

# Tabu Search Algorithms for Extended Models of the Resource-Constrained Project Scheduling Problem

Koji Nonobe and Toshihide Ibaraki

The resource-constrained project scheduling problem (RCPSP) is known as a general model that can handle many of the real-world scheduling problems. The goal is to determine start times of given activities so as to minimize the makespan (latest completion of all activities), subject to resource constraints on quantities consumed and precedence constraints between activities. Due to its importance in practice, many exact and heuristic algorithms have been developed for the RCPSP. Various extensions of the RCPSP model have also been proposed [1], because problems arising in real applications contain different types of activities and constraints.

Our research has produced a suite of tabu search algorithms for this problem and its extensions. As subsequently described, these algorithms have succeeded in improving the best known solutions of many benchmark instances, and are capable of solving models of additional generality.

The first important extension is the multimode RCPSP (MRCPSP) model, in which a set  $M_j$  of modes is associated with each activity  $j$ , and exactly one mode in  $M_j$  is chosen to process  $j$ , where each mode determines the processing time and resource requirements of  $j$ . In this model, in addition to ordinary renewable resources, nonrenewable resources can also be included in constraints. The model MRCPSP/max further extends the MRCPSP, in that minimum and maximum time-lags between activities can be specified.

Additional extensions are developed in [2] and [3]. The model in [2] is more general than MRCPSP in the sense that renewable resource capacities and requirements can be time-varying and that any linear inequalities on 0-1 variables  $x_{jm}$  (describing whether  $j$  is processed in mode  $m$  or not) are allowed to define nonrenewable resource constraints. The immediate precedence constraint can also be used to impose that, between activities  $i$  and  $j$  using renewable resource  $r$ , no other activity using  $r$  can be scheduled. This makes it possible, for example, to formulate setup activities which should be processed immediately before a given activity  $j$ . The model in [3] is an extension of MRCPSP/max in the following sense. Processing of an activity on a resource can be suspended for some time, during which processing of the same activity on other resources may be continued. It is also possible to shorten the processing time of an activity by splitting it into sub-activities and processing some of them in parallel.

Tabu search algorithms for these models were proposed in [2, 3]. In both algorithms, a solution is represented by a pair of mode assignment and activity list (that is, a permutation of all activities), from which a schedule is constructed by assigning the minimum start time to each

activity, chosen one by one in the order of the given activity list, under the condition that the feasibility of the tentative partial schedule is maintained. If the construction cannot continue without violating constraints, some activities already scheduled are canceled and rescheduled. The tabu search is used to find mode assignments and activity lists that can produce good schedules. The neighborhood of a solution in the tabu search is defined as a set of solutions obtained (i) by changing the mode of an activity, or (ii) by modifying the activity list by moving an activity from the current position to another. To keep the neighborhood size small, a limited number of the above operations are selected on the basis of evaluations obtained by critical path analysis.

The algorithms were computationally tested on benchmark instances of RCPSP, MRCPSP and MRCPSP/max, as well as more complicated scheduling problems in real applications [2, 3]. They succeeded in updating the then best known solutions of many benchmark instances, even though they are designed for more general models.

## References

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