Tabu Search for Quadratic Coloring and Real World Exam Scheduling

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1 Introduction

During a written exam, the teacher usually prepares some variants of a test to be solved by the students. It normally happens that some variants are quite similar while others are completely different. We have considered the problem of assigning the test variants to the desks of a classroom in such a way that desks that are close-by receive different variants. The problem arose from a real need at the University of Bologna, where some classrooms are equipped with networked desktop computers, and the teacher can remotely assign the test variants to each desk.

More precisely, the problem can be modeled as a Vertex Coloring problem on an undirected graph, where vertices correspond to desks and edges connect desks which are close-by. A positive weight, associated with each edge, represents the proximity of the vertices at the endpoints, the weight is increasing with the proximity, so as to model the "temptation" that a student may have in taking inspiration from its neighbor. Colors are associated with the exam variants: for every pair of colors, a positive weight defines the similarity of the corresponding variants. By defining the *vicinity* of an edge-color assignment as the edge weight times the similarity of the colors assigned to the endpoints, the problem asks to color a subset of the graph vertices, whose cardinality N corresponds to the number of students attending the exam, in such a way that the *overall vicinity* is minimized.

The problem has a natural quadratic structure because the penalty associated with a coloring depends on the colors given to couples of vertices. Possible quadratic models and exact solution methods are discussed in [2].

In order to obtain good quality solutions in short computing time, as required by the application, we tackle the problem with a Tabu Search algorithm. The algorithm moves between solutions where all vertices are colored (which is not strictly necessary, since we have to color only N of them). Given a solution, we define its neighborhood as the set of solutions which can be obtained by changing the color of a specified vertex i. This neighborhood was originally proposed by Hertz and de Werra [1] for the first Tabu Search algorithm for the classical Vertex Coloring problem.

The Tabu Search algorithm maintains an ordered list of the vertices, according to the nonincreasing value of their contribution to the overall vicinity (which is the objective function to be minimized) in the current coloring. At each iteration, a vertex i within the first ones in the ordering is randomly selected, and its color is changed so as to minimize its contribution to the vicinity function. For the following iterations, vertex i is considered *tabu*, and thus cannot be selected for recoloring. In order to compute the value of the new solution obtained by changing the color of the vertex i, the algorithm iteratively selects the vertex j with the largest contribution to the vicinity function and uncolors it (so its contribution is null), thus obtaining a solution with only N colored vertices.

The Tabu Search was computationally tested on a set of instances derived from 4 real classrooms. The classrooms have from 20 to 79 desks, and are usually used for written exams. The Tabu Search algorithm is able to produce good approximate solutions for the considered problems, with computing times ranging from few seconds to less than two minutes for the largest considered instances. The algorithm is currently used to assign tests to the desks of the classrooms in the Engineering Faculty of the University of Bologna.

References

[1] A. Hertz and D. de Werra. Using tabu search techniques for graph coloring. *Computing*, 39:345–351, 1987.

[2] J. Duives, A. Lodi and E. Malaguti. Test-Assignment: A Quadratic Coloring Problem. *Technical Report OR-10-12, DEIS - University of Bologna*, Italy, 2010.