R enters u_S.
$\widehat{m}$	The cutoff value on the m continuum (between 0 and 1), indicating the m value for the marginal friend who gets a recommendation (with a homogeneous population).
α	A parameter capturing the degree of overlap in the network; an α close to 1 indicates very little overlap, nearly a tree structure; an α close to 0 indicates extensive overlap, so that each potential customer is a friend of many existing customers.
β	The coefficient on j in u_S; this parameter represents how much concern a customer has for a friend.
β_d	The β for distant friends.
β_c	The β for close friends.
q	The percentage of the population who are distant friends to a customer.
$\widehat{m}_d$	The cutoff value on the m continuum (between 0 and 1), indicating the m value for the marginal distant friend who gets a recommendation.
$\widehat{m}_c$	The cutoff value on the m continuum (between 0 and 1), indicating the m value for the marginal close friend who gets a recommendation.
π	Firm profit.
φ_d	Consumer surplus for distant friends.
φ_c	Consumer surplus for close friends.
$\sigma^2/(2R)$	The risk-adjusting term; σ^2 is the variance of x and R is the risk tolerance parameter of the exponential utility function.

something about the other party's tastes. These conditions create an information asymmetry. The customer has awareness of the product as well as knowledge about the match between the friend and the product. The friend may have better knowledge about his preferences than the customer, but absent product knowledge, it is difficult for him to have a precise assessment of his own value. Thus, he relies on a personalized recommendation to infer his value.

We capture this information asymmetry between a customer and a friend in an admittedly highly simplified model. Our abstraction aims to capture an essential feature of referrals as recommendations and assessments: the person making the recommendation has information not known by the person receiving the recommendation.

Our model of information endowments is as follows. The customer has private information (which is usually referred to as his "type" in signaling models) about the value the friend will get from using the product, which we denote m. Here, the private information is the mean of the distribution of the random variable x, the friend's value in using the product. In contrast, the friend is less well informed. He has a prior on his expected value (m) but does not know that value precisely. We analyze the case of a uniform $[0, 1]$ prior on m.

3.3. Utilities

The utility of the friend is based on whether he buys the product. If he does, the (ex post) value is the difference between the value he gets from the product (x) and the price of the product (p). The friend's utility u_R is given by $u_R = a_R(x - p)$.

As the title of our paper suggests, the utility of the customer u_S depends on both firm-offered and interpersonal incentives. If the friend does buy based on a customer's recommendation, then the customer's utility depends on the bonus the firm offers and the utility that the friend gets from the product. In §2, we explained how this idea is consistent with the theory of "social motives" that combine self-interest and altruism. In addition, the combination of a firm-offered incentive and an interpersonal incentive is reminiscent of the "embedded markets" studied by Frenzen and Davis (1990) and Frenzen and Nakamoto (1993), where utility includes a component related to the social relationship in which a transaction is embedded.

The customer's utility is represented as $u_S = a_S a_R[B + j(u_R)]$, where B is the referral bonus and the j function represents the customer's concern for the friend's outcome. The bonus is nonnegative but if the friend has a bad outcome, u_R and $j(u_R)$ can be negative, implying that u_S can be negative. The utility u_S is zero if the friend does not buy based on the customer's recommendation or if no recommendation is made. We start by analyzing j linear in u_R in §4 and then analyze a concave j representing recommendation risk aversion in §5.

3.4. Equilibrium

We use perfect Bayesian equilibrium (PBE) as a solution concept for the game. The PBE is useful in signaling games because it allows the receiver of the signal to have prior beliefs and to update them based on the signal, but also to have residual uncertainty. Treating j generally for now (i.e., not specifying a functional form), we describe a PBE of this signaling game.

In this equilibrium, a customer will recommend to friends at the upper end of the distribution on value for the product. The customer is selective in his recommendations, and the result is that the recommendations are credible and that the friends optimally accept them. Conditions on the price are required to ensure this credibility. This equilibrium is formalized in the following proposition.

Proposition 1. *For m uniformly distributed between 0 and 1, if $2p-1 \le \hat{m} \le 2p$, where $\hat{m}$ is the m that solves $B + E_x[j(u_R) \mid m] = 0$ and $E_x[j(u_R) \mid m]$ is increasing in m, the following strategy profile and beliefs comprise a PBE*:

$$\textit{Customer S's strategy}: \quad \begin{cases} 0 \le m < \hat{m}\colon\ a_S = 0, \\ \hat{m} \le m \le 1\colon\ a_S = 1; \end{cases}$$

$$\textit{Friend R's strategy}: \quad \begin{cases} a_S = 0\colon\ a_R = 0, \\ a_S = 1\colon\ a_R = 1; \end{cases}$$

$$\textit{Beliefs}: \quad \mu(m \mid a_S) = \begin{cases} \mu(m \mid a_S = 0) = 1/\hat{m} \\ \quad \textit{for } 0 \le m < \hat{m}, \\ \mu(m \mid a_S = 1) = 1/(1-\hat{m}) \\ \quad \textit{for } \hat{m} \le m \le 1. \end{cases}$$

The proofs of the propositions are in the appendix.

The mechanics of the customer's behavior are as follows. In this equilibrium, a customer will recommend to friends whose mean value in use (m) is high enough at or above some $\hat{m}$, i.e., those friends most likely to benefit from the product. However, just because a friend is in the "most likely" group does not actually mean that the friend will like the product. Referral bonuses may encourage the customer to make some negative expected value recommendations ($m < p$).

The mechanics for the friend are simpler: the friend follows the recommendation. The constraints $2p-1 \le \hat{m} \le 2p$ are required for the recommendation to be credible. The constraint $2p-1 \le \hat{m}$ ensures that the friend will optimally buy a product that is recommended. The constraint $\hat{m} \le 2p$ ensures that the friend will optimally not buy a product that is not recommended.

Intuitively, this equilibrium can be understood as follows. A customer realizes value from two sources, a referral bonus (if offered) and the friend's outcome with a purchased product. A customer will only make a recommendation when these two sources together create positive value: he wants to have his recommendation accepted.[2] Therefore, he is going to be discriminating in his recommendations. This discrimination lends credibility to his recommendation. However, when there is a bonus, the customer may exaggerate to some friends. Therefore, not all recommendations will necessarily have positive expected value, but the need for the recommendations to be credible limits the extent of this exaggeration.

4. Recommendation Risk Neutrality

In this section, we look at the firm's decisions about the optimal price and bonus for a heterogeneous population, building on the signaling model from §3. Then, we look at the results for firm profit and consumer surplus.

We include two types of heterogeneity in the population. First, we consider variation across the population in the mean value for the product (m). This first aspect captures the idea that different people have different tastes for the product. Second, we consider two types of relationships between customers and friends: close and distant. This second aspect captures how sensitive a customer's utility is to a friend's utility.

4.1. Model

We start with a simple model of a customer caring about the outcome of his recommendation for the friend: the friend's utility (value from the product minus the price, if he buys it) enters the customer's utility function in a linear way. In other words, for every dollar of value that a friend realizes, the customer who made the recommendation vicariously experiences β units of value. Following our earlier notation, $j(u_R) = \beta u_R$. Even this simple model allows us to make many of the main arguments of the paper.

We assume that the "level" β of the relationship is common knowledge for the customer and the friend. The firm knows the distribution over the possible values of β in the population but does not know the β that characterizes any one particular relationship.

Following our analysis in §3, we assume that there is a uniform distribution over $[0, 1]$ of mean values for the product (m) in the population. For analytical tractability, we use a two-point model of the distribution on β in the population, representing people's distant (β_d) and close (β_c) relationships, $\beta_d < \beta_c$. For a fraction q of the population, $j(u_R) = \beta_d u_R$, and for the rest, $j(u_R) = \beta_c u_R$. The distinction between the categories is that customers value outcomes for close friends more highly than they do for distant friends.

We make the following additional assumptions for tractability. First, we assume that the two types of heterogeneity, on m and on β, are independent. Second, we assume a simple network structure in which each customer knows the same size group of friends (normalized to one), with overlaps in those groups symmetric across the customers. To capture the level of overlap, we use a parameter, $0 < \alpha \le 1$. An α close to one is a treelike structure with little overlap in friends.

[2] We have also considered the case in which u_S incorporates "pain avoided" for the "not recommended" case, capturing the idea that the sender realizes utility from having his friend accept his recommendation, even if the recommendation is to not buy the product. The basic structure of our results is maintained with this variation. Details can be found in the electronic companion to this paper, available a part of the online version that can be found at http://mktsci.pubs.informs.org.

Third, we analyze the case of zero marginal costs. Finally, our analytical results are from a single-period model, but we provide evidence of the robustness of the results to a multiperiod setting in the electronic companion.

4.2. Analysis and Results

Customers make recommendations if they expect to have an overall positive value, including the firm-offered referral bonus and the interpersonal incentives. That is, customers in the β_i, $i \in \{d, c\}$, type of relationship will recommend to anyone with $m \geq \hat{m}_i$, where $\hat{m}_i$ is the m that solves $B + E_x[j(u_R) \mid m] = 0$. For the $j(u_R) = \beta_i u_R \equiv \beta_i(x - p)$ case considered in this section,

$$\hat{m}_i = p - B/\beta_i.$$

If the firm does, in fact, sell to both close and distant friends (and we consider the necessary and sufficient conditions in the appendix), the firm's profit maximization problem as a function of decision variables price p and bonus B is as follows:[3]

$$\begin{aligned} &\max_{p,B} \quad \alpha(p-B)[q(1-\hat{m}_d)+(1-q)(1-\hat{m}_c)] \\ &\text{subject to} \quad 2p-1 \leq \hat{m}_d \leq 2p, \quad 2p-1 \leq \hat{m}_c \leq 2p, \\ &\qquad 0 \leq \hat{m}_d \leq 1, \quad 0 \leq \hat{m}_c \leq 1, \\ &\qquad p \geq 0, \quad B \geq 0. \end{aligned} \tag{1}$$

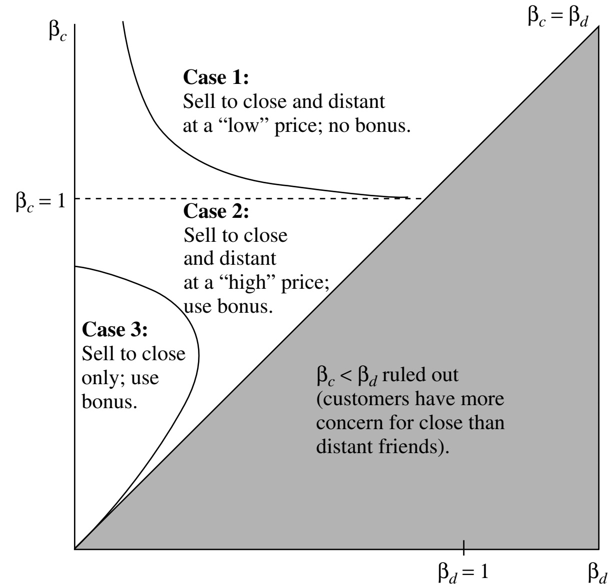
The expression $(p - B)$ is the profit margin on each unit sold. The expression $\alpha[q(1-\hat{m}_d)+(1-q)(1-\hat{m}_c)]$ gives the quantity sold. It combines the percentage of the distant friends who buy $\alpha(1-\hat{m}_d)$ and the percentage of close friends who buy $\alpha(1-\hat{m}_c)$, based on the proportion of distant friends (q).

The constraints arise both from the credibility concerns in the signaling model and the logical structure of the problem. Proposition 1 shows credibility constraints that arise from the recommendation as an informational signal. There are four constraints like that: $2p-1 \leq \hat{m}_d \leq 2p$ and $2p-1 \leq \hat{m}_c \leq 2p$. Because the m themselves are distributed between 0 and 1, the objective function in (1) is based on $0 \leq \hat{m}_d \leq 1$ and $0 \leq \hat{m}_c \leq 1$. Finally, we require that p and B be nonnegative.

In the appendix, we show how to determine which of the constraints is active under different conditions and provide the formulation of the other case (target close friends only). The results of the optimization are summarized in Proposition 2 and illustrated in Figure 1. We define two boundaries, $q_{1,2} \equiv \beta_d(\beta_c - 1)/(\beta_c - \beta_d)$ and $q_{2,3} \equiv (\beta_c - \beta_d)(1-\beta_c)/((\beta_c-\beta_d)+2\beta_c+\beta_c^2+\beta_d\beta_c)$, to help describe the results.

[3] In this analysis, we do not explicitly consider other forms of awareness of the product beyond recommendations. Including those other avenues would mean that the price considered here is the "price for recommended customers."

Figure 1 **Regions from Proposition 2**



Note. The boundary between Cases 1 and 2 is given by $q = q_{1,2}$ and between Cases 2 and 3 by $q = q_{2,3}$.

Proposition 2. *If customers are risk neutral in their recommendations to both distant and close friends, the optimal bonus and price are as follows.*

Case	*Conditions*	*Optimal bonus and price*
Case 1	$q < q_{1,2}$	$B^* = 0, \quad p^* = 1/2;$
Case 2	$q \geq \max\{q_{1,2}, q_{2,3}\}$	$B^* = \dfrac{\beta_d}{2(1+\beta_d)}, \quad p^* = 1 - \dfrac{1}{2(1+\beta_d)};$
Case 3	$q < q_{2,3}$	$B^* = \dfrac{\beta_c}{2(1+\beta_c)}, \quad p^* = 1 - \dfrac{1}{2(1+\beta_c)}.$

Furthermore, in Case 2, *the optimal bonus and price are increasing in β_d. In Case* 3, *the optimal bonus and price are increasing in β_c.*

The cases from Proposition 2 are portrayed in Figure 1, drawn in (β_d, β_c) space, for a very low q (0.05), i.e., a population where 95% of the friends are close friends. The low q was chosen for Figure 1 to graphically illustrate all three of the cases. (The region for Case 3 shrinks rapidly with q; it disappears completely when $q \geq 1/3$.)

In Case 1, customers have high concern relative to Cases 2 and 3 for close and distant friends (and are more concerned for close friends than themselves; i.e., $\beta_c > 1$), so referral bonuses should not be used.

Without a bonus, the optimal price is relatively low, and even though the group of distant friends may be small, all friends are targeted with the low price. In Case 2, customers have lower concern for friends compared to Case 1, and there are sufficient numbers of distant friends, so the firm sets the price and bonus to appeal to both distant and close friends. In Case 3, customers have a relatively low concern for distant friends, but there are insufficient numbers of those distant friends (i.e., low enough q) to warrant targeting them. In this case, recommendations to distant friends are not credible, so the firm just sells to customers' close friends.

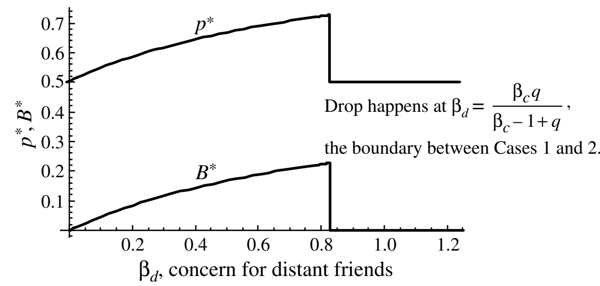
If the population is homogeneous in β, that is, made up of only one type of friend, then the regions simplify greatly. (Homogeneity in β is equivalent to $q=1$ and $\beta_d=\beta_c\equiv\beta$.) For $\beta\le 1$, Cases 2 applies (and Case 3 disappears); for $\beta>1$, Case 1 applies.[4] The $B^*=0$ versus the $B^*>0$ distinction does not require heterogeneity in β.

4.2.1. Optimal Price and Bonus. Within each case in Proposition 2, the optimal bonus and price are increasing (at least weakly) in the "concern" coefficients, β_d and β_c. In other words, over some ranges, as the level of concern in relationships increases, the optimal price and bonus increase. With higher β_i, the signal embodied in the recommendation is more reliable and more discriminatory: there is a smaller chance of a negative expected value recommendation *and* there is a higher level of value implied by the recommendation. A more discriminatory signal implies greater selectivity, that is, a lower quantity of recommendations. The firm can counteract that smaller quantity by increasing the bonus or reducing the price. However, reducing the price would fail to take advantage of the higher implied value. Therefore, with a higher β_i, the firm leverages the upside of the selectivity—the higher implied value—and mitigates the downside—a lower quantity—by optimally using a higher bonus and a higher price.

However, if the customer cares a great deal about friends' outcomes, then it is inefficient for the firm to reward the customer; the firm is better off allocating that money directly to the friends via price. This switch in strategy from bonus to no bonus is illustrated in Figure 2. In Case 1, a bonus is a less efficient tool than price for expanding quantity; thus, a bonus should not be used.

4.2.2. Profit and Consumer Surplus. Proposition 2 presented the results about the optimal firm strategy. Now, we turn our attention to welfare consequences—firm profit and consumer surplus. Consumer surplus can be decomposed into the friends' surplus and the customers' surplus; here, we focus on friends' surplus, denoting distant friends' surplus as φ_d and close friends' surplus as φ_c. The expressions for firm profit and friends' surplus at the optimal solutions are presented in Proposition 3.

Figure 2 **Optimal Price and Bonus as a Function of β_d, with $\beta_c>1$ and Linear Utility**



Note. The figure is drawn with $\beta_c=1.25$ and $q=0.5$.

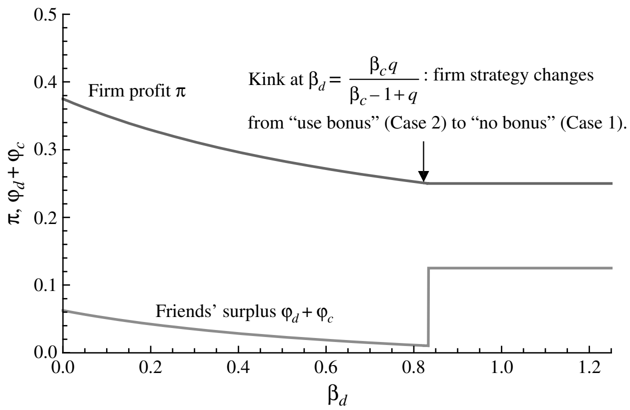
Proposition 3. *If customers are risk neutral in their recommendations to both distant and close friends, using the cases defined in Proposition 2 the optimal profit and friends' surplus are given in the table below.*

Case	*Profit*	*Friends' surplus*
Case 1	$\pi=\alpha/4$	$\varphi_d=\alpha q/8$, $\varphi_c=\alpha(1-q)/8$;
Case 2	$\pi=\alpha\dfrac{\beta_d(1-q)+\beta_c(1+q)}{4\beta_c(1+\beta_d)}$	$\varphi_d=0$, $\varphi_c=\dfrac{\alpha(1-q)(\beta_c^2-\beta_d^2)}{8\beta_c^2(1+\beta_d)^2}$;
Case 3	$\pi=\alpha\dfrac{(1-q)}{2(1+\beta_c)}$	$\varphi_d=0,\quad \varphi_c=0.$

A first conclusion from Proposition 3 is that when the firm uses a bonus (Cases 2 and 3), profit tends to decrease in the level of concern. In Case 2, profit is decreasing in β_c and is decreasing in β_d for $q>(1-\beta_c)/(1+\beta_c)$. In Case 3, profit is decreasing in β_c. With greater concern, the increased discrimination in recommendations, which yields fewer recommendations, hurts the firm's profits. The firm optimally charges a higher price but also pays a higher bonus. The optimal margin, p^*-B^*, in all three cases is equal to 1/2. The same margin, on a lower quantity, provides lower profit. In Figure 3, we see profit strictly decreasing in β_d until β_d reaches the boundary between Cases 1 and 2, at which point profit is no longer a function of β_d. In addition, profit from alternative approaches to promotion such as investment in advertising would also be independent of β_d.

[4] Note that in Proposition 2, the boundary between Cases 1 and 2 is written in terms of q, but it is derived from $q/\beta_d+(1-q)/\beta_c=1$. This original form makes it easier to see the simplification to $q=1$ for the homogeneous case.

Figure 3 **Firm Profit and Friends' Surplus as a Function of β_d, with $\beta_c = 1.25$, $q = 0.5$, $\alpha = 1$**



Therefore, we would still expect to see the strictly then weakly decreasing profit pattern when considering such alternatives.

A second conclusion from Proposition 3 is that when the firm uses a bonus (Cases 2 and 3), surplus is zero for the marginal group of friends, those on the border between buying and not. The marginal group is the distant friends if both close and distant are targeted, and the close friends if they are the only ones targeted. With linear utility, when a bonus is used there are equal numbers of people in the marginal group who get positive and negative expected value recommendations. This balance allows the recommendation to be credible but just (i.e., friends in the marginal group interpret recommendations as "this product is *just* worth the price"). In Case 2, in which both close and distant friends get recommendations, close friends in aggregate do realize positive surplus φ_c. In Case 3, only close friends are targeted, so they are the marginal group and realize no surplus.

A third conclusion is that in Case 2, friends' surplus $\varphi_d + \varphi_c$ (which equals φ_c in Case 2 because $\varphi_d = 0$) is decreasing in β_d but increasing in β_c. A greater concern for distant friends, β_d, raises the optimal price and bonus, decreases the quantity of distant friends who get a recommendation, and increases the quantity of close friends who get a recommendation. The increased quantity of close friends has a lower average value and pays a higher price. Even though there are more close friends buying, the lower average surplus produces lower total friends' surplus. In contrast, a higher β_c benefits close friends in aggregate because a more select group of them get a recommendation at a price determined only by β_d, providing each one who does buy with a higher net value.

The preceding discussion and Figure 3 used just the friends' surplus from buying the product. More inclusively, we could also include vicarious utility or the aggregate bonuses earned by the customers. With the broadest definition of consumer surplus, greater concern tends to boost consumer surplus because of the link between greater concern and higher bonuses.

5. Recommendation Risk Aversion

In this section, we expand the model of interpersonal utility by studying recommendation risk aversion. Recommendation risk aversion captures the idea that customers are loath to make recommendations that may turn out poorly for their friends. Applying the classic idea of risk aversion, a customer weighs possible negative outcomes for the friend more than equivalent-sized positive ones.

5.1. Model

To model recommendation risk aversion, we use an exponential utility function for the utility associated with the customer's recommendation, based on the uncertain value the friend will get from using the product. The exponential utility function is one of the standard ways to model risk aversion. Its wide use can be attributed to two useful properties (Howard 1988; Luenberger 1998, p. 464). First, this functional form of a concave utility function is characterized by a single parameter, the "risk tolerance," which captures the curvature of the utility function. Second, the exponential utility function is wealth independent, meaning that constant values such as the referral bonus B and the price p are simply added to the overall dollar value.

The exponential utility is a function of the friend's utility if he buys the product, $u_R = x - p$. The functional form is $1 - e^{-(\tilde{x}-p)R}$, where x is the uncertain value the friend will get from the product, p is the price, and R is the risk tolerance parameter of the exponential utility function. (The degree of risk aversion, therefore, can be represented by $1/R$.) To combine referral bonuses and expected utility from a recommendation, we use the Pratt (1964) approximation for a risky prospect's certainty equivalent CE (or dollar value), $CE = m - \sigma^2/(2R)$, where m is the mean of the uncertainty quantity x (consistent with our earlier notation), σ^2 is the variance of that uncertain quantity, and R is the risk tolerance parameter of the exponential utility function.[5] This is a tractable and familiar mean-variance form of risk aversion, where the overall dollar value is the mean value less a factor related to the variance of the uncertain quantity.

Returning to the notation from §3, we define $j(u_R) = \beta(1 - e^{-(\tilde{x}-p)R})$. From the signaling model, a recommendation risk-averse customer will make recommendations to friends whose mean value m is above some value $\widehat{m}$. The $\widehat{m}$ is the m that solves $B + E_x[j(u_R) \mid m] = 0$

[5] This is an approximation that holds for small variance gambles. It is exact when the utility function is exponential and the distribution on the uncertainty is normal for any size variance.

or $B+\beta(\widehat{m}-\sigma^2/(2R)-p)=0$. Solving for $\widehat{m}$ gives $\widehat{m}=\sigma^2/(2R)+p-B/\beta$.

5.2. Analysis and Results

In this section, we maintain the earlier model of distant and close relationships—using β_d and β_c as in §4—but now the close relationships are also characterized by risk aversion. If the firm sells to both distant and close friends, the objective function and the constraints are written the same as in the risk-neutral analysis in §4, given by (1), except that now $\widehat{m}_d=p-B/\beta_d$ and $\widehat{m}_c=\sigma^2/(2R)+p-B/\beta_c$.

This "sell to both groups" formulation is only one of the possibilities. As in the risk-neutral case, it may be optimal to sell only to close friends when there are insufficient numbers of distant friends to warrant targeting them. It may also be optimal to target only the distant friends if the degree of risk aversion is so high that the price would have to be unprofitably low for customers to take the risk to recommend to close friends. For each of these three possibilities—targeting both groups, targeting close friends only, and targeting distant friends only—we must consider "positive bonus" and "no bonus" approaches. Out of the six combinations (three possible targets times two bonus approaches), only one is not possible (targeting close friends only with no bonus). The five remaining combinations correspond to regions of the parameter space, $(\beta_d, \beta_c, q, \sigma^2/(2R))$. The details of all five regions and the proof of Proposition 4 are in the electronic companion.

Proposition 4 presents the patterns from these findings. Some of the results in the proposition, such as Parts 1 and 4, confirm and extend findings from §4. Other parts, such as Part 3, provide points of contrast. The qualifier "at least locally" in the results below refers to behavior within each of the five regions.

Proposition 4. *If the customers are recommendation risk neutral for their distant friends and recommendation risk averse for their close friends*:

(1) *A bonus should not be used* ($B^*=0$) *if* β_d *and* β_c *are each above a threshold*.

(2) *If a bonus is optimal* ($B^*>0$), *it is strictly increasing, at least locally, in* β_d *if distant friends are targeted and/or* β_c *if close friends are targeted*.

(3) *If a bonus is optimal* ($B^*>0$) *and close friends are targeted, the bonus is strictly increasing, at least locally, in* $\sigma^2/(2R)$.

(4) *If a bonus is optimal* ($B^*>0$) *and distant friends are targeted, price is strictly increasing, at least locally, in* β_d.

(5) *If a bonus is optimal* ($B^*>0$) *and only close friends are targeted, price is strictly increasing, at least locally, in* β_c *and* $\sigma^2/(2R)$; *if both close and distant friends are targeted, price is strictly decreasing, at least locally, in* β_c *and* $\sigma^2/(2R)$.

In general, concern and risk aversion increase the optimal bonus when a bonus is used. In contrast, the effects on price depend on which groups are targeted. Concern for the marginal group of friends (distant or close) raises prices, but concern and risk aversion for the inframarginal group (close only) decreases prices. We further discuss the intuition behind these results in §5.4, where we make a comparison across risk-neutral and risk-averse cases.

5.3. Profit and Consumer Surplus

We see similar patterns in the optimal firm profit between the risk-averse and risk-neutral cases. In particular, in both settings, if no bonus is optimally used (i.e., $B^*=0$), the optimal profit is not dependent on the levels of concern β_d and β_c. We also tend to see that the greater the level of concern, the lower the firm profit. (However, the conditions are more complex with risk aversion than with risk neutrality.)

There is an important distinction for consumer surplus between the two cases: recommendation risk *neutrality* always leads to some exaggerated recommendations, but recommendation risk *aversion* does not always lead to some. This can be seen from $\widehat{m}_d$, the cutoff on friends' value that determines who gets a recommendation. That quantity is $\widehat{m}_d=p-B/\beta_d$, which implies that when a bonus is used, $\widehat{m}_d\le p$; i.e., some distant friends will get exaggerated recommendations. (This will also be true under recommendation risk neutrality to close friends.) However, with recommendation risk aversion, the cutoff is $\widehat{m}_c=\sigma^2/(2R)+p-B/\beta_c$, which allows for the possibility that $\widehat{m}_c>p$. If that is the case, all close friends who get a recommendation have a positive expected value; in other words, recommendation risk aversion can mean that customers are conservative with their close friends and they do not even make *all* positive expected value recommendations possible. Therefore, when the firm optimally acts to expand the group of close friends who get a recommendation (i.e., lower $\widehat{m}_c$), the additional close friends who get the recommendation are all generating positive expected surplus.

5.4. Conclusions

Table 2 summarizes some of the key results from the risk-neutral and risk-averse cases. Table 2 presents the effects of model parameters on optimal price and bonus for moderate recommendation risk aversion, low enough β_i (which includes all of $\beta_d<\beta_c<1$), and a high enough proportion of distant friends q. These conditions on the parameters focus attention on cases in which bonuses are used and where both close and distant friends get recommendations.

Table 2 reveals two patterns. First, for distant friends, increased concern raises both the price and the bonus. Second, for close friends, the level of concern (at least weakly) raises the bonus but (at least weakly) lowers the price. The level of concern can be represented by the β_i coefficient or by the risk-adjusting term $\sigma^2/(2R)$. We note that if it is the friends who are risk averse and not the customers, then higher β_i still yields higher price and bonus but higher $\sigma^2/(2R)$ has the opposite effect. (We present the analysis in the electronic companion.)

Why does the level of concern systematically raise the bonus in these cases? An increase in β_i raises B because raising the bonus is a more efficient way to expand recommendations than lowering the price in these cases. We discussed this result for the risk-neutral case in §4.2.1, and the logic applies to the other cases as well.

Why are there systematic differences between the effects of distant and close friends? A key point is that the credibility of recommendations to distant friends is more strained. In our notation, a recommendation to a distant friend is credible if $(1+\widehat{m}_d)/2 \geq p$, and a recommendation to a close friend is credible if $(1+\widehat{m}_c)/2 \geq p$. Because $\widehat{m}_d < \widehat{m}_c$, the former condition is stronger. In all the models, we see that the customers' additional concern for the marginal new customers raises the optimal price and bonus.

Why does risk aversion change the effect of the close friends on price and bonus? With risk aversion, concern for close friends puts downward pressure on price. The price decreasing in β_c (or $\sigma^2/(2R)$) supports the idea that the more a customer cares about friends' outcomes, the better deal (i.e., the lower the price) the firm should offer the friends.

As the caption for Table 2 says, the results summarized there hold in part of the parameter space. Another consistent finding beyond those conditions is that when concern (as represented by the β_i) is very high (e.g., $1 < \beta_d < \beta_c$, which is sufficient but not necessary), it is no longer optimal for a firm to use a bonus program at all. In those cases, the motivation of a customer wanting a good outcome for a friend makes price reductions the best tool for the firm to expand recommendations.

6. Discussion

In this section, we present the managerial implications that arise from our work, discuss the limitations of our analysis, and conclude our paper.

6.1. Managerial Implications

Conceptually, we have defined the most relevant domain for our discussion as experience goods in which user taste is important. Practically speaking, customers must have a well-defined identity to the firm to implement referral programs. In services (e.g., cable TV, Internet service, online poker), customers have accounts that can be credited when referrals are made.

In addition to this implementation concern, our work offers guidance to a manager making decisions about referral programs. Below, we discuss managerial implications for category sensitivity and exaggerated recommendations.

6.1.1. Category Sensitivity. Briefly stated, our finding is that—if the category is not extremely sensitive—the more sensitive the category, the higher the bonus should be. If the category *is* extremely sensitive, referral bonuses should not be used at all.

In our analysis, we have generically described the β_i (how much a customer cares about friends' outcomes) as being a feature of the relationship itself. However, we can also interpret the β_i as related to the sensitivity of the product category. For example, highly personal or sensitive products such as services related to areas like child care, mental health, or reproduction will be characterized by a strong concern by the customer about the outcome for the friend. In less sensitive areas, such as phone or Internet service, cable television, and grocery store choices, a referral is much less worrisome. Categories like personal trainers or hair salons will be somewhere in the middle.

We found that in some ranges, the more sensitive the product, the higher the optimal referral bonus should be. Increased sensitivity will lead to more selectivity in recommendations, leading to a higher

Table 2 Summary of Results for Low Enough β_i, Low Enough $\sigma^2/(2R)$, and High Enough q

	Heterogeneous risk neutral (Proposition 2)		Distant risk neutral, close risk averse, $R_d = \infty$, $R_c = R$ (Proposition 4)		Heterogeneous risk averse (electronic companion)	
Friends	p	B	p	B	p	B
Distant						
β_d	+	+	+	+	+	+
$\sigma^2/(2R_d)$	NA	NA	NA	NA	+	+
Close						
β_c	0	0	−	+	−	+
$\sigma^2/(2R_c)$	NA	NA	−	+	−	+

optimal price and bonus. However, when concern for friends makes price the more efficient lever, the firm should not offer a bonus to customers at all.

The analytical results we presented have a normalized scale $[0, 1]$ for mean product value (m), so our conclusions about higher bonuses are conclusions about bonus levels relative to mean value. More sensitive categories may also tend to have more at stake, having a wider range on the mean value. In the risk-neutral setting, by allowing that range to vary across categories (i.e., m is uniformly distributed over zero to an arbitrary upper bound), we can show that the bonus is increasing in proportion to the size of the stakes (the upper bound). That extension is consistent with the positive relationship between sensitivity and bonus size.

Furthermore, we find that bonus programs are most profitable in the least sensitive categories. Therefore, bonus plans make more sense in more mundane categories. In those categories, the optimal bonuses are low, so the most profitable bonus programs entail small bonuses. The following two examples illustrate less and more sensitive categories.

In our investigation of national Internet service providers (spanning DSL, cable, satellite and dial-up technologies), we found that 18 out of 24 identified providers offered some kind of referral bonus. A typical program was a free month of service or monetary equivalent for the referring customer. This category is not highly sensitive, so the existence and low bonus level (compared to the overall lifetime price paid by the new customer) are consistent with our theory.

A friend of one of the authors referred several of his friends to his LASIK eye surgeon, and much joking (among the friends) about referral bonuses ensued. At that center, and at most LASIK centers we investigated, there was not a referral bonus. This product category has an extreme range of outcomes: perfect vision without glasses is a really great outcome and permanently impaired vision is a really terrible outcome. Because of the risk in the referrals, this is a highly sensitive category, and laser eye surgery centers are better off running pricing promotions directly than using referral bonuses.

6.1.2. Issues with Exaggerated Recommendations. Should firms avoid bonuses because they encourage exaggerated recommendations? We do find that part of the profit from bonus programs may come from exaggerated recommendations. Bonuses may be profit maximizing but there is a concern that this is a short-term perspective, especially when combined with the idea that a person's estimate of how much others will like a product depends on how delighted that person was with the product (cf. Biyalogorsky et al. 2001). However, the answer to the question posed is "not necessarily." When firms use bonuses to overcome conservatism that comes from recommendation risk aversion, they can actually expand recommendations within the "positive expected value" segment of the market.

Apartment complexes are an example of a category that could experience some recommendation risk aversion. Residents of apartment complexes may be reluctant to recommend their apartment complex to a friend because if the friend does not have a good experience living there, that could be quite salient to the recommender if he or she lives near the dissatisfied party and sees them frequently. We surveyed local apartment complexes and estimate that approximately 60% of them do offer some kind of referral program. A typical program offers a free month of rent to the referring party.

We interpret this bonus acting as a "nudge" to help people overcome a possible reluctance to recommend rather than a "bribe" that encourages recommendations likely to yield unhappy customers. In less risky categories, bonuses are more likely to encourage customers to exaggerate on the margin. Firms need to be attuned to the longer-term effects of dissatisfied customers in those situations.

6.2. Model Limitations

Before concluding, we comment on some of the assumptions we made for analytical tractability in our analysis. An implicit assumption in our work is that the customer is the only source of information about the new product. That assumption is more strained for products that have some objective and possibly publicly available data, such as procedure success rates for physicians or *Consumer Reports*-type reviews of new products. In our models, we considered two types of heterogeneity (closeness and value); the availability of more information suggests a third type, that of varied information endowments.

Our model of the network structure is also related to this information endowment issue. We have used a simple, symmetric network structure, characterized by the parameter α, which represents the degree to which customers have unique friends. We have not explicitly modeled how friends treat multiple recommendations. This possibility could be formulated as an information gathering problem, similar to that in McCardle (1985), in which more recommendations would move the friend closer to sufficient information to make a decision about adopting or rejecting the new product.

Another simplification we made was to focus on the interplay between customers and potential new customers, and not on the forces operating between competing firms. Firms with novel or highly differentiated products may have some market power, but in other industries, the competitive forces will be more important. Therefore, not analyzing competition must be considered a limitation of the current work.

Some readers have suggested that the existence of a bonus can be a deal breaker: some people have a stance that they will not make a recommendation if they will receive a bonus; other people will not accept a recommendation when a bonus is at stake. The agents in our model have compensatory values—adding a firm-offered incentive (bonus) with value derived from concern for a friend's outcome. A noncompensatory model of behavior by customers could certainly reduce the appeal of bonuses to the firm. In our analysis, a distant friend is correct to be wary of bonus-induced recommendations, as his expected value is zero.

We see these areas as potential directions for further work. Heterogeneity in information endowments, multiple recommendations, and questions about competition would lend themselves to analytical modeling approaches. The notion of a "bonus boycott" is suggestive of a behavioral approach. In fact, there are many aspects of our model (such as the interpersonal incentives and risk aversion) that could be investigated further with behavioral approaches.

6.3. Conclusion

In writing about third-party product reviews, Chen and Xie (2005) explain how communication of credible information has been an ongoing challenge for firms. Marketing scholars have studied many avenues for addressing the difficulty, including whether price can be used as a signal of quality (Gerstner 1985), using price and advertising as a signal (Zhao 2000), and using product line design (Villas-Boas 2004). In this paper, we have studied a different strategy for communication of credible information, stimulating word of mouth through referral bonuses. Our work contributes to this area by building on the work of Biyalogorsky et al. (2001). We examined the case in which referrals do not just spread awareness but also serve as recommendations. These recommendations contain signals about the value of the product to a friend who is less able to judge the value of something new.

Some of our findings are consistent with those of Biyalogorsky et al. (2001). For example, they find that when customers are more discriminating in their recommendations (which they conceptualize with a higher "delight threshold"—how thrilled a customer has to be with the product before he will recommend it), profits are driven down (see, e.g., Biyalogorsky et al. 2001, Figures 3 and 6). In our work, we also find that when customers are more discriminating in their recommendations (which we conceptualize with higher-concern coefficients, different from the delight thresholds), the firm's profit suffers. In addition, like us, Biyalogorsky et al. (2001) find that above a high enough level of discrimination, the firm should not use a referral bonus.

However, fundamental differences come from the recommendation's role as an informative signal. Biyalogorsky et al. (2001) find that referral rewards are not useful when the delight threshold is too low; in contrast, we find the low-concern cases are the most profitable ones in which to use a referral bonus. In addition, when the delight threshold is high, Biyalogorsky et al. (2001) find that referral bonuses should not be used; however, they also find that the price should be high. In contrast, we argue that with high concern, price is a more efficient way to increase the quantity of referrals and should be set low. These discrepancies suggest that by including recommendation credibility, the firm must consider not just how the optimal price and bonus affect how happy existing and future customers are with the product, but also how customers trade off conflicting social motives of self-interest and altruism in choosing to engage in word of mouth.

7. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mktsci.pubs.informs.org/.

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Appendix

Proof of Proposition 1. To prove that this strategy profile and these beliefs comprise a PBE, first we show that each player's strategy is a best response to the other's; then we show the beliefs are consistent with the strategies.

(1) Is R optimizing given S's strategy? For that to be true, R has to want to not buy when the signal is 0, which is the case when $\hat{m}/2 \leq p$. In addition, R has to want to buy when the signal is 1, which is the case when $(1+\hat{m})/2 \geq p$. Together, these form the condition $2p-1 \leq \hat{m} \leq 2p$.

(2) Is S optimizing given R's strategy? First, look at $0 \leq m < \hat{m}$. Under this strategy, S says $a_S = 0$ and R does not buy, so S gets 0. Can S change its strategy for any of the m in $0 \leq m < \hat{m}$ to get more than 0? Look at $\hat{m} - \varepsilon$. Should S say buy ($a_S = 1$) for that m? Given that $\hat{m}$ is the m that solves $B + E_x[j(u_R) \mid m] = 0$ and $E_x[j(u_R) \mid m]$ is increasing in m, $B + E_x[j(u_R) \mid m - \varepsilon] < 0$. So, S does not want to change his strategy for any m in $0 \leq m < \hat{m}$. Now, look at $\hat{m} \leq m \leq 1$. This is where $a_S = 1$. Does S want to change? No. With this strategy, S has a nonnegative value, so he does not want to change to a zero value.

(3) Are the beliefs consistent with the strategies? S's strategy is contingent on whether m is above or below $\widehat{m}$. Conditioning on that distinction, the updated probability distributions become uniform over $\widehat{m} \leq m \leq 1$ or $0 \leq m < \widehat{m}$, respectively. □

Proof of Proposition 2, Including Derivation of Risk Neutral Cases (§4). First, we present and solve the formulations of the firm's profit maximization problem for both risk-neutral cases: targeting both groups of friends and targeting only close friends. Second, we show when each case applies. Third, we prove the claims at the end of Proposition 2 about price and bonus being increasing in β_i in Cases 2 and 3.

Sell to Both Groups. The optimization problem in which the firm sells to both close and distant friends is given in the text (1). There are 10 constraints in that formulation, but when we eliminate the redundant ones, we are left with only $\widehat{m}_d \geq 2p-1$, $\widehat{m}_d \geq 0$, and $B \geq 0$. Using $\widehat{m}_d = p - B/\beta_d$, the first two reduce to $p \leq 1 - B/\beta_d$ and $p \geq B/\beta_d$, respectively.

The behavior of the (unconstrained) objective function determines which constraints are binding. The difference between the value of the objective function at $(p+\Delta, B+\Delta)$ to that at (p, B) is $\alpha(p-B)\Delta(q/\beta_d + (1-q)/\beta_c - 1)$. For $p > B$, that difference is positive when $q/\beta_d + (1-q)/\beta_c > 1$. Therefore, when $q/\beta_d + (1-q)/\beta_c > 1$, simultaneous increases in p and B increase the objective function without bound, and upper bounds on p and B are binding: the constraint $p \leq 1 - B/\beta_d$ is binding. Furthermore, if $q/\beta_d + (1-q)/\beta_c < 1$, then simultaneous *decreases* in p and B increase the objective function without bound, and a lower bound will be binding: $B \geq 0$.

Plugging these binding constraints into the objective function yields the following:

$$\text{If } \frac{q}{\beta_d} + \frac{1-q}{\beta_c} > 1, \quad B^* = \frac{\beta_d}{2(1+\beta_d)}, \quad p^* = 1 - \frac{1}{2(1+\beta_d)};$$

$$\text{If } \frac{q}{\beta_d} + \frac{1-q}{\beta_c} < 1, \quad B^* = 0, \quad p^* = 1/2.$$

Sell Only to Close Friends. If the number of distant friends is too small (i.e., q low), then the firm may decide that it is not worth pursuing that segment at all. If the firm targets only close customers, the profit maximization problem is

$$\max_{p,B} \alpha(p-B)(1-q)(1-\widehat{m}_c)$$

subject to

$2p-1 \leq \widehat{m}_c \leq 2p$ (signal to close friends must be credible),

$\widehat{m}_d \leq 2p-1$ (signal to distant friends not credible),

$0 \leq \widehat{m}_d \leq 1, \quad 0 \leq \widehat{m}_c \leq 1$

(logical restriction based on $m_i \in [0,1]$),

$p \geq 0, \quad B \geq 0$ (price and bonus are nonnegative),

with $\widehat{m}_c = p - B/\beta_c$ and $\widehat{m}_d = p - B/\beta_d$ (however, because of $\widehat{m}_d \leq 2p-1$, the recommendations to distant friends are not credible).

Once again, eliminating redundant constraints reduces the set to $\widehat{m}_d \leq 2p-1$ (equivalently, $p \geq 1 - B/\beta_d$), $\widehat{m}_c \geq 2p-1$ (equivalently, $p \leq 1 - B/\beta_c$), $\widehat{m}_d \geq 0$ (equivalently, $p \geq B/\beta_d$), and $B \geq 0$. In this case, the objective function is unbounded in simultaneous increases in p and B for $\beta_c < 1$ (and for simultaneous decreases for $\beta_c > 1$). So, for $\beta_c < 1$, an upper bound on p and B applies, namely, $p \leq 1 - B/\beta_c$. Using $p = 1 - B/\beta_c$ and optimizing the objective function gives the solution $B^* = \beta_c/(2(1+\beta_c))$, $p^* = 1 - 1/(2(1+\beta_c))$, which satisfies the other constraints. For $\beta_c > 1$, $p^* = 1/2$ and $B^* = 0$, but that solution violates the "signals not credible to distant friends" constraint.

Finally, we comment that for recommendation risk neutrality, we do not have to consider the possibility of selling only to distant friends. Setting a price to appeal to distant friends will allow for credible recommendations to close friends as well.

Compare the Profit in the Two Cases. Comparing the optimal profit expressions in the two cases gives the condition that selling to both beats selling to close only when $\alpha(1-q)/(2(1+\beta_c)) < \alpha(\beta_d(1-q) + \beta_c(1+q)/(4\beta_c(1+\beta_d)))$. That expression reduces to the condition given in Proposition 2 as the boundary between Cases 2 and 3: $q > (\beta_c - \beta_d)(1-\beta_c)/((\beta_c - \beta_d) + 2\beta_c + \beta_c^2 + \beta_d\beta_c)$.

Price and Bonus Increasing in Concern. For the final line of the proposition, to see that B^* and p^* are increasing in β_i (where $i = d$ for Case 2 and $i = c$ for Case 3), we look at the first derivatives: $dB^*/d\beta_i = dp^*/d\beta_i = 1/(2(1+\beta_i)^2)$.

That expression is positive, so the optimal B and p are increasing in β_i in Cases 2 and 3. □

Proof of Proposition 3. We derive the expressions for profit and the different components of consumer surplus below.

Case 1. From Proposition 2, $B^* = 0$ and $p^* = 1/2$, which implies that $\widehat{m}_d = 1/2$ and $\widehat{m}_c = 1/2$.

Profit: $\pi = \alpha(p-B)[q(1-\widehat{m}_d) + (1-q)(1-\widehat{m}_c)]$ yields $\pi = \alpha/4$.

Distant friends' surplus: $\varphi_d = \alpha q \int_{\widehat{m}_d}^1 (m-p)\,dm = \alpha q/8$.

Close friends' surplus: $\varphi_c = \alpha(1-q)\int_{\widehat{m}_c}^1 (m-p)\,dm = \alpha(1-q)/8$.

Case 2. From Proposition 2, $B^* = \beta_d/(2(1+\beta_d))$ and $p^* = 1 - 1/(2(1+\beta_d))$, which implies that $\widehat{m}_d = \beta_d/(1+\beta_d)$ and $\widehat{m}_c = (\beta_c - \beta_d + 2\beta_d\beta_c)/(2\beta_c(1+\beta_d))$.

Profit: $\pi = \alpha(p-B)[q(1-\widehat{m}_d) + (1-q)(1-\widehat{m}_c)]$ yields $\pi = \alpha(\beta_d(1-q) + \beta_c(1+q)/(4\beta_c(1+\beta_d)))$.

Distant friends' surplus: $\varphi_d = \alpha q \int_{\widehat{m}_d}^1 (m-p)\,dm = 0$.

Close friends' surplus: $\varphi_c = \alpha(1-q)\int_{\widehat{m}_c}^1 (m-p)\,dm = \alpha(1-q)(\beta_c^2 - \beta_d^2)/(8\beta_c^2(1+\beta_d)^2)$.

Case 3. From Proposition 2, $B^* = \beta_c/(2(1+\beta_c))$ and $p^* = 1 - 1/(2(1+\beta_c))$, which implies $\widehat{m}_c = \beta_c/(1+\beta_c)$.

Profit: $\pi = \alpha(p-B)[(1-q)(1-\widehat{m}_c)]$ yields $\pi = \alpha((1-q)/(2(1+\beta_c)))$.

Distant friends' surplus: $\varphi_d = 0$ (they do not buy).

Close friends' surplus:

$$\varphi_c = \alpha(1-q)\int_{\widehat{m}_c}^1 (m-p)\,dm = 0. \quad \square$$

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