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Invited Review

Contributions of Professor William W. Cooper in Operations Research and Management Science

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ABSTRACT

Over a long and remarkably productive career, Professor William W. (Bill) Cooper has made many pioneering contributions to Operations Research and Management Science (OR/MS), with notable forays into the areas of (a) linear and non-linear programming, (b) goal programming, (c) chance-constrained programming, (d) data envelopment analysis, and (e) manpower planning, among others. His legendary partnership with Abraham Charnes has provided results whose connections go back to the 18th century, bearing on problems conceived but left unsolved by Laplace and Gauss. We document cross-fertilizing links among Bill Cooper's multiple research focuses, and their impacts on other researchers. A trace of his work discloses a web of influence that has produced a wide range of advances in OR/MS by those who follow in his footsteps, representing a productive *tour de force* that shows no sign of abating.

1. Introduction

We undertake a review of the contributions of Professor William W. Cooper (University of Texas at Austin) in celebration of his 95th birthday. Professor Cooper has dedicated his life to the development of various methodologies and concepts in business education and research. He had made major contributions that were formative in launching the fields of linear programming, non-linear programming, goal programming, chance-constrained programming, manpower planning and multi-objective optimization. Later, he extended the technique of linear programming and non-linear programming into the development of Data Envelopment Analysis (DEA) which has been widely applied to performance analysis in public and private sectors. Professor Cooper has also developed important business-related concepts and research in areas such as management science, managerial accounting, economics, management, marketing, and auditing, all of which serve currently as pedagogical and research bases in modern business and business education along with public policy.

Professor Cooper's first published article was an economic analysis entitled "The Yardstick of Public Utility Regulation" that appeared in *the Journal of Political Economy*, June 1943 LI, no. 3, pp. 258–262. In fact, still earlier in 1938 he published a proceedings paper for the Committee on Capital Gains Taxation of the National Tax Association, which later became an article entitled "Costs, Prices and Profits – Accounting in the War Program," and published in *The Accounting Review*, July, l945, val. 20, no. 3, pp. 267–308 along with E.L. Kohler. On August 31, 1945, the American Institute of Accountants (currently the American Institute of CPAs where CPAs stand for Certified Public Accountants) chose his article as the most significant contribution to accounting in the year. Since then, over a period spanning seven decades, Professor Cooper has now published 27 books and more than 520 articles in leading international journals. His research includes significant contributions to accounting, economics, management, public policy and other research areas[1] as well as to Operations Research and Management Science (OR/MS), though we focus here only on his contributions in OR/MS.

The structure of this review paper is organized as follows: Section 2, immediately following, classifies Professor Cooper's contributions into six groups and discusses relationships among them. This section corresponds to publications listed in the references of this article. Section 3 discusses the historical path of his contributions from the development of L1 regression to DEA. The article on L1 regression was also the first research effort that identified the formulation, subsequently referred to as goal programming (GP). It is widely known

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[1] The special issue of *Production and Operations Management* (2008) 17 pp. i–ii provides short curriculum vitae of Professor Cooper. The contributions of Professor Cooper in accounting, economics and public policy will be prepared by Y. Ijiri and T. Sueyoshi. Their review article on his contributions in the areas will be published in *Journal of Accounting and Public Policy*.

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that GP serves an important methodological basis for multi-objective optimization. This section describes how the development of GP influences the research of Professor Cooper. Section 4 reviews Cooper's contributions in the area of DEA, to which he has devoted a major part of his research during the past three decades. Section 5 concludes the review and summarizes some of the key elements of Professor Cooper's work.

2. Research classification

Professor Cooper has a long list of publications. This review eliminates most of his contributions related to accounting, management, economics, public policy and other research areas. Furthermore, he was a recipient of many awards. For example, Professor Cooper has been a fellow of the econometric society since 1956. He was the first president of The Institute of Management Science (TIMS) in 1954 and was appointed as the Accounting Hall of Fame (the most prestigious award in Accounting) in 1995. This review does not describe a long list of his awards and professional contributions. Rather, this review focuses upon his research contributions in OR/MS and these influences to other researchers.

Table 1 classifies the OR/MS publications of Professor Cooper into six categories: linear/non-linear programming, manpower planning, GP, chance-constrained programming, DEA and others. Each research area is further classified by modeling, theory, algorithms and applications. The number in each cell corresponds to the reference number of this article. A reference number may be listed in several cells when an article with the reference number belongs to multiple features. For example, [66] is the first book on linear programming in the OR/MS community that covers both its modeling and theoretical aspects. The book also describes algorithmic aspects of the simplex method and various types of applications. Hence, [66] belongs to all categories from modeling to application, as listed in the first row of Table 1. In a similar manner, [123] has used GP as a method for manpower planning, but Table 1 classifies it as his contribution in manpower planning. The separation is based upon the purpose of each study.

DEA[2] contains the largest number of his publications in Table 1. The linear/non-linear programming[3] is the second, and the chance-constrained programming (CCP)[4] is the third in his publication record. The remaining research areas include GP, manpower planning[5] and the others.[6]

The cell in Table 1, that contains the largest number of his publications, is the theoretical work on DEA. Nevertheless, one of Professor Cooper's most seminal contributions, viewed in terms of its impact on applications and on a broad range of investigations by other researchers, has been the introduction of GP. This review examines his contributions on GP in the next section.

3. Goal programming and data envelopment analysis: origins and influences

3.1. History of L1 regression

To describe the historical context out of which GP has arisen, we go back to the science of the 18th century and then trace forward to consider how GP influences modern statistics and mathematical programming. Fig. 1 depicts the history of L1 regression, GP and DEA. The article [163] prepared by Professor Cooper describes relationships between GP and DEA,[7] but we are motivated to cover additional aspects of GP not examined there.

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[2] Professor Cooper increased his publications in OR/MS after the first DEA article [142] was published in 1978. The development of DEA gave him an opportunity to increase his publication rate. At that time, he was already recognized as an international researcher in OR/MS so that many researchers paid attention to DEA. Furthermore, the practicality of DEA also invited much research interest and activities among many researchers in OR/MS, economics, accounting, management and other business-related areas in many different countries. The contribution of DEA can be found in many places. For example, the research [142] is selected as the most influencing article in EJOR Celebrating the 30th Anniversary of Euro. See the web site of this journal (http://www.elsevier.com/authored_subject_sections/S03/Anniversary/30th_anniversary.htm). Several review articles published in *Socio-Economic Planning Sciences* report that DEA has more than 4000 contributions. Such research efforts are due to the DEA development of Professor Cooper and his associates.

[3] Professor Cooper and his associates produced many contributions in the initial stage of linear programming and non-linear programming. Professor Cooper established his reputation by his works in the research area because linear programming is the main stream of OR/MS. For example, his works include various methods to solve transportation problems [56,65], network models [67,76], reformulation of a fractional model [70], geometric programming [97,115,116] and non-linear programming [60,111–114]. All of these studies make an important foundation of modern OR/MS.

[4] The topic of chance-constrained programming (CCP) was first introduced in [151]. The research [63,68] established theoretical foundations on CCP. The approach is a transformation method that changes a stochastic problem to an equivalent linear programming problem. The reformulation of CCP needs to prescribe a tolerance (or satisficing) level of risk. As a result of the reformulation, we can solve the stochastic problem by linear programming algorithm. The reformulation process of CCP is related to the three models (an expectation model, a probability model and a variance model). Professor Cooper applied the CCP to accounting and DEA in many decisional cases. For example, Refs. [31,32] discussed the relationship among, cost, volume and profit within an accounting framework with a time horizon. See Refs. [45–48] for the CCP applications to finance and accounting. The CCP method was used as an analytical method in these studies. Furthermore, Refs. [166,167,172] discussed stochastic DEA where the CCP added a stochastic feature to the radial models (CCR and BCC).

[5] Professor Cooper was very interested in manpower planning. Maybe, his interest was influenced by his wife (Ruth Cooper, J.D., Esq.). She was the first woman lawyer in the state of Pennsylvania (USA) who fought for women's social issues and supported low income families. During World War II, Dr. Ruth Cooper worked in a Japanese concentration camp. She realized at that time that US policy was not always in justice. Then, she decided to help underprivileged people as a woman lawyer. In USA, many women work as layers and corporate executives nowadays. Dr. Ruth Cooper was the first individual who understood the importance of the woman's issue from World War II. She dedicated her life for helping women and underprivileged families. Influenced by Ruth, funding opportunities from US Navy made Professor Cooper to explore manpower planning issues as part of his concern toward public policy. For example, Refs. [126,135,136,138,140,141] discussed EEO (Equal Employment Opportunity) affirmative action planning by combining CCP with GP in a Markov process of EEO. Furthermore, the research [123–125] discussed EEO and military manpower planning by combining GP with generalized network as an assignment problem.

[6] Professor Cooper was interested in public policy. He established the School of Urban & Public Affairs and serviced as the first Dean (l969–l975) at Carnegie-Mellon University (CMU). He was also interested in the development of statistical methods such Khinchin–Kullback–Leibler estimation [146,158]. The CCP has a statistical linkage with risk [68]. The research [149] explored how to deal with multi-collinearlity in regression analysis. His interest in statistics was because he taught statistics at CMU.

[7] From Table 1, we find that GP does not have the number of publications that adequately reflect its influence on multiple criterion optimization. Thus, the number of publications produced by Professor Cooper does not reflect the level of its scientific contribution. Furthermore, Professor Cooper shifted his research effort from GP to DEA even though he clearly understood the importance of GP. However, there is a close linkage between GP and DEA. The research [163] provides an exact analytical characterization on the relationship between GP and DEA. The article was written by Professor Cooper in response to his receiving a Gold Medal Award from the International Society for Multi-Criteria Decision Making in 2004.

Table 1
Research classification of Professor Cooper's contributions

Research area	Modeling	Theory	Algorithm	Application
Linear/non-linear programming	[57,59,64,66,74,97,101,104,131]	[27,54,60–62,65,66,70,71,73, 94,101,111–116,130,133,152]	[54,56,58,66,67,77,84,101]	[59,66,75,76,95,101,107,109,129,132,159]
Manpower planning	[123–126,135,137]	[136,140,141,187]	[139]	[106,138,160]
Goal programming	[79,80,96,100,127]	[149,163]	[110]	[6,26,91,92,106,134,150,160,168,177]
Chance constrained programming	[31,32,45,46,48,53,63,166,167, 172,173]	[68,72,83,108,118,153,155]	[154]	[28,47,151,162]
Data envelopment analysis	[3,5,14,16,20,21,25,35,86,98,102, 103,105,122,142,143,147,166, 167,172–174,182–185, 189–191,195]	[9–17,19–21,30,34,44,52,85,102, 103,105,121,122,148,163–165, 169,170,174,175,178,182,186, 189–195,199,200,202]	[156,157,189–191,195]	[1,2,4,5,7,8,18,22–24,33,36–38,40,43, 49–51,93,122,128,164,176,188–191,195]
Others	[55,87–89,120]	[29,41,69,81,119,144–146,158]		[39,42,78,82,90,99,117,161,171, 179–181,196–198,201]

The number within [ ] corresponds to the reference number at the end of this article.

According to the studies on statistical history,[8] regression analysis was initially investigated as far back as the 16th century. In the 18th century, Roger J. Boscovich (1711–1787) established a use of L1 regression analysis as a research methodology. To understand his idea, consider a data set that contains an independent variable (x) and a dependent variable (y). The data set has n observations ($j = 1, \ldots, n$), and we seek to fit a regression model $y = \beta_0 + \beta_1 x$ to the data set. Boscovich first proposed the minimization of the sum of absolute deviations as a regression criterion that is expressed mathematically by the following equation:

$$\text{minimize} \sum_{j=1}^{n} |y_j - (\beta_0 + \beta_1 x_j)|. \tag{1}$$

Using (1), Boscovich measured the meridian near Rome in 1757.[9] Eq. (1) is the original form of L1 regression, or so-called "least absolute value regression".

Influenced by Boscovich's research, Laplace (Pierre Simon Marquis de Laplace: 1749–1827) considered (1) to be the best criterion for regression analysis and used it until approximately 1795. [Laplace undertook to serve as a diplomat for Napoleon at that time.] However, Laplace thereafter discontinued his examination of the topic because his algorithm for the criterion had a computational difficulty.[10]

To describe Laplace's algorithm for solving (1), assume that $\beta_0 = 0$ and $x_j > 0$ ($j = 1, \ldots, n$), and reorganize the data set so that $y_1/x_1 \geqslant y_2/x_2 \geqslant \ldots \geqslant y_n/x_n$. If there is an integer number (h) that satisfies the following condition:

$$x_1 + x_2 + \cdots + x_{h-1} < x_h + x_{h+1} + \cdots + x_n \quad \text{and} \quad x_1 + x_2 + \cdots + x_h > x_{h+1} + x_{h+2} + \cdots + x_n,$$

then the estimate of β_1 is measured by $\hat{\beta}_1 = y_h/x_h$.[11] A drawback of Laplace's algorithm is that it can be used to solve only a very small L1 regression problem. Consequently, Laplace stopped studying the L1 regression after 1795 because of its computational difficulty.

According to historical record,[12] the first researcher to overcome the computational difficulty encountered by Laplace was Gauss (Carl Friedrich Gauss: 1775–1855). In 1795, when he was only 20, Gauss first proposed the minimization of the sum of squared deviations that was mathematically expressed by

$$\text{minimize} \sum_{j=1}^{n} [y_j - (\beta_0 + \beta_1 x_j)]^2. \tag{2}$$

The regression criterion is nowadays referred to as L2 regression, or "ordinary least squares". The most important feature of (2) is that it is differentiable. Hence, we can easily obtain the parameter estimates to optimize (2) even if (2) is applied to a data set with a large sample size. The regression criterion (2) was a trivial contribution in Gauss' estimation. Hence, he did not publish the least squares method until 1821. According to the literature,[13] the first researcher who published the method of least squares was Legendre (Adrien Marie Legendre: 1752–1833), who introduced both the regression criterion and the well-known "least squares normal equations" in 1805.

After the discovery of the least squares method, Gauss studied the probability distribution of errors from 1797 to 1798 and found the "normal distribution". Gauss proved that least squares estimates became maximum likelihood estimates if the distribution of errors follows the normal distribution. [Maximum likelihood estimation was already studied by Daniel Bernoulli (1700–1782) at that time.] Unfortunately, Gauss did not publish his findings until 1809. After Gauss published his research results in 1809, Laplace published immediately the central limit theorem.[14] [The L1 regression produces maximum likelihood estimates under the double exponential distribution.]

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[8] Harter, H.L. "The Method of Least Squares and Some Alternatives" *International Statistical Review* (1974) 42 p. 147, pp. 235–264 and (1975) 43 pp. 1–44.

[9] Stigler, S.M. "Studies in the History of Probability and Statistics XXXII" *Biometrika* (1973) 60 pp. 439–445.

[10] Eisenhart, C. "The Meaning of 'Least' in Least Squares" *Journal of the Washington Academy of Science* (1964) 54 pp. 24–33.

[11] The equation indicates that L1 regression determines a parameter estimate on the median of an observed data set. Consequently, the L1 estimate has robustness to the presence of an outlier. In contrast, L2 regression is influenced by an outlier because its estimate is a mean. Many researchers such as Boscovich and Laplace in the 18th century understood the property of robustness in the L1 regression. Therefore, they investigated the L1 criterion, not the L2 criterion, in the initial stage of regression analysis.

[12] Harter, H.L. "The Method of Least Squares and Some Alternatives" *International Statistical Review* (1974) 42 p. 147, pp. 235–264 and (1975) 43 pp. 1–44.

[13] Eisenhart, C. "The Meaning of 'Least' in Least Squares" *Journal of the Washington Academy of Science* (1964) 54 pp. 24–33.

[14] Stigler, S.M. "Studies in the History of Probability and Statistics XXXII" *Biometrika* (1973) 60 pp. 439–445.

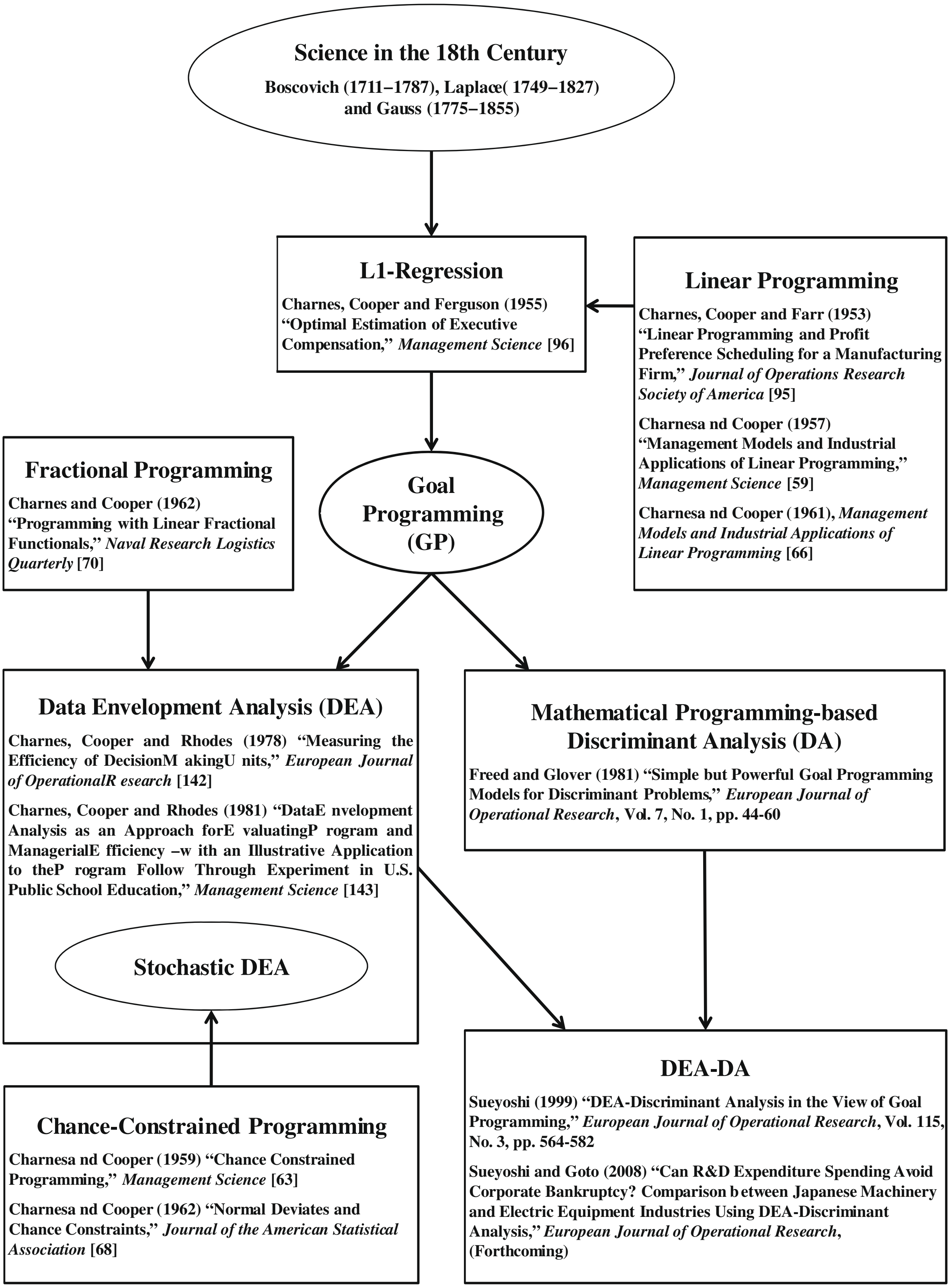

Fig. 1. History of L1 regression, goal programming and data envelopment analysis.

As a result of the discovery of the ordinary least squares method, the normal distribution, the least squares normal equations and the central limit theorem, the L2 regression (or least squares method) has become the main stream of regression analysis in modern statistics. L1 regression almost disappeared from the history of statistics because of the computational difficulty posed by its lack of differentiability. Most scientists religiously believed that the method of least squares under the normal distribution is the best regression criterion, and even today, most statistical textbooks at the college level discuss only the least squares method for regression analysis.

3.2. Origin of goal programming (GP)

The computational difficulty related to L1 regression was first overcome in the work of Charnes et al. [96] who transformed the L1 regression into an equivalent linear programming problem. Returning to Fig. 1, the computer algorithm for linear programming was developed in the beginning of 1950s. Along with the development of a computer system, the research [96] first proposed a mathematical formulation for L1 regression.

To describe their reformulation, we generalize (1) in the following manner:

$$\text{Min} \sum_{j=1}^{n} |y_j - X_j\beta|. \tag{3}$$

Here, the dependent variable (y_j) is to be fitted by a reference to a row vector of m independent variables $X_j = (1, x_{1j}, \ldots, x_{mj})$ for all observations $(j = 1, \ldots, n)$. The regression model is expressed by $X_j\beta$ where $\beta = (\beta_0, \beta_1, \ldots, \beta_m)^{\mathrm{T}}$ is a column vector with $m + 1$ parameters.

Following the research [80], positive and negative parts of each error are introduced for (3) in the following manner:

$$\delta_j^+ = 1/2\{|y_j - X_j\beta| + (y_j - X_j\beta)\} \quad \text{and} \quad \delta_j^- = 1/2\{|y_j - X_j\beta| - (y_j - X_j\beta)\} \quad (j = 1, \ldots, n). \tag{4}$$

Here, the two equations indicate positive and negative parts of the jth error, respectively. Based upon (4), we have

$$\delta_j^+ + \delta_j^- = |y_j - X_j\beta| \quad \text{and} \quad \delta_j^+ - \delta_j^- = y_j - X_j\beta \quad (j = 1, \ldots, n). \tag{5}$$

Using (5), the L1 regression is reformulated by the following original GP model:

$$\text{Min}\left\{ \sum_{j=1}^{n} (\delta_j^+ + \delta_j^-) \middle| X_j\beta + \delta_j^+ - \delta_j^- = y_j, \delta_j^+ \geqslant 0, \delta_j^- \geqslant 0 \ (j = 1, \ldots, n) \right\}. \tag{6}$$

An important feature of (6) is that it can incorporate prior information on parameter estimates into (6) as additional inequality constraints. Such an estimation capability cannot be found in the conventional regression methods in statistics. For example, Ref. [96] incorporated such side constraints in order to model hierarchy-based salary consensus (e.g., the salary of the president is not lower than that of any secretary).

Formulation (6) is a special case of the GP model,[15] which incorporates weights (indicating the importance among goals) in the objective function. A weighted GP model can be expressed by

$$\text{Min}\left\{ \sum_{j=1}^{n} (w_j^+\delta_j^+ + w_j^-\delta_j^-) \middle| X_j\beta + \delta_j^+ - \delta_j^- = g_j, \delta_j^+ \geqslant 0, \delta_j^- \geqslant 0 \ (j = 1, \ldots, n) \right\}. \tag{7}$$

Here, the jth observation on the dependent variable (y_j) is replaced by the jth goal (g_j). Each deviation has a weight (w) to express the importance of each goal.

The GP terminology was first introduced in [66] and soon the GP model became widely used as one of the principal methods in the general realm of multi-objective optimization. Furthermore, an important application of the L1 regression can be found in [150]. The research discussed the existence of a methodological bias in research. The methodological bias implies that "different methods often produce different results." Therefore, we need to examine different methods to make a conclusion. The concern is very important in particular when we make a policy suggestion from empirical evidence for guiding a large policy issue such as the divestiture of the Bell system. No previous study except [150] mentioned the issue of the methodological bias in the OR/MS literature.

3.3. Origin of data envelopment analysis (DEA)

Fig. 1 indicates that GP has served as a basis for the development of DEA with a linkage via fractional programming [163]. The reformulation from a fractional model[16] to a linear programming equivalence was first proposed in [70] and the reformulation was widely used in the OR/MS literature. Furthermore, the reformulation was used to develop the first DEA model, often referred to as in "CCR ratio form" [142,143] because of the relation to "fractional programming". Here, CCR stands for Charnes–Cooper–Rhodes.

CCR: The CCR ratio firm (for input-oriented measurement) has the following formulation to determine the DEA-based technical efficiency of the kth organization:

$$\begin{aligned} \text{Min}\Bigg\{ \theta - \varepsilon\left(\sum_{i=1}^{m} d_i^x + \sum_{r=1}^{s} d_r^y \right) \Bigg| & \sum_{j=1}^{n} x_{ij}\lambda_j + d_i^x = \theta x_{ik} (i = 1, \ldots, m), \ \sum_{j=1}^{n} y_{rj}\lambda_j d_r^y = y_{rk} \ (r = 1, \ldots, s), \\ & \theta : \text{URS}, \ \lambda_j \geqslant 0 \ (j = 1, \ldots, n), \ d_i^x \geqslant 0 \ (i = 1, \cdots m), \ \text{and} \ d_r^y \geqslant 0 \ (r = 1, \ldots, s). \Bigg\}. \end{aligned} \tag{8}$$

Here, the DEA model evaluates n organizations $(j = 1, \ldots, n)$ in relation to each other. Each organization, referred to as decision making unit (DMU) in DEA, uses m inputs $(i = 1, \ldots, m)$ to produce s outputs $(r = 1, \ldots, s)$. x_{ij} is an observed value related to the ith input of the jth organization and y_{rj} is the observed value related to its rth output. Slacks (deviations) related to inputs and outputs are d_i^x and d_r^y, respectively. The ε is a non-Archimedean small number. The scalar (λ_j), often referred to as "structural" or "intensity", is used to make a linkage among DMUs in a data space. The DEA-based efficiency score is measured by the variable θ that is unrestricted in (8). The status of technical efficiency is confirmed when both $\theta = 1$ and the condition that all slacks are zero.

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[15] The formulation in (6) was initially referred to as an "inequally constrained regression" in [96] and was subsequently changed to GP in [66].

[16] Schaible S. "Fractional Programming" in S.I. Gass and C.M. Harris, *Encyclopedia of Operations Research and Management Science*, Norwell, Mass., Kluwer Academic Publishers (1996) provides a discussion of the large literature on fractional programming evolved after [70].

Professor Cooper and his associates developed various types of DEA models as extensions of the CCR ratio form. The succeeding sections discuss the major instances of such model extensions, including "stochastic DEA" [173], which arises by incorporating CCP into DEA [63,68]. In addition to its role as an underpinning of DEA, the GP model also plays a major part in the mathematical programming-based discriminant analysis (DA) of Freed and Glover in 1981. Sueyoshi has developed DEA-DA in 1999 and 2008, as well.[17] Thus, GP has served as a methodological base for the development of various other mathematical programming models for statistical analysis and performance analysis. Fig. 1 visually describes some of the key scientific influences of GP on other research in optimization, although we stress that the figure does not cover all such influences.

3.4. Relationships between goal programming and data envelopment analysis

DEA researchers have long paid attention to the additive model [98][18] which aggregates input-oriented and output-oriented measures to produce a single measure for technical efficiency. The efficiency of the kth organization is measured as follows:

$$\mathrm{Max}\left\{\sum_{i=1}^{m} d_i^x + \sum_{r=1}^{s} d_r^y \,\middle|\, \sum_{j=1}^{n} x_{ij}\lambda_j + d_i^x = x_{ik}\ (i=1,\ldots,m),\ \sum_{j=1}^{n} y_{rj}\lambda_j - d_r^y = y_{rk}\ (r=1,\ldots,s), \right.$$
$$\left. \sum_{j=1}^{n}\lambda_j = 1,\ \lambda_j \geqslant 0\ (j=1,\ldots,n),\ d_i^x \geqslant 0\ (i=1,\ldots,m),\ \text{and}\ d_r^y \geqslant 0\ (r=1,\ldots,s).\right\}. \tag{9}$$

The additive model determines the level of technical efficiency by an existence of a slack(s): (a) full efficiency ↔ all slacks are zero and (b) inefficiency ↔ at least one slack(s) is non-zero.

To discuss a linkage between the additive model and GP, the research [163] proposes the following GP model for two groups as an extended model of (7):

$$\mathrm{Min}\left\{\sum_{i=1}^{m}(\delta_i^+ + \delta_i^-) + \sum_{r=1}^{s}(\delta_r^+ + \delta_r^-) \,\middle|\, \sum_{j=1}^{n} a_{ij}\lambda_j + \delta_i^+ - \delta_i^- = g_i\ (i=1,..,m),\ \sum_{j=1}^{n} b_{rj}\lambda_j + \delta_r^+ - \delta_r^- = g_r\ (r=1,..,s),\right.$$
$$\left.\lambda_j \geqslant 0\ (j=1,\ldots,n), \delta_i^+ \geqslant 0, \delta_i^- \geqslant 0\ (i=1,\ldots,m)\ \text{and}\ \delta_r^+ \geqslant 0,\ \delta_r^- \geqslant 0\ (r=1,\ldots,s).\right\}. \tag{10}$$

The GP model incorporates two different groups: the first group (for inputs) has m goals g_i $(i=1,..,m)$ and the second group (for outputs) has s goals g_r $(r=1,..,s)$. The positive and negative deviations between each goal and its real achievement are specified by δ_i^+ and δ_i^-, respectively, for the first group. In a similar manner, the deviations δ_r^+ and δ_r^- are for the second group. [The deviations in the constraints of (10) have opposite signs to the formulation proposed in [163]. We use the deviation signs in order to maintain consistency with (7). However, such a change does not influence an optimal solution.] The GP model (10) has n unknown decision variables λ_j $(j=1,\ldots,n)$. The ith input of the first group has the jth observation expressed by a_{ij} and the rth output of the second group has the jth observation expressed by b_{rj} in (10). The replacement of $a_{ij},\ b_{rj},\ g_i$ and g_r by $x_{ij},\ y_{rj},\ x_{ik}$ and y_{rk}, respectively, changes (10) to the following model:

$$\mathrm{Min}\left\{\sum_{i=1}^{m}(\delta_i^+ + \delta_i^-) + \sum_{r=1}^{s}(\delta_r^+ + \delta_r^-) \,\middle|\, \sum_{j=1}^{n} x_{ij}\lambda_j + \delta_i^+ - \delta_i^- = x_{ik}\ (i=1,..,m),\ \sum_{j=1}^{n} y_{rj}\lambda_j + \delta_r^+ - \delta_r^- = y_{rk}\ (r=1,..,s),\right.$$
$$\left.\lambda_j \geqslant 0\ (j=1,\ldots,n), \delta_i^+ \geqslant 0,\ \delta_i^- \geqslant 0\ (i=1,\ldots,m)\ \text{and}\ \delta_r^+ \geqslant 0,\ \delta_r^- \geqslant 0\ (r=1,\ldots,s).\right\}. \tag{11}$$

The research [163] incorporates $d_i^x = \delta_i^+ - \delta_i^-$ and $|d_i^x| = \delta_i^+ + \delta_i^-$ for all i as well as $d_r^y = \delta_r^+ - \delta_r^-$ and $|d_r^y| = \delta_r^+ - \delta_r^-$ for all r in (11) along with $\delta_i^+\delta_i^- = 0$ and $\delta_r^+\delta_r^- = 0$ for all i and r. The conditions on the products avoids the simultaneous occurrence of $\delta_i^+ > 0$ and $\delta_i^- > 0$ for all i as well as $\delta_r^+ > 0$ and $\delta_r^- > 0$ for all r. The research [80] explains why we can omit the non-linear conditions.

Using the absolute value characterization on deviations, [163] documents the non-linear equivalent model to (10) as follows:

$$\mathrm{Min}\left\{\sum_{i=1}^{m}|d_i^x| + \sum_{r=1}^{s}|d_r^y| \,\middle|\, \sum_{j=1}^{n} x_{ij}\lambda_j - d_i^x = x_{ik}(i=1,..,m),\ \sum_{j=1}^{n} y_{ij}\lambda_j + d_r^y = y_{rk}(r=1,..,s),\ \lambda_j \geqslant 0\ (j=1,\ldots,n),\right.$$
$$\left. d_i^x \geqslant 0\ (i=1,\ldots,m)\ \text{and}\ d_r^y \geqslant 0\ (r=1,\ldots,s).\right\}. \tag{12}$$

The minimization of $\sum_{i=1}^{m}|d_i^x| + \sum_{r=1}^{s}|d_r^y|$ can be replaced by the maximization of $\sum_{i=1}^{m} d_i^x + \sum_{r=1}^{s} d_r^y$ in (12) because $\mathrm{Min}\sum_{i=1}^{m}|-d_i^x| + \sum_{r=1}^{s}|-d_r^y| = \mathrm{Max}\sum_{i=1}^{m} d_i^x + \sum_{r=1}^{s} d_r^y$. In this case, the signs of the deviations become opposite in the constraints of (12). Furthermore, we incorporate $\sum_{j=1}^{n}\lambda_j = 1$ as an additional side constraint in (12). Then, (12) becomes the additive model (9).

The first DEA publication is generally regarded to be the article [142] that appeared in 1978 (see also Cooper [163, pp. 5–6]) although Professor Cooper and his associates presented DEA at the TIMS Hawaii conference in 1977. Viewing DEA as an extension of GP (and fractional programming) and in consideration of the historical linkage between GP and L1 regression, we also see that DEA has historical links with L1 regression. In this respect, the history of DEA is connected in a roundabout fashion with developments in the 18th century, as manifested in the work of Laplace and Gauss, because they attempted to develop algorithms for the L1 regression.

4. Contributions in data envelopment analysis

Fig. 2 summarizes contributions of Professor Cooper to DEA model developments. As visually summarized in Fig. 1, Professor Cooper has mainly dedicated to the development of DEA after 1978. Hence, this section describes his contributions on the development of DEA models

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[17] See Fig. 1 that lists the references of the three articles.

[18] A linkage between the additive model and the CCR ratio form is well known. See, for example, the research works [189–191].

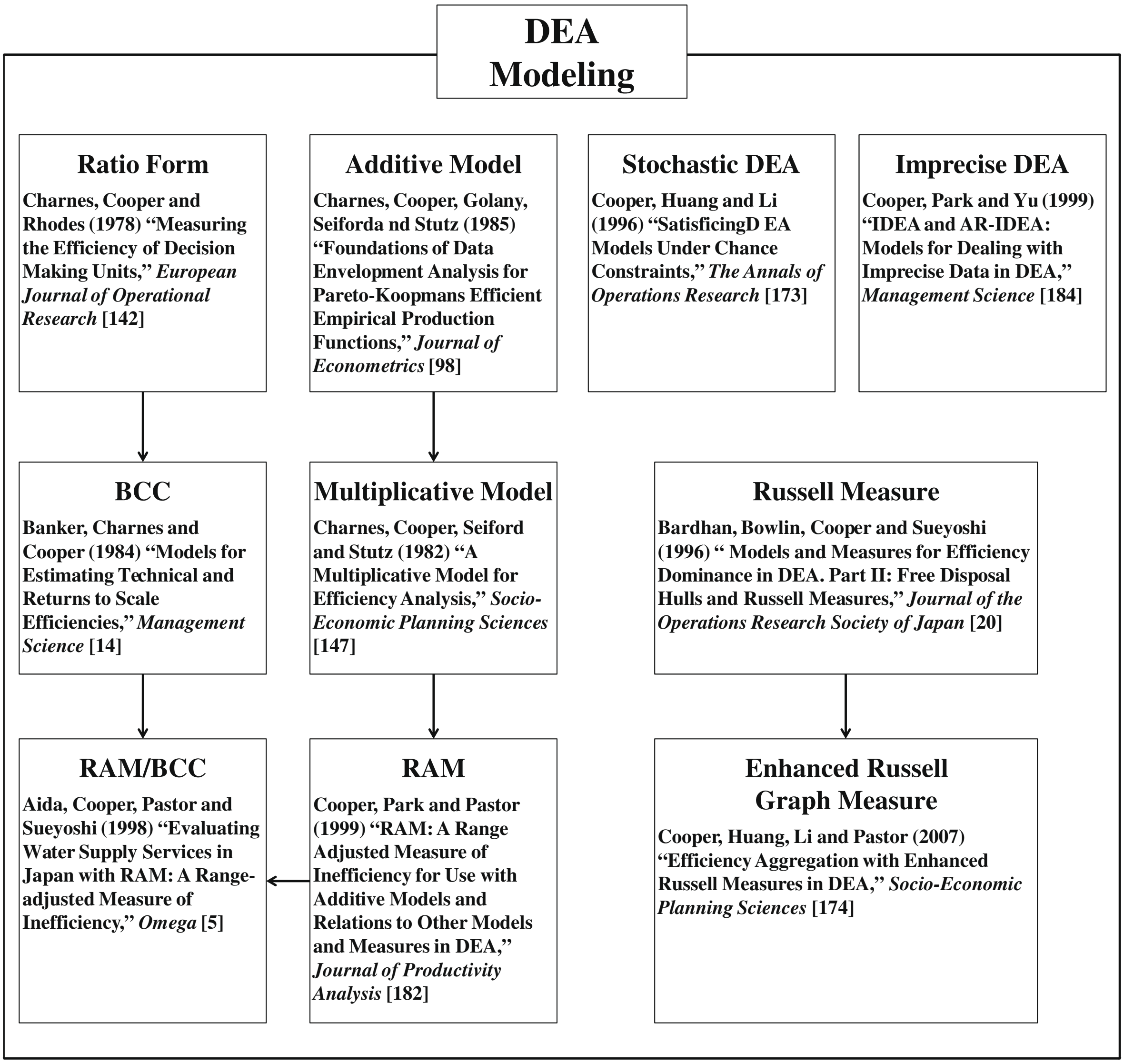

Fig. 2. Contributions of Professor Cooper in DEA models.

and related to theories in detail. We are fully aware of the existence of DEA applications, as summarized in Table 1. This review drops its description because the DEA applications have many variations.

BCC: The BCC model [14] incorporates the additional side constraint $\sum_{j=1}^{n}\lambda_j = 1$ into the CCR model (8). Here, BCC stands for Banker–Charnes–Cooper. Although the only difference between CCR and BCC is the additional constraint, there is a major conceptual difference between them. The CCR ratio form assumes constant RTS (Returns to Scale) while the BCC model is formulated under the assumption of variable RTS. Both CCR and BCC are referred to as "radial models" in the DEA community because they have an efficiency score (θ) that is measured radially on an efficiency frontier.

Additive model: DEA researchers have long paid attention to the additive model as an alternative to the radial models, because the additive model aggregates input-oriented and output-oriented measures to produce a single non-radial measure for technical efficiency [98]. A practical difficulty of the additive model (9) is that it does not have a satisfactory efficiency measure. Hence, we need to determine the efficiency measure in terms of a total amount of slack variables obtained from the additive model. According to [182], we measure the technical efficiency for the additive model by $1 - \frac{1}{m+s}\left(\sum_{i=1}^{m} d_i^{x*}/x_{ik} + \sum_{r=1}^{s} d_r^{y*}/y_{rk}\right)$, where all x_{ik} and y_{rk} are assumed to be positive. The slack values d_i^{x*} and d_r^{y*} are obtained from an optimal solution of (9). Each slack is divided by the corresponding input or output and their sum is divided by the total number of inputs and outputs. Consequently, the second part of the equation indicates the level of total inefficiency. The efficiency measure is therefore determined by subtracting the total inefficiency from unity.

Multiplicative model: If inputs and outputs are expressed in terms of natural logarithm, the additive model is referred to as "a multiplicative model" that corresponds to a generalization of Cobb-Douglas type of production function [147,148]. Mathematically, the model changes a data set (x_{ij}, y_{rj}) to $(\hat{x}_{ij}, \hat{y}_{rj})$, where $\hat{x}_{ij} = \ln(x_{ij})$ and $\hat{y}_{rj} = \ln(y_{rj})$. Thus, the inputs and outputs have the following Cobb-Douglas relationship:

$$x_{ik} = \Pi_{j=1}^{n} x_{ij}^{\lambda_j}\, \mathrm{e}^{d_i^x}\ \ (i = 1, 2, \ldots, m) \quad \text{and} \quad y_{rk} = \Pi_{j=1}^{n} y_{rj}^{\lambda_j}\, \mathrm{e}^{d_r^y}\ \ (r = 1, 2, \ldots, s). \tag{13}$$

The formulation of the multiplicative model is the same as the additive model (1), except the fact that the data are stated logarithmically. Furthermore, while some (but not all) inputs and outputs may be zero in the additive model, the multiplicative model requires that all inputs and outputs are strictly positive.

RAM (range adjusted measure): By modifying the additive model, the research in [182,183] proposed the following RAM model:

$$\begin{aligned}&\text{Max}\Bigg\{\frac{1}{m+s}\left[\sum_{i=1}^{m} d_i^x/R_i^x+\sum_{r=1}^{s} d_r^y/R_r^y\right]\Bigg|\sum_{j=1}^{n} x_{ij}\lambda_j + d_i^x = x_{ik}\ (i=1,\ldots,m),\ \sum_{j=1}^{n} y_{rj}\lambda_j - d_r^y = y_{rk}\ (r=1,\ldots,s),\\ &\quad\sum_{j=1}^{n}\lambda_j = 1,\ \lambda_j \geqslant 0\ (j=1,\ldots,n),\ d_i^x \geqslant 0\ (i=1,\ldots,m),\ \text{and}\ d_r^y \geqslant 0\ (r=1,\ldots,s).\Bigg\}.\end{aligned} \tag{14}$$

Here, the RAM specifies the ranges for inputs and outputs as follows:

$$R_i^x = \bar{x}_i - \underline{x}_i\ (i=1,2,\ldots,m) \quad \text{and} \quad R_r^y = \bar{y}_r - \underline{y}_r\ (r=1,2,\ldots,s) \tag{15}$$

and

$$\begin{aligned}&\bar{x}_i = \max_j\{x_{ij}\}, \quad \underline{x}_i = \min_j\{x_{ij}\} \quad (i=1,2,\ldots,m),\\ &\bar{y}_r = \max_j\{y_{rj}\}, \quad \underline{y}_r = \min_j\{y_{rj}\} \quad (r=1,2,\ldots,s).\end{aligned} \tag{16}$$

Thus,

$$R_i^x = \bar{x}_i - \underline{x}_i = (\bar{x}_i + a_i) - (\underline{x}_i + a_i) \quad \text{and} \quad R_r^y = \bar{y}_r - \underline{y}_r = (\bar{y}_r + b_r) - (\underline{y}_r + b_r). \tag{17}$$

Since the objective of (14) has the numerators and denominators stated in the same units, the efficiency measure of RAM is units invariant. Furthermore, the measure is affine and invariant, as well. This means that RAM is invariant to transformation from $x_i' = a_i + b_i x_i\ (i=1,..,m)$ and $y_r' = a_r + b_r y_r\ (r=1,..,s)$, as in the transformation between "Centigrade" and "Fahrenheit" in temperature measures.

To determine the technical efficiency, we obtain slack values d_i^{x*} and d_r^{y*} from an optimal solution of (14). Then, the measure for efficiency $(\Gamma *)$ is given by

$$\Gamma^* = 1 - \frac{1}{m+s}\left(\sum_{i=1}^{m} d_i^{x*}/R_i^x + \sum_{r=1}^{s} d_r^{y*}/R_r^y\right). \tag{18}$$

Since (14) has $0 \leqslant d_i^{x*} \leqslant R_i^x$ and $0 \leqslant d_r^{y*} \leqslant R_r^y$, the range of technical efficiency satisfies $0 \leqslant \Gamma^* \leqslant 1$. If $\Gamma^* = 1$, then RAM has $d_i^{x*} = d_r^{y*} = 0$ and the kth DMU is fully efficient. Furthermore, RAM defines slacks as $d_i^x = x_{ik} - \sum_{j=1}^{n} x_{ij}\lambda_j \leqslant R_i^x$ and $d_r^y = \sum_{j=1}^{n} y_{rj}\lambda_j - y_{rk} \leqslant R_r^y$. If $\Gamma^* = 0$, then RAM has $d_i^{x*} = R_i^x$ and $d_r^{y*} = R_r^y$ at optimality. Therefore, the kth DMU is fully inefficient.

RAM/BCC: Although RAM serves as an important measure for efficiency, it has a practical difficulty in determining the level of efficiency. For example, the study of Ref. [5] reported that most inefficient organizations belonged to a very small range of inefficiency value. Indeed, the distribution variance of efficiency scores was small and close to unity in the RAM application, which was mathematically reasonable but unacceptable from a practical standpoint. To overcome the practical difficulty, Aida et al. [5] proposed a combined use of RAM/BCC as follows:

$$\begin{aligned}&\text{Min}\Bigg\{\theta - \frac{1}{m+s}\left[\sum_{i=1}^{m}\frac{d_i^x}{R_i^x} + \sum_{r=1}^{s}\frac{d_r^y}{R_r^y}\right]\Bigg| -\sum_{j=1}^{n} x_{ij}\lambda_j + \theta x_{ik} - d_i^x = 0\ (i=1,\ldots,m),\ \sum_{j=1}^{n} y_{rj}\lambda_j - d_r^y = y_{rk}\ (r=1,\ldots,s),\\ &\quad\sum_{j=1}^{n}\lambda_j = 1,\ \lambda_j \geqslant 0\ (j=1,\ldots,n),\ d_i^x \geqslant 0\ (i=1,\ldots,m),\ \text{and}\ d_r^y \geqslant 0\ (r=1,\ldots,s),\ \theta:\text{URS}.\Bigg\}.\end{aligned} \tag{19}$$

The efficiency measure $(\Gamma^*_{\text{RAM-BCC}})$ of the kth DMU, specified relative to an optimal solution of (19), becomes

$$\Gamma^*_{\text{RAM-BCC}} = \theta^* - \frac{1}{m+s}\left[\sum_{i=1}^{m} d_i^{x*}/R_i^x + \sum_{r=1}^{s} d_r^{y*}/R_r^y\right]. \tag{20}$$

The efficiency measure (θ^*) indicates the level of input-oriented efficiency. The second part of (20) gives the slack-based adjustment due to inefficiency. If the efficiency measure is unity, then the organization is fully efficient. The paper [183] replaced the objective in (19) by $\theta - (\varepsilon/(m+s))\left(\sum_{i=1}^{m} d_i^x/R_i^x + \sum_{r=1}^{s} d_r^y/R_r^y\right)$. Here, ε is a non-Archimedean small number that is smaller than any positive number.

Russell measure: The Russell measure [19,20] for the kth DMU is measured by the following model

$$\begin{aligned}&\text{Min}\Bigg\{\frac{1}{m+s}\Big(\sum_{i=1}^{m}\theta_i + \sum_{r=1}^{s}\frac{1}{\phi_r}\Big)\Bigg| -\sum_{j=1}^{n} x_{ij}\lambda_j + \theta_i x_{ik} \geqslant 0\ (i=1,\ldots,m),\ \sum_{j=1}^{n} y_{rj}\lambda_j - \phi_r y_{rk} \geqslant 0\ (r=1,\ldots,s),\\ &\quad\lambda_j \geqslant 0\ (j=1,\ldots,n),\ \theta_i \leqslant 1 (i=1,\ldots,m),\ \text{and}\ \phi_r \geqslant 1\ (r=1,\ldots,s).\Bigg\},\end{aligned} \tag{21}$$

where variables (θ_i and ϕ_r) indicate the efficiency measures related to the ith input and the rth output, respectively.

An important feature of (21) is that it involves a non-linear programming formulation. Hence, to solve (14), the representation (21) is reformulated in [19,20] as a linear programming problem. The approach proposed by Professor Cooper and his associates in [19,20] produces an approximate value for the Russell measure.[19]

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[19] Many economists and DEA researchers refer to (21) as "Russell measure". However, Russell did not document (21) and did not discuss how to solve (21). The first article that discussed (21) was [20]. The second author of this review paper remembers that Professor Cooper studied (21) from 1985 to 1986, but he could not publish his research results because none understood the importance of (21) in the western DEA community. Hence, the second author brought it to Japan and published the research results from (21) in *Journal of the Operations Research Society of Japan* (1996). Therefore, the second author believes that (21) should be referred to as "Cooper measure". See Ref. [174] for the recent contribution on the topic.

Enhanced russell graph measure (*ERGM*): To overcome the difficulty of computing and interpreting (21) (stemming from its non-linear formulation), an "enhanced russell graph measure (ERGM)" was introduced in Ref. [174]:

$$\text{Min}\left\{\left(\sum_{i=1}^{m}\theta_i/m\right)\bigg/\left(\sum_{r=1}^{s}\phi_r/s\right)\middle|-\sum_{j=1}^{n}x_{ij}\lambda_j+\theta_i x_{ik}\geqslant 0\ (i=1,\ldots,m),\ \sum_{j=1}^{n}y_{rj}\lambda_j-\phi_r y_{rk}\geqslant 0\ (r=1,\ldots,s),\right.$$
$$\left.\lambda_j\geqslant 0\ (j=1,\ldots,n),\ \theta_i\leqslant 1\ (i=1,\ldots,m),\ \text{and}\ \phi_r\geqslant 1\ (r=1,\ldots,s)\right\}. \tag{22}$$

The efficiency measure obtained by ERGM attains full efficiency if the objective of (22) is unity at optimality. The result is identified when there is no positive slack. An important feature of ERGM is that (22) can be solved by any linear programming algorithm.

Stochastic DEA: DEA efficiency is addressed in the context of CCP [166,167,172,173]. The resulting DEA model is referred to as "stochastic DEA" and has the following formulation:

$$\text{Max}\left\{P\left[\left(\sum_{r=1}^{s}w_r\bar{y}_{rk}\right)\bigg/\left(\sum_{i=1}^{m}v_i\bar{x}_{ik}\right)\geqslant\beta_j\right]\middle|P\left[\left(\sum_{r=1}^{s}w_r\bar{y}_{rj}\right)\bigg/\left(\sum_{i=1}^{m}v_i\bar{x}_{ij}\right)\geqslant\beta_j\right]\geqslant 1-\alpha_j\ (j=1,\ldots,n).\right\} \tag{23}$$

Here, P stands for "probability" and the "overbar" notation (–) indicates that inputs and outputs are random variables with a known joint probability distribution. The multipliers to be determined by (23) are $w_r\ (r=1,\ldots,s)$ and $v_i\ (i=1,\ldots,m)$ with $w_r\geqslant 0$ and $v_i\geqslant 0$. The value $\alpha_j\ (j=1,\ldots,n)$ indicates the probability of achieving the value regarding a prescribed minimum level of efficiency (β_j) according to the choice of multipliers. Thus, $1-\alpha_j$ indicates the probability of failing to attain the indicated value. The stochastic model (23) corresponds to the CCR ratio form. It is possible to apply the stochastic context to the other types of DEA. The contribution of [166,167,172,173] is to transform (23) into an equivalent linear programming model.

Imprecise DEA: A method for dealing with imprecise data in DEA is introduced in [184] where "imprecise" means that some data lie within prescribed bounds but the exact value taken within these bounds is unspecified. The model for imprecise DEA is as follows:

$$\text{Max}\left\{\sum_{r=1}^{s}u_r y_{rk}\middle|\sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_i j\leqslant 0\ (j=1,..,n),\ \sum_{i=1}^{m}v_i x_{ik}=1\ \ y_{rj}\in D_{rj}^{+}\ (j=1,\ldots,n),\ \ x_{ij}\in D_{ij}^{-}\ (j=1,\ldots,n),\right.$$
$$\left.u_r\geqslant 0\ (r=1,..,s),\ \ v_i\geqslant 0\ (i=1,\ldots,s)\right\}, \tag{24}$$

where D_{rj}^{+} indicates the upper and lower bounds of the rth output of the jth organization and D_{ij}^{-} indicates the upper and lower bounds of the ith input of the jth organization. The data ranges are expressed by $\underline{y}_{rj}\leqslant y_{rj}\leqslant\bar{y}_{rj}\ (r=1,\ldots,s)$ and $\underline{x}_{ij}\leqslant x_{ij}\leqslant\bar{y}_{ij}\ (i=1,\ldots,m)$, respectively. The contribution of [184] is to transform (24) into an equivalent linear programming model by using the cone ratio approach discussed next.

Reviewing all components of Fig. 2, this study confirms that Professor Cooper has been involved in the development of nearly all of the currently used DEA models and solution methods. Thus, he has had a profound impact on the theoretical extensions and applications by the development of various DEA models.

Next, shifting from the development of various DEA models, Fig. 3 visually summarizes the contributions of Professor Cooper in DEA theory and related algorithms. These contributions include DMU characterization, RTS (Returns to Scale) measurements, congestion, cone ratio analysis, comparisons between DEA and regression analysis, a theoretical linkage to game theory, and other properties related to technical efficiency.

DMU characterization/classification: DMU classifications were investigated in [156,157]. Special computer codes to solve DEA models fully utilize the proposed DMU classifications for their algorithmic developments. The classification partitions the set J of all DMUs into two subsets: $J=J_d\cup J_n$. The subset J_d is a dominated set and the subset J_n is non-dominated. To specify the two groups, the research [156,157] uses a concept of dominance that is mathematically specified by

$$[x_{1j},\ldots,x_{mj},-y_{1j},\ldots,-y_{sj}]^{\mathrm{T}}\geqslant[x_{1j'},\ldots,x_{mj'},-y_{1j'},\ldots,-y_{sj'}]^{\mathrm{T}}. \tag{25}$$

Here, $\geqslant$ implies that at least one component has the relationship (>). Using (25), the two subsets become:

$$J_d=\{j\in J|\ \text{there is a DMU}j'\ \text{that satisfies (25)}\}\ \text{and}\ J_n=J-J_d. \tag{26}$$

After classifying J into the two subsets, each subset is separated further as follows:

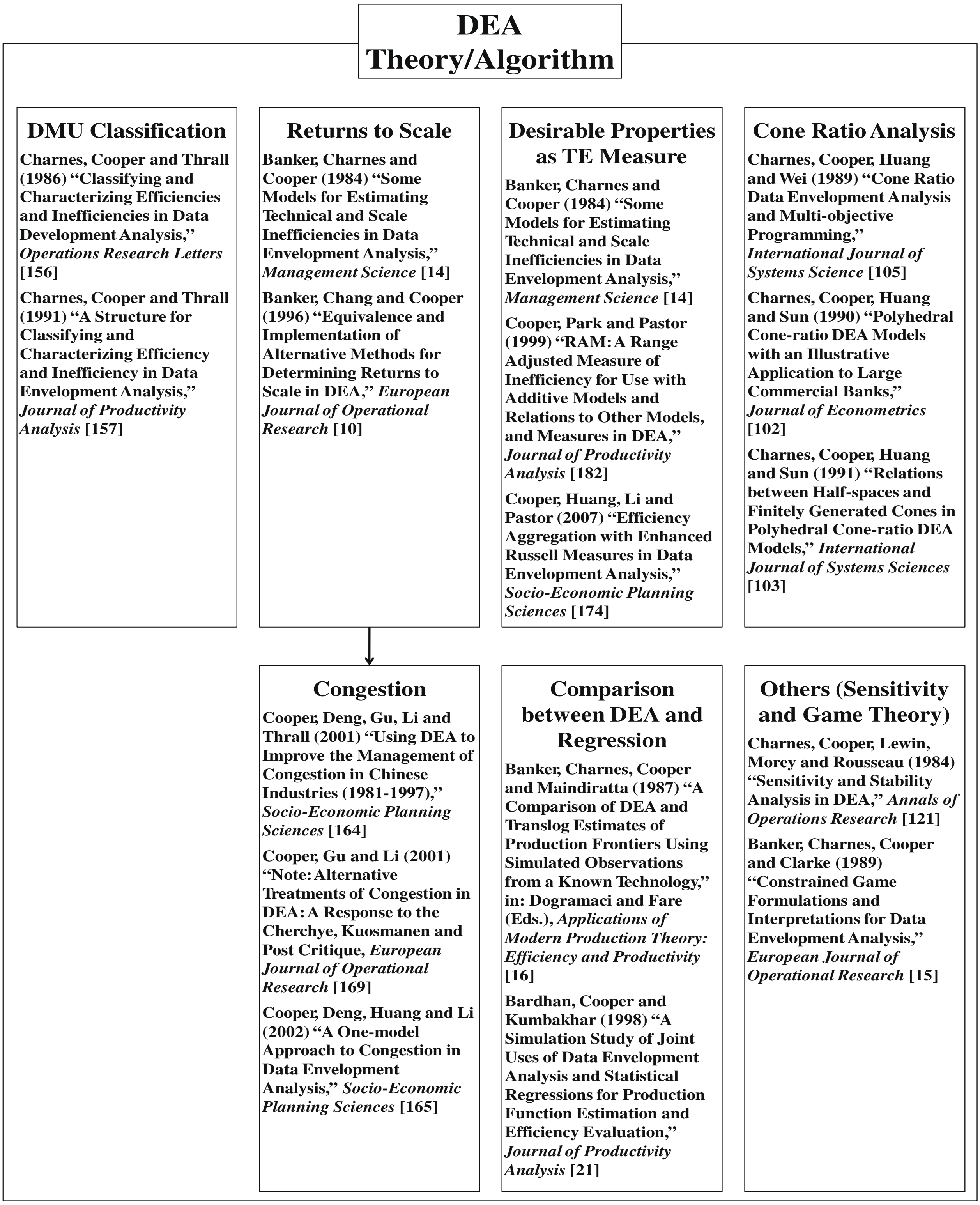
$$J_n=E\cup E'\cup IE'\cup IF'\quad\text{and}\quad J_d=IE\cup IF. \tag{27}$$

Here

$$E=\{k\in J_n|\theta_k^*=1,\lambda_k^*=1,\lambda_j^*=0\ \ (\forall j\neq k\in J_n)\ \text{and all slacks are zero}\},$$
$$E'=\{k\in J_n|\theta_k^*=1,\lambda_k^*<1,\lambda_j^*>0\ \ (\exists j\in E),\ \text{all slacks are zero and multiple solutions}\},$$
$$IE'=\{k\in J_n|\theta_k^*<1,\lambda_k^*=0,\lambda_j^*>0\ \ (\exists j\in E)\},$$
$$IF'=\{k\in J_n|\theta_k^*=1\ \text{and at least one slack is positive}\},$$
$$IE=\{k\in J_d|\theta_k^*<1,\lambda_k^*=0,\lambda_j^*>0\ \ (\exists j\in E)\},\ \text{and}$$
$$IF=\{k\in J_d|\theta_k^*=1\ \ \text{and at least one slack is positive}\}.$$

Special computer codes are designed to identify E at an early stage of DEA computation and determine the technical efficiency of the other DMUs based upon E. Consequently, we can drop many DMUs from a computational process of DEA and thereby reduce the computational time. Note that radial models (CCR and BCC) have IF and IF′, while non-radial models do not have such subsets.

Returns to scale (*RTS*): RTS is an important DEA concept that relates a proportional increase in outputs to a unit increase in inputs. Conceptually, RTS measures point elasticity $e_p=(y/x)/(\mathrm{d}y/\mathrm{d}x)$ where average production (y/x) is divided by marginal production $(\mathrm{d}y/\mathrm{d}x)$. The contribution of Professor Cooper and his associates is that they extend the concept to the measurement of multiple outputs, including consideration of multiple solutions.



DEA Theory/Algorithm

DMU Classification

Charnes, Cooper and Thrall (1986) "Classifying and Characterizing Efficiencies and Inefficiencies in Data Development Analysis," *Operations Research Letters* [156]

Charnes, Cooper and Thrall (1991) "A Structure for Classifying and Characterizing Efficiency and Inefficiency in Data Envelopment Analysis," *Journal of Productivity Analysis* [157]

Returns to Scale

Banker, Charnes and Cooper (1984) "Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis," *Management Science* [14]

Banker, Chang and Cooper (1996) "Equivalence and Implementation of Alternative Methods for Determining Returns to Scale in DEA," *European Journal of Operational Research* [10]

Desirable Properties as TE Measure

Banker, Charnes and Cooper (1984) "Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis," *Management Science* [14]

Cooper, Park and Pastor (1999) "RAM: A Range Adjusted Measure of Inefficiency for Use with Additive Models and Relations to Other Models, and Measures in DEA," *Journal of Productivity Analysis* [182]

Cooper, Huang, Li and Pastor (2007) "Efficiency Aggregation with Enhanced Russell Measures in Data Envelopment Analysis," *Socio-Economic Planning Sciences* [174]

Cone Ratio Analysis

Charnes, Cooper, Huang and Wei (1989) "Cone Ratio Data Envelopment Analysis and Multi-objective Programming," *International Journal of Systems Science* [105]

Charnes, Cooper, Huang and Sun (1990) "Polyhedral Cone-ratio DEA Models with an Illustrative Application to Large Commercial Banks," *Journal of Econometrics* [102]

Charnes, Cooper, Huang and Sun (1991) "Relations between Half-spaces and Finitely Generated Cones in Polyhedral Cone-ratio DEA Models," *International Journal of Systems Sciences* [103]

Congestion

Cooper, Deng, Gu, Li and Thrall (2001) "Using DEA to Improve the Management of Congestion in Chinese Industries (1981-1997)," *Socio-Economic Planning Sciences* [164]

Cooper, Gu and Li (2001) "Note: Alternative Treatments of Congestion in DEA: A Response to the Cherchye, Kuosmanen and Post Critique, *European Journal of Operational Research* [169]

Cooper, Deng, Huang and Li (2002) "A One-model Approach to Congestion in Data Envelopment Analysis," *Socio-Economic Planning Sciences* [165]

Comparison between DEA and Regression

Banker, Charnes, Cooper and Maindiratta (1987) "A Comparison of DEA and Translog Estimates of Production Frontiers Using Simulated Observations from a Known Technology," in: Dogramaci and Fare (Eds.), *Applications of Modern Production Theory: Efficiency and Productivity* [16]

Bardhan, Cooper and Kumbakhar (1998) "A Simulation Study of Joint Uses of Data Envelopment Analysis and Statistical Regressions for Production Function Estimation and Efficiency Evaluation," *Journal of Productivity Analysis* [21]

Others (Sensitivity and Game Theory)

Charnes, Cooper, Lewin, Morey and Rousseau (1984) "Sensitivity and Stability Analysis in DEA," *Annals of Operations Research* [121]

Banker, Charnes, Cooper and Clarke (1989) "Constrained Game Formulations and Interpretations for Data Envelopment Analysis," *European Journal of Operational Research* [15]

Fig. 3. Contributions of Professor Cooper to DEA theory and algorithms.

To describe briefly how to measure RTS within the DEA framework, we formulate the dual model of RAM (14) as follows:[20]

$$\min\{V^{\mathrm{T}}X_k - W^{\mathrm{T}}Y_k + \sigma \mid V^{\mathrm{T}}X_j - W^{\mathrm{T}}Y_j + \sigma \geqslant 0\ (j = 1, \ldots, n),\ \ V \geqslant R^x, W \geqslant R^y\}. \tag{28}$$

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[20] Banker, R.D., Cooper W.W., Seiford, L.M., Thrall, R.M. and Zhu, J. "Returns to Scale in Different DEA Models" *European Journal of Operational Research* (2004) 154 pp. 345–362.

Here, $V = (v_1, \ldots, v_m)^T$ and $W = (w_1, \ldots, w_s)^T$ are two column vectors of dual variables related to the first and second sets of constraints in (14). A dual variable (σ) is associated with the last constraint of (14).

Using an optimal solution (V^*, W^*, σ^*) obtained from (28), we have the following RTS classification:

(a) increasing RTS prevails at a projected point if and only if $\sigma^* < 0$ for all optimal solutions,
(b) decreasing RTS prevails at a projected point if and only if $\sigma^* > 0$ for all optimal solutions, or
(c) constant RTS prevails at a projected point if and only if $\sigma^* = 0$ for at least one optimal solution.

Professor Cooper and his associates have examined the upper and lower bounds of σ^* to examine the status of RTS in the presence of multiple solutions. See Refs. [9–11,14] for Professor Cooper's multi-dimensional discussions on RTS.

Congestion: The economic concept of "congestion" is captured in a novel way by RTS. Congestion is a widely observed phenomenon in which inefficiency is identified in such a manner that a reduction in an input results in an increase in a maximum possible output without worsening other inputs and outputs [195]. Fig. 4 depicts such an occurrence of congestion in a production function having arguments in a single input (x) and a single output (y). Fig. 5 specifies the two types of RTS under congestion: No RTS at "A" and Negative RTS at "*F*". The conventional use of point elasticity for the RTS classification indicates that dy/dx becomes zero at "A" and negative at "*F*". Hence, the elasticity also becomes negative. Thus, Figs. 4 and 5 indicate a theoretical linkage between congestion and RTS.

DEA-based congestion is extensively investigated in [34,44,164,165,169,195], whose contributions include: (a) developing an analytical scheme capable of identifying an occurrence of congestion, (b) producing a measure related to the degree of congestion, (c) distinguishing the degree of congestion from other components of technical efficiency, and (d) additionally extending the concept of congestion by associating it with other economic concepts such as marginal productivities and rates of substitutions.

Desirable properties: The works of Refs. [14,182,174] discuss the following desirable properties for technical efficiency:

(a) *Homogeneity*: An output-based efficiency measure should be homogeneous of degree one in output quantities, while an input-based efficiency measure should be homogeneous of degree minus one in input quantities. For example, if we double all the input quantities, then the input-based efficiency measure should be cut in half.
(b) *Strict monotonicity*: A measure for technical efficiency should be non-decreasing in output quantities and non-increasing in input quantities along with an efficient output (input) vector.

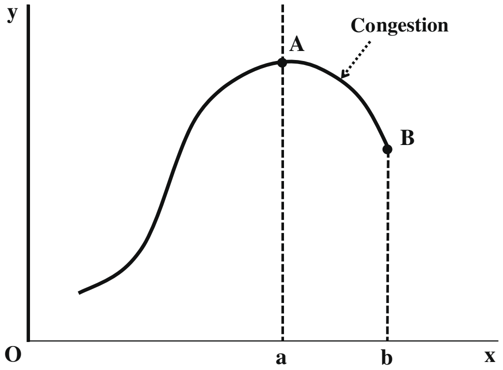

Fig. 4. Congestion in production.

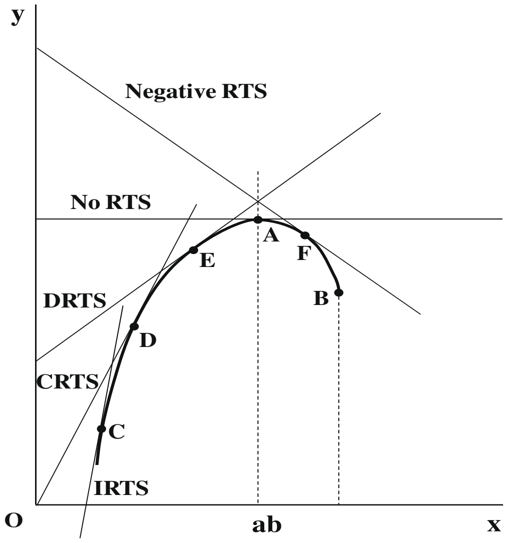

Fig. 5. Five types of RTS. IRTS: Increasing RTS; CRTS: Constant RTS; DRTS: Decreasing RTS.

(c) *Efficiency requirement*: An efficiency measure should be between zero and one, where "zero" implies full inefficiency and "one" implies full efficiency.
(d) *Unique projection for efficiency comparison*: An efficiency measure compares each input (output) vector with an efficient input (output) vector. The uniqueness of vector comparison is very desirable. For example, a radial measure minimizes an input amount that can be shrunk along a projection ray, while holding the output quantities constant. However, the non-radial measures do not have the ray, so that these measures have an infinite number of projection vectors for efficiency comparison.
(e) *Aggregation*: A jointly measured efficiency among all inputs and all outputs is desirable for performance analysis. The property indicates that the aggregation of inputs and outputs should not influence the efficiency measurement of all entities.
(f) *Unit invariance*: The unit of inputs and outputs should not influence an efficiency measure.
(g) *Invariance to alternate optima*: An occurrence of multiple solutions on DEA should not influence an efficiency measure.
(h) *Translation invariance*: An efficiency measure should not be influenced even if inputs and/or outputs are shifted toward a same direction by adding or subtracting a specific real number.

The foregoing properties provide a foundation for comparing different DEA models and serve as a guide in selecting an appropriate DEA model in each application.

Multiplier restriction by cone ratio analysis: A problem in applying DEA to performance analysis is that DEA assigns zero values to many multipliers. A zero assignment implies that a corresponding input or output is not fully utilized in the DEA evaluation. Moreover, the occurrence of many zero-valued multipliers implies that there are many efficient organizations. The result is mathematically acceptable but managerially problematic because managers and corporate leaders are interested in the ranks of their firms within a common industry. To reduce the number of efficient DMUs and the number of zero-valued multipliers, cone ratio analysis has been proposed in Refs. [102–105,202] to restrict the multipliers to a prescribed region within a data domain. Usually, such a restriction is based upon prior information such as previous experience. Cone ratio analysis adds new mathematical sophistication to previous work on assurance region analysis as introduced by Thompson et al.[21] The computational effort required by cone ratio analysis is less than that of assurance analysis and the approach additionally reduces the number of efficient DMUs and zero multipliers.[22] Moreover, cone ratio analysis provides a computational framework for carrying out both efficiency and effectiveness measurements.

Comparison between DEA and regression analysis: Conceptually, performance analysis based upon technical efficiency is broadly classified into parametric and non-parametric analyses. Parametric analysis needs to assume a functional form for a production frontier which is usually stochastic. We typically assume two types of errors to estimate the stochastic production frontier: an observational error and a managerial error in stochastic frontier regression.[23] The observational error is usually assumed to follow a two-sided error like a normal distribution and the managerial error follows a single-sided error like a half-normal distribution and an exponential distribution. There are many different combinations between the observational error and the managerial error. Moreover, maximum likelihood estimation is used to obtain parameter estimates of the stochastic production function. The level of technical efficiency indicates how much each observation deviates from the stochastic production frontier.

DEA constitutes a form of non-parametric analysis,[24] enabling it to avoid the specification of the production function and removing the need to assume an error distribution. The investigations of Refs. [16,21] compare stochastic production analysis with DEA from the perspective of technical efficiency measurement and production analysis.

Other theoretical contributions in DEA: It is beyond the scope of this review to describe all of many additional contributions of Professor Cooper in the DEA area. However, we briefly mention his works in sensitivity analysis and in establishing connections with game theory. An incisive treatment of sensitivity and stability analysis within DEA is provided in [121], providing important mathematical insights into the manner in which the structure of a data set relates to performance analysis. For example, when applying DEA to examine an occurrence of a frontier shift between multiple periods, sensitivity analysis makes it possible to know whether DEA can produce a feasible solution or not. Such analysis also makes it possible to predict the level of efficiency in a period following the current period. Professor Cooper's theoretical linkage between DEA and game theory in [15] discloses how an optimal DEA solution can improve the efficiency of organizations within a framework of game where two different groups are interested in their strategies for the enhancement of their profits.

Finally, as visually summarized in Fig. 3, Professor Cooper has been involved in the development of almost all DEA theoretical works. His research effort on DEA theory and its usages has served as an important basis for various algorithmic developments. In recognition of these contributions, Professor Cooper and his associates obtained the John Von Neumann Theory Prize (the most prestigious award in the OR community) jointly from TIMS and ORSA (Operations Research Society of America) in 1982.

5. Conclusion

Professor Cooper's pioneering contributions to the fields of OR/MS have left an enduring legacy, making a particular impact on business education and research. He was instrumental in fashioning fundamental concepts and models in the initial stages of linear programming,

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[21] Thompson, R.G., Singleton, F.D., Thrall, R.M. and Smith, B.A. "Comparative Site Evaluations for locating a High-Energy Physics Lab in Texas," *Interface* (1986) 16 pp. 35–49.

[22] The research [186] produces a method that maximizes the number of non-zero multipliers in a set of alternative optima. The same authors of [186] then show how to apply the approach to the evaluation of basketball players in the Spanish basketball league. Therefore, the research provides a basis for extending DEA in evaluating sport activities. See, Cooper, W.W., Ruiz, S.L. and Sirvent, I. "Selecting Non-zero Weights to Evaluating Basketball Players with DEA,"*European Journal of Operational Research* (Forthcoming in 2008) which is now available from the EJOR web site.

[23] The stochastic frontier regression is usually referred to as "stochastic frontier analysis". See Kumbhakar, S.C. and Lovell, C.A.K., *Stochastic Frontier Analysis*, Cambridge University Press, Cambridge, UK.

[24] According to a well-known statistician such as Huber P.J. *Robust Regression*, John Wiley (1981) p. 6 "A procedure is called non-parametric if it is supposed to be used for a broad, non-parametrized set of underlying distributions. For instance, the sample mean and the sample median are the non-parametric estimates of the population mean and the sample mean, respectively. A test is called distribution-free if the probability of falsely rejecting the null hypothesis is the same for all possible underlying continuous distributions." This review considers that DEA is non-parametric because it measures estimates (λ^*) of a piece-wise linear production function, not parameter estimates of a production function as found in the stochastic frontier analysis. Hence, the definition of non-parametric in this review paper is different from the definition of the conventional statistics.

non-linear programming, GP, CCP, manpower planning and multi-objective optimization. Later, he extended his contributions to linear programming and non-linear programming to launch the area of DEA which has been widely applied to performance analysis in the public and private sectors. Professor Cooper also developed critical business-related concepts and research areas within OR/MS, management, marketing, managerial accounting, auditing, economics, and public policy, all of which currently serve as pedagogical and research bases in modern business and business education.

This review work has left an indelible imprint on many domains, and serves as a continuing foundation for investigations by researchers around the globe. In this celebration of his 95th birthday, it is appropriate to pay Professor William W. Cooper's special tribute for his many advances that have lifted the realm of optimization and its applications to new levels, and that have helped to pave the foundation of modern analysis and applications.

Acknowledgement

This research is dedicated to Ruth Cooper, J.D., Esq. She had long supported Professor Cooper as his best friend and wife. She was the first woman lawyer in the state of Pennsylvania (USA) who fought for women's social issues and supported low income families. She was a true lawyer who worked for the American society. We thank Dr. Robert Dyson (Warwick University) who gives us an opportunity to prepare this review article.

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