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THE PASSENGER-MIX PROBLEM IN THE SCHEDULED AIRLINES

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ABSTRACT. Deregulation has opened up many opportunities and challenges in the transportation industry — opportunities to increase profits and challenges to keep from being outflanked by competition. A goal of particular interest to the scheduled airlines is to set prices more adaptively and to change them more rapidly. A difficult problem arises when many passengers with different itineraries compete for a limited number of seats on a single-flight segment. The problem is complicated by the existence of different fare classes, many flight segments, and different demands across time. For any given set of prices, flight-segment capacities, and passenger-carrying demand, there is some number of passengers at each fare class on each flight segment that will optimize revenue. Knowledge of such an optimum can be used not only in pricing analysis but also in setting policies to influence the passenger fare-class mix so that the optimum will be more nearly achieved in actual practice.

We describe a method for identifying the optimum fare-class mix and the design of a system for that purpose which we built and implemented for Frontier Airlines. The recognition and formulation of the problem has become even more important as the number of aircraft in the sky has been reduced and the competition for a limited number of seats has become more intense.

Prior to deregulation, competition among carriers was limited by the Civil Aeronautics Board (CAB) in two of the three major areas of airline marketing — route authority and pricing — leaving only the amount of capacity (number of flights) to be made available by any one carrier over any one route up to individual carrier management judgment. Competition, therefore, was limited to frills (fancy meals) and flights (departures every hour).

Pricing policies were generally viewed and analyzed from an industry standpoint because the CAB would not permit any carrier to offer a lower fare that was uneconomic for the industry as a whole. Thus even though a particular fare might benefit a particular carrier at the expense of another carrier, the CAB would not permit the offering of the proposed fare without an extremely strong justification

TRANSPORTATION; AIR

showing clearly that the fare would benefit the general public. The carriers who stood to lose revenue as a result of the lower fare would argue in rebuttal that their loss of revenue would have to be offset by a general fare increase in all fares, thereby harming the general public by requiring them to pay a higher fare and in effect subsidizing those few passengers who would benefit from the lower fare proposed by their competitor.

With the advent of deregulation, carriers suddenly found themselves facing new forms of competition. New, low-cost, nonunion carriers sprung up in major markets offering transportation at unrestricted fares priced from 30 to 75% below existing fares. Smaller regional carriers whose route structure had been limited by the CAB to short-haul, feeder-type operations hubbed around a single major airport such as Denver or St. Louis began to expand into other major cities and compete with the large trunk carriers whose route structures had been designed by the CAB to carry passengers over the longer distances between the major hub cities.

In addition to expanding their services to large cities beyond their old route structures, the regional airlines realized that they could also compete effectively for a portion of the long-haul pool of traffic that had historically traveled on the trunk carriers' long-haul nonstop flights by offering lower fares on their multistop or connecting flights.

Since the individual airlines were no longer limited to an industry orientation with regard to their pricing policies, true price competition expanded dramatically. The rewards associated with filling seats (that would otherwise be empty) with low-fare passengers that an airline would otherwise not have carried must be balanced against the risks of displacing higher-fare passengers that would otherwise have been carried.

The problem is complicated by (a) the existence of a multitude of prices (fares) with varying degrees of restrictions limiting the availability of all but the highest priced seat; (b) numerous flights operated by a number of airlines over various routings, any one of which (or combination of two or more) can be used by passengers to get to their destinations; and finally, (c) varying degrees of demand for the seats on any one airline's flight segment over time, depending upon the number of city pairs that can be reasonably serviced by the particular flight, the season of the year, day of week, time of day, quality of service offered (nonstop, one-stop, connection) for a particular passenger's routing vis-a-vis alternative flights either of the same or competitive carriers.

The Passenger Mix Problem

The problem faced by the airlines then may be termed the "pricing and passenger mix" problem. The problem has relevance not only for airlines but also for other segments of the carrier industry. One might substitute the term "load mix" in alternative settings; e.g., for a trucking company or steamship company that has a set of regularly scheduled routes and which faces decisions of the type elaborated below.

With the elimination of certain government restrictions, airlines now have more opportunity to explore different pricing and routing options and to seek for each the best mix of passengers [Murphy, 1980]. The determination of this mix provides two major outcomes: (1) it enables the airline to structure its reservation system more effectively, setting appropriate limits and priorities governing the number of passengers at different fare classes traveling on different flights; (2) it allows different price/route scenarios to be evaluated in consideration of the profit generated from the best passenger mix, relative to a given scenario.

The passenger mix problem may therefore be viewed as serving both a "tactical" (reservation monitoring) objective and a "strategic" (price/route setting) objective. To meet these objectives, a computer-based system embodying a convenient, user-friendly model and a highly efficient solution method is needed. A system that fails to exhibit these characteristics will not only incur undesirable costs in terms of human and computer resources but will also seriously inhibit scenario analysis and responsiveness to changing conditions [Dembo and Mulvey, 1976; Glover and Klingman, 1978; Glover, McMillan, and Taylor, 1977].

This paper describes the development of a system based on a network-related model that meets the dual criteria of convenience and efficiency, a system implemented for Frontier Airlines. We will first provide a description of the passenger pricing and passenger mix problem and some of its practical implications, and then develop the network-related model by reference to a simplified illustration. Finally, we will describe supporting software features that provide additional user convenience and report preliminary computational experience.

Features of the Pricing and Passenger Mix Problem

The profitability of a passenger to the carrier depends on the length of the trip and the fare class he travels. While revenue *per mile* is generally less for passengers traveling long distances, the total revenue to the carrier is greater for those passengers.

Associated with each passenger on a given flight segment is an opportunity cost, in that each passenger on a given flight segment occupies a seat that might have gone to another passenger traveling a more profitable itinerary or at a more profitable fare class. Thus on a flight which connects terminals A, B, and C (in that order), a *local* passenger traveling only from B to C occupies a seat that might have gone to a passenger traveling A to C, a *through* traveler. The traveler from B to C may therefore be responsible for an empty seat on segment A to B of that flight or on some segment of another flight involved in the passenger itinerary (PI) which includes segment B to C. On each segment of each flight in a carrier's network there may be many PI's, passengers traveling from many different origins to many different destinations and in different fare classes.

Given a forecast of the demand for PI's on any one day at the various fare classes and over the carrier's entire network, there exists a theoretically optimal mix of PI's at the various fare classes. The optimal mix is that mix of passenger itineraries and associated fare classes which maximizes the carrier's total revenue that day. The optimal mix of PI's can be expressed in terms of the best number of PI's in various fare classes on each segment of each flight; that is, *the optimal occupancy* of the available seats on each segment of each flight.

Much of the planning done at the operations level in the scheduled airlines focuses on PI's and the demand for them at the various fare classes. Marketing managers endeavor to design fare class structures, in association with PI's, so as to increase occupancy and thus increase revenue. It is a complex business in that fare class modification and the offering of special discounts may result in "spill" and "diversion."

Spill is the movement of passengers to other flights, either the same or competing carriers. Diversion occurs when a passenger who would have stayed with the same carrier at the original higher fare takes advantage of a discount fare which was offered to stimulate increased occupancy, thus generating less revenue for the carrier.

In modifying the fare class structure the carrier tries to control spill and diversion through pricing and by adding restrictions such as the timing of the travel, the length of stay, and the length of time between the purchase of the ticket and the departure of the flight. Carriers also try to minimize spill and diversion through "capacity control." This is done chiefly through specifying the number of seats on each flight segment that are reserved for each of the various fare classes.

If the carrier had perfect capacity control, seats could be reserved on each flight segment by specific PI's at the associated fare classes. Given a forecast of the demand for the various PI's at the various fare classes, he could forego accepting a reservation for a local traveler going from B to C in anticipation of utilizing that seat for a through traveler going from A to B to C, etc. But carriers' reservation allocation systems do not commonly make that possible today, and instead capacity control is exercised by restricting fare classes rather than PI's. By controlling the number of seats allocated to each of the various fare classes on each segment of each flight, the carrier has some measure of capacity control. This provides a means for dealing with spill and diversion and for minimizing the displacement of higher-revenue-producing traffic by lower-revenue-producing traffic.

Based on the foregoing considerations, the passenger mix problem may be posed as follows: "Given each day's forecast of the demand for PI's at the various fare classes, what PI and associated fare-class mix, on each segment of each flight, will maximize revenue for that day?" The answer to that question equips the carrier to determine the optimal reservation allocation, among the various fare classes, on each segment of each flight.

The opportunity cost associated with each passenger on each segment of each flight depends on his PI and fare class and the demand for all other PI's and associated fare classes that are impacted by his occupying a seat. Therefore, the optimal PI/fare class mix, on each segment of each flight, must be determined in the context of the entire problem; a global optimum is required.

Knowing the revenue associated with the optimal mix, the carrier is then equipped to assess the propriety of modifying the fares, for the various classes, in pursuit of greater demand which altered fares might generate.

Model Formulation

We formulated the problem as a minimum cost (maximum profit) network flow problem with special side constraints. In the network portion, one set of arcs corresponds to segments of flights, and another set corresponds to PI's differentiated by fare classes. Flow on the former (forward arcs) represents the number of passengers on a flight segment, and flow on the latter (back arcs) represents the number of passengers on each PI at each of the various fare classes.

The formulation is illustrated in Figures 1 and 2.

FIGURE 1. PASSENGER CAPACITY AND LOADING SHOWN AS ARC FLOWS.

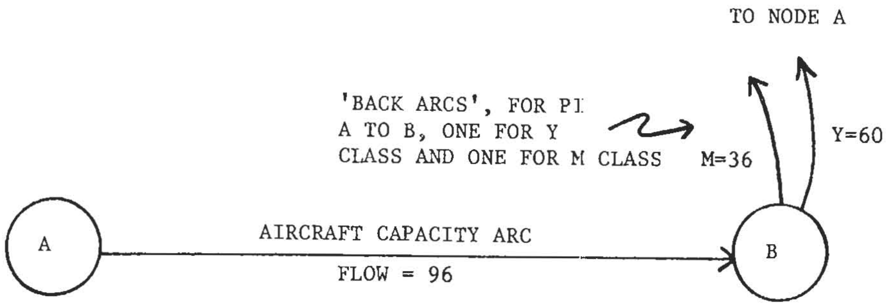
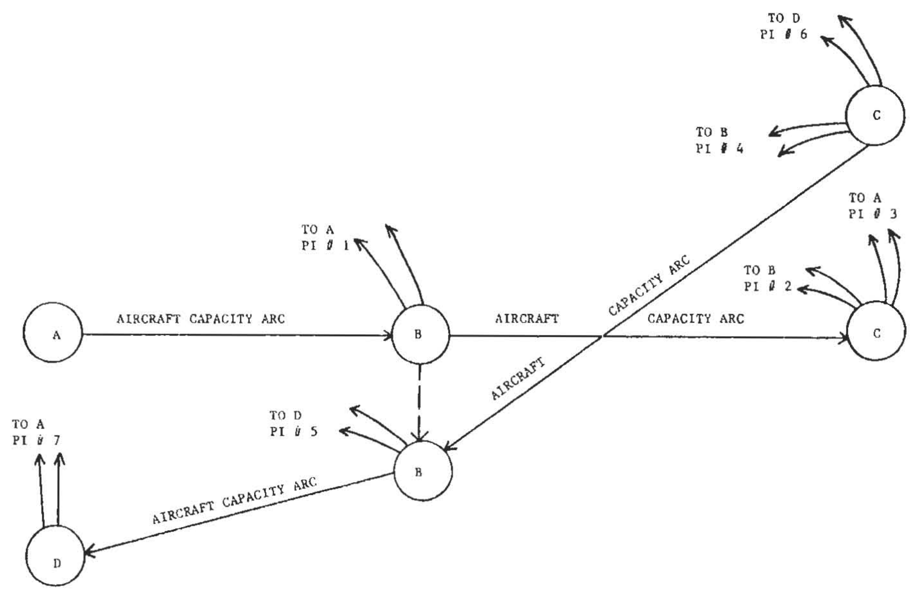


FIGURE 2.
FLIGHT 1. A TO B TO C, ARRIVING AT B 10:00 AM.
FLIGHT 2. C TO B TO D, ARRIVING AT B 11:00 AM.



One segment of a flight connects terminals A and B in Figure 1. A flow of 96 units on the arc connecting A to B indicates 96 passengers, on various itineraries, traveling on segment A to B of that flight. Two fare classes are possible in this example, Y and M. The flow Y = 60 on one "back arc" connecting B to A indicates 60 passengers on a PI traveling at fare class Y from A to B. M = 36 indicates 36 passengers traveling on the same PI at fare class M.

In Figure 2, two flights are represented schematically: Flight 1 connects A to B to C, in that order, and Flight 2 connects C to B to D. Flight 1 arrives at B one hour prior to Flight 2's arrival at B, so that passengers on the first segment of Flight 1 can transfer to Flight 2 at B for continued travel to D.

The possible PI's, in this example, are:

PI #	Itinerary	
1	A to B	on Flight 1
2	B to C	on Flight 1
3	A to B to C	on Flight 1
4	C to B	on Flight 2
5	B to D	on Flight 2
6	C to D	on Flight 2
7	A to B (on Flight 1) then B to D (on Flight 2)	

Back arcs in Figure 2 connect B to A for the two fare classes associated with PI #1; C to B for PI #2; C to A for PI #3; C to B for PI #4; B to D for PI #5; C to D for PI #6; and D to A for PI #7, representing the two fare classes for the PI which connects A to B on Flight 1 and to D on Flight 2.

Two nodes are required for B; one representing the time period associated with Flight 1's arrival at B and the other Flight 2's arrival at B. (More precisely, the nodes represent terminal B over the span between arrival and departure times for each flight.) Flow on the dashed arc connecting B to B represents passengers transferring at B from Flight 1 to Flight 2. Similarly, two nodes are required for C one for the time period associated with Flight 2's departure from C, and one for Flight 1's arrival at C.

Flow on each forward arc is limited by the capacity of the aircraft, and flow on each back arc is limited by the demand for the PI and the fare class that back arc represents. These limits translate into simple upper bounds on the arcs. An increment of revenue is associated with each unit of flow on each back arc, equal to the price of the ticket at the fare class that arc represents.

This network formulation by itself can be shown to represent the problem accurately under a variety of circumstances — as, for example, if no sequence of transfer arcs and flight segment arcs can create a cycle, directed or otherwise. More narrowly, it suffices if each PI/fare-class arc lies on at most (hence exactly) one simple directed cycle. The latter condition is satisfied if (but not only if) there is no way to transfer to obtain two different directed routes (of transfer and segment arcs) from a node in one flight to a node in the same or a different flight.

In the general setting of the problem, and in the application we dealt with, the network formulation by itself was inadequate to represent the problem with complete fidelity. The existence of a number of simple directed cycles containing the same PI/fare-class arc (for a number of such arcs) caused the network formulation to be too loosely constrained. The existence of more than one cycle containing the same back arcs may cause the sum of PI flows to exceed the capacity of flight segment arcs intended to be used by the PI's. The additional side constraints required to deal with this inadequacy stipulate that the sum of flows on PI/fare-class arcs may not exceed the intended flight segment arc capacities.

Solution Procedure and Computational Experience

A network optimizing component, in the system we designed and built, finds that flow on each arc which maximizes revenue on the carrier's network, without violating the aircraft capacity constraints and the upper bounds posed by the demands forecast for the various PI's at their associated fare classes.

If the special non-network side constraints are satisfied, the solution is optimal. If not, the procedure identifies exactly the side constraints that are violated and enforces them by a successive restriction approach [see, e.g., Gill and Murray, 1974; Murphy, 1980].

The system built for Frontier Airlines was designed to accommodate a network of 600 flights and 30,000 PI's with up to five fare classes per PI. The number of special side constraints ranges from about 1800 to 2400, but generally our procedure finds that only 40 to 60 of these require explicit handling. Execution time on a 16 bit minicomputer bears a linear relationship to the number of arcs and nodes and is brief enough to make use of the system in an interactive mode quite manageable.

Following a run of the optimizer, a post-processor extracts and accumulates the flows representing fare-class loads among the PI's identified as optimal for each segment and reports the optimal fare class allocation for each flight segment. Additional analysis produces a report identifying both satisfied and unsatisfied demand by flights and city pairs or "markets."

To provide a basis for comparison, an LP formulation of the problem would involve some 200,000 variables and 3,000 constraints (excluding simple bounds) and would be expected to require several hours to solve with the best available LP methods, making interactive analysis completely out of the question.

Because of the marked efficiency of this network-based method and the tailored pre- and post-processors built into it for easy manipulation of data and assumptions, frequent interaction with the system is possible. As demand forecasts are modified and updated and revenue figures are changed to reflect various policies and operating conditions, the system makes it possible to provide information for better pricing and reservation allocation decisions.

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April 15, 1982

Dr. R. E. D. Woolsey
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Dear Dr. Woolsey:

I am writing to inform you that the decision support system described in the paper, "The Passenger Mix Problem in the Scheduled Airlines," by Glover and others was in fact constructed and has been implemented at Frontier Airlines.

We are using that system presently, and it has improved our capability of pricing our product and developing our discount inventory. It has also given us the ability to give valuable comments to our scheduling department as well as to provide input to our planning function.

Sincerely,

Mark S. Schneider
Director - Pricing & Capacity Control

MSS:ctb