VALUING REAL OPTIONS: INSIGHTS FROM COMPETITIVE STRATEGY

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INTRODUCTION

Growth opportunities and future strategies can comprise a significant proportion of a firm’s valuation. At the end of 2006, the median company in the S&P 500 and Russell 3000 had 25% and 40% of their valuation, respectively, attributed to Future Growth Value (FGV®), the capitalized value of future profit growth. Acquisition premiums can also be interpreted as estimates of value creation attributed to new tactics and operational improvements under a new regime. Unfortunately, managers often find static Net Present Value tools and trading multiples to be too rigid to evaluate the contingent nature of strategic decisions and the cash flow recovery profiles associated with possible outcomes. For example, Microsoft was willing to develop its Xbox platform at a loss because it expected subsequent game and peripheral offerings linked to it to generate significant profits. Similarly, commodities producers frequently choose to delay extraction until output prices swing in their favor. Academics and practitioners have recognized the similarities of payoff functions between such contingent decisions about real assets, classic examples of “real options,” and those of financial securities whose value is derived from the price of something else. The Black-Scholes model and Binomial Lattices have emerged as the most frequently prescribed and used tools for evaluating real options within both capital budgeting and enterprise valuation contexts. With the classic real option decision growing increasingly complex, and managers becoming more sophisticated, a frank assessment of modern valuation tools is timely.

Section I provides an overview of commonly used approaches for pricing contingent claims on financial securities. Section II reviews how practitioners and academics have extended these

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1 FGV is calculated as Enterprise Value – Capital in place – Current Year’s EVA®/WACC. S&P data for the 2006 year end was downloaded from Bloomberg; and Russell 3000 and WACC data pulled from The 2007 Russell 3000 EVA/MVA Annual Ranking Database
approaches to the basic application of real option pricing. Section III, the core focus of this chapter, scrutinizes the assumptions made in the extension of models built for financial securities to real projects. A number of refinements are presented that attempt to better address the strategic realities of the firm and, in doing so, generate a more robust valuation. A summary and discussion of areas of future research conclude the chapter.

I. OVERVIEW OF OPTION PRICING FOR FINANCIAL SECURITIES

Options give an investor the right, but not the obligation, to buy or sell a security according to predetermined terms during some period or at some specific point. Stock option contracts can be divided into two categories: calls and puts. A call option gives the holder the right to purchase ("call") stock from a counterparty at a fixed exercise price at or before a specific date. A put option gives the holder the right to sell ("put") stock to a counterparty at a fixed exercise price at or before a specific date. At contract expiry, the call holder makes money when the price of the underlying is above the exercise price, while the opposite holds for the put holder. When the exercise is only permitted upon option contract expiry, it is termed “European”; when exercise is permitted at any point up to and upon expiry, it is termed “American”. Figure 1 depicts the payoff function for call and put option positions where the exercise price is $100.

Figure 1: Payoff Diagrams for Stock Options

Stock options can serve a number of important purposes:
1. Given their low price relative to the underlying stock, call and put options can be used to make a leveraged bet on future returns. For the same up-front cost as a single stock, a number of call options can be purchased, resulting in more than a dollar-for-dollar change in wealth for each dollar change in stock price.

2. Call and put options can provide an inexpensive way to hedge positions in firms with similar exposures, or holdings in the stock itself. Investors locked into a long position in a stock can get insurance on their position by purchasing a put option.

3. Finally, call options are instruments frequently used in executive compensation to align the long-term interests of management and the firm. For example, Table 1 shows FedEx disclosed in its 2007 Annual Report the granting of options to its management.

Table 1: FedEx Option Grants

<table>
<thead>
<tr>
<th>Expected lives</th>
<th>2007</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected volatility</td>
<td>5 years</td>
<td>5 years</td>
<td>4 years</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
<td>22%</td>
<td>25%</td>
<td>27%</td>
</tr>
<tr>
<td>Dividend yield</td>
<td>4.879%</td>
<td>3.704%</td>
<td>3.559%</td>
</tr>
<tr>
<td>Dividend yield</td>
<td>0.3023%</td>
<td>0.3229%</td>
<td>0.3215%</td>
</tr>
</tbody>
</table>

The weighted-average Black-Scholes value of our stock option grants using the assumptions indicated above was $31.60 per option in 2007, $25.78 per option in 2006 and $20.37 per option in 2005. The intrinsic value of options exercised was $145 million in 2007, $191 million in 2006 and $126 million in 2005.

The following table summarizes information about stock option activity for the year ended May 31, 2007:

<table>
<thead>
<tr>
<th>Stock Options</th>
<th>Shares</th>
<th>Weighted-Average Exercise Price</th>
<th>Weighted-Average Remaining Contractual Term</th>
<th>Aggregate Intrinsic Value (in millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outstanding at June 1, 2006</td>
<td>17,099,526</td>
<td>$60.82</td>
<td>5.9 years</td>
<td>$656</td>
</tr>
<tr>
<td>Granted</td>
<td>2,094,873</td>
<td>110.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exercised</td>
<td>(2,332,845)</td>
<td>49.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forfeited</td>
<td>(270,153)</td>
<td>89.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outstanding at May 31, 2007</td>
<td>16,590,601</td>
<td>$58.22</td>
<td>5.9 years</td>
<td>$656</td>
</tr>
<tr>
<td>Exercisable</td>
<td>10,418,072</td>
<td>54.75</td>
<td>4.6 years</td>
<td>$577</td>
</tr>
<tr>
<td>Expected to Vest</td>
<td>5,672,529</td>
<td>90.37</td>
<td>8.0 years</td>
<td>$109</td>
</tr>
</tbody>
</table>

Options contracts are written on all sorts of other underlying assets and variables such as bonds, interest rates, exchange rates, and commodities. They also have been tailored in their mechanics, with options termed Asian, Barrier, Bermuda, and Digital characterizing different payoff rules.
The Black-Scholes Model

The growth of innovation and volume in option trading has coincided with advances in pricing approaches for stock options. One of the most influential approaches was published by Fischer S. Black and Myron S. Scholes in 1973. The “Black-Scholes” model employed Geometric Brownian Motion, a domain of stochastic calculus, to simulate the price path of the underlying stock. Options had been traded on exchanges as far back as the 17th century, but Figure 2 shows it wasn’t until after the Black-Scholes model and equally important research by Robert C. Merton (1973) was published that the market truly took off.

Figure 2: Size of the Derivatives Market

![Graph showing the growth of the derivatives market from 1987 to 2008.](image)

*Source: International Swaps and Derivatives Association. Figures include interest rate swaps, currency swaps, credit default swaps, and equity derivatives.*

The Black-Scholes model prices stock options with five variables:

- The price of the underlying (S)
- The exercise (“strike”) price (X)
- The risk-free rate (r)
- The option contract horizon, in years (T)
- The annual return volatility of the underlying (σ).

Figure 3 summarizes how the option value responds to changes in the above variables.
Consider the logic behind the valuation relationships in Figure 3. The right to buy (call) stock is worth more when you have to pay less than what it’s worth; and the right to sell (put) stock is worth more when you are entitled to receive more than what it’s worth. This refers to the intrinsic value of the option, or the current spread between the stock and exercise price. Since option contracts endow a right but not an obligation of exercise, having a more uncertain underlying price path increases the possible upside, while the potential downside (you forfeit the cost of the call when exercise is not worthwhile) remains limited. Finally, having more time to enjoy price swings in your favor increases the value of the option to you. Black-Scholes synthesizes the relationships between these inputs to produce European call (C) and put (P) option values as follows:

\[ C = S^* N(d_1) - X * e^{-rt} * N(d_2) \]

\[ P = X^* e^{-rt} N(-d_2) - S^* N(-d_1) \]

Where

\[ d_1 = \left[ \ln(S / X) + ((r + \sigma^2) / 2)t \right] / \sigma \sqrt{t} \]

and

\[ d_2 = d_1 - \sigma \sqrt{T} \]
One significant finding of Black and Scholes is that there exists a relationship between the option value, the current value of the underlying stock, and the return on a risk-free security. The “put-call parity” relation is

\[ C + X * e^{-r} = P + S \]

Exercise 1:

a) Price a European call option issued on stock with an exercise price of $15, value of $13, annual return volatility of 25%, and horizon of one year. Assume the risk-free rate is 5%.

b) Use the put-call Parity relationship to price a put option on the same stock.

c) Another one year European call issued on a different stock is priced at $1.20. It has an exercise price of $20, a current value of $17, and the risk-free rate is still 5%. Use the Black-Scholes model to calculate the annual return volatility implied by the current price.

Solution:

a) $0.81

b) $2.08

c) 28.74%

The configuration of the Black-Scholes model shown above assumes no dividends are paid on the underlying stock. When dividends are paid, call option holders suffer because they don’t participate in the payment, and the underlying stock price falls by approximately the amount of the dividend. For this same reason, put holders are better off—the spread between the exercise and underlying price increases. Figure 4 outlines the effect of dividends on call and put option values.
To handle cases where the underlying stock has a non-zero dividend yield, the Black-Scholes model can be applied as follows:

\[
C = S \cdot e^{-yt} \cdot N(d_1) - X \cdot e^{-rt} \cdot N(d_2)
\]

Where y equals the annual dividend yield (the dividend divided by S), and d_1 and d_2 now equal

\[
d_1 = \left[ \ln \left( \frac{S}{X} \right) + \left( (r + y - \sigma^2) / 2 \right)t \right] / \sigma \sqrt{t}
\]

\[
d_2 = d_1 - \sigma \sqrt{t}
\]

The value of the put can be determined using the same put-call parity relationship for non-dividend-paying stocks outlined earlier. Note that the dividend yield represents a continuous payment; consistent with the time mechanics assumed in the Black-Scholes model.

**Exercise 2:**

a) Refer back to the excerpt of the FedEx annual report at the beginning of this section. Use the horizon, volatility, risk-free rate, and dividend yield assumptions from the 2007 column to price a call option on FedEx stock issued “at the money” (S=X) when the stock is worth $108.75.

**Solution:**

a) $31.57
Binomial Lattices

Whereas the Black-Scholes model applies continuous time dynamics, Binomial Lattices use discrete time dynamics. This approach, developed by Cox, Ross, and Rubinstein (1977), is particularly useful for analyzing the effects on option values of one-time events such as bankruptcies or mergers; and recurring events such as quarterly dividends; in addition to modeling the American option exercise. The same five variables (price of the underlying, strike price, risk-free rate, option contract horizon, and return volatility of the underlying) play a role in valuing the option. In fact, as the Binomial Lattice is geared with smaller and smaller time increments, the option price will converge to the Black-Scholes value.

A Binomial Lattice works as follows. The period leading up to option expiration is split into subperiods, marked by “nodes.” Price discovery for the underlying occurs at each node—the stock rises or falls by a specific amount depending on its return volatility. The stock increase scalar “u” which set the magnitude of the price rise is equal to $e^{α\Delta t}$ and the stock decrease scalar “d” equals $1/u$. Readers familiar with decision trees will recognize the mechanics of this framework, presented in Figure 5.

**Figure 5: Simple Binomial Lattice for Stock**
Exercise 3:

a) Using the volatility assumptions of Exercise 1a), model the price path of the stock for one year using a Binomial Lattice with nodes every three months.

Solution:

a)

Once the price path of the underlying has been simulated, the option can be valued at each ending node. The option is worth the greater of $S - X$ and 0 (given the investor can choose not to exercise).

For the call option, the value is the greater of 0 and $S - X$: for the put option, the value is the greater of 0 and $X - S$. 
The option value today is then estimated by working recursively through the tree, assuming risk neutrality. This means that future payoffs are discounted at the risk-free rate, where the probability of a “u” step in the underlying is:

\[ p_u = \frac{e^{(r-y)T} - d}{(u-d)} \]

And the probability of a “d” step is simply \( 1 - p_u \)

The value of the options at each of the second-to-last nodes can be calculated:

For each node prior to the second-to-last node, using the same recursive approach:

When the stock pays a discrete dividend (as the vast majority of dividends are paid in practice), it can be modeled in the lattice by simply subtracting the dividend from the underlying when it is paid.
Exercise 4:

a) The stock from Exercise 3 pays a dividend of $0.10 at the end of each quarter.

Recalculate the price path of the stock using the Binomial Lattice approach.

b) How does the new price path change the value of the call option?

Solution:

a)

b)

The call is worth $0.13 less.

II. BASIC OPTION PRICING APPLICATIONS FOR REAL ASSETS

The pricing tools for call and put options have been extended to the valuation of all sorts of other contracts including interest rate derivatives, futures contracts, and exotic variations of vanilla
calls and puts. Largely because of the explicit modeling of state-dependent decisions, they have also proven useful for evaluating real options, where the firm holds the right but not the obligation to make some business decision. Traditional valuation approaches such as NPV and Internal Rate of Return are useful for applying to the “as-is” perspective of the firm. However, the total worth of a firm comprises not just the value of the current operations, but also the value derived from the ability to expand projects progressing successfully; abandon ones revealing themselves to be unsuccessful; and take advantage of the learning, information, and market position gained in both scenarios. Net Present Value (NPV) and Internal Rate of Return (IRR) analyses often are too rigid to maneuver the dynamic and flexible nature of such strategic options.

The classic capital budgeting exercise involves discounting expected cash flows at an appropriate risk-adjusted rate. When only point estimates of the most likely scenario are used in valuation; and the payoffs from dynamic and flexible strategies are ignored or improperly measured; the valuation exercise is incomplete (Myers 1977, 1984). How can the ability to expand, abandon, and collect information be priced? What is the conceptual connection to the options framework?

Production Flexibility and Platform Investments

Let’s start with a call option. There are many situations where possessing the right, but not the obligation, to make a production or investment decision creates value for the firm. Consider the investment and extraction opportunities of an oil company. While spot (current) market prices are a major determinant of project valuation, the possibility of delaying production until prices may be higher can be a material source of value as well. The greater the uncertainty in the output
market, the more the right to delay is worth. The value of this flexibility, combined with the present value of the project in current conditions, can be compared to the fixed investment cost. A manufacturer may look at its capacity decisions with a similar perspective. Maintaining excess capacity or inventory can allow the firm to capture enormous profits during periods of peak demand that more than offset holding costs and the opportunity cost of the capital tied up.

Yet another analogy can be made to companies that invest significant amounts in intangible assets. Much of their focus is on searching for growth opportunities where platform investments, often developed at considerable expense, spawn profitable offspring. For example, Microsoft’s ownership of the Windows platform allowed them to develop follow-on products such as the Office software, MSN Messenger application, and subsequent versions of Windows. Had Microsoft overlooked the value of follow-on products and made the decision to develop the first Windows product based solely on its stand-alone profits, the company might not have invested so heavily or placed so much faith in the development of what would be one of the richest real option platforms in history. Note that Microsoft succeeded with another platform investment in the Xbox video game system, sold at a considerable loss (as described in Figure 6), expecting to more than recover through the sale of games and hardware that are linked exclusively to the system (Ivan, 2007).
Information and Organizational Learning

There are scenarios in which the information revealed or the organizational learning following a decision can be used to maximize the potential of platform investments. Sometimes the uncertainty surrounding the future success of a new product release can be managed in such a way to make even the biggest “Hail Mary” of offerings worth taking on. A film studio can phase its investments in advertising, distribution, and development of sequels for a movie by staging its expenditures; pending the reaction of critics and other viewers at early screenings. While new releases tend to have a low probability of success, successful releases generate vast payoffs not just from screening revenues, but DVDs, merchandise, and sequels. Therefore, being able to accelerate or decelerate spending after the results of early screenings are known is like letting the studio participate in a real options lottery, and keeping the cost of their participation at a minimum. Figure 7 provides a hypothetical investment decision roadmap for film studios.
The investments made by a pharmaceutical company can be thought of in terms of embedded real options as well. Like the film studio, it can similarly stage spending— in this case the research and development of drugs, pending clinical trials and regulatory approval. Furthermore, it attaches value to learning, often firm-specific and proprietary, that occurs even when a given product fails to make it to market. The research and development for one project may generate knowledge and capabilities useful for others that would not have necessarily been enjoyed had the initial project not been carried out. Breakthroughs and patents spawned from a losing effort can be applied to support related projects, and mistakes made in previous trials avoided.

**Project Abandonment as an Option**

The right to abandon or scale back an investment is also worth something to the firm, and this is what typifies the put variety of real options. Take a minerals company that gets hit with an extended period of declining prices for its output. Rather than continue to operate at a loss, it can sell its property and equipment, and use the proceeds to invest elsewhere. There are many
analogous situations in which shutting down a business that runs into hard times is a much better alternative to “letting it ride.”

Each of the investment scenarios above can be characterized by a common element. Being able to make or unwind an investment at a future date, pending the realization of some event, is a valuable right. When relating the realization to the price path of the underlying security for a call or put option, the analogy between the real investment and the financial derivative, for the purpose of real option pricing, is complete. To the extent one can model the variables that determine the value of the investment—be it the price path of oil, the popularity of a movie, or the success of follow-on projects—the Black-Scholes or Binomial Lattice framework can be applied. The remainder of this section demonstrates how the tradeoffs between the benefits and costs of investment flexibility and information are quantified in each of these frameworks.

**Using the Black-Scholes Model to Value Real Assets**

Recall the example of the oil company that possesses reserves, and thinks about market price volatility in timing extraction. This situation naturally extends itself to Black-Scholes valuation. The inputs originally taken from the opportunity to invest in a financial security can be drawn from the characteristics of the real investment opportunity as illustrated in Figure 8.

**Figure 8: Real Option Valuation Drivers Using Black-Scholes**

<table>
<thead>
<tr>
<th>Black-Scholes Input</th>
<th>Stock Option</th>
<th>Real Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>Stock price</td>
<td>Value of reserves today, given quantity and price of oil, and variable cost structure</td>
</tr>
<tr>
<td>X</td>
<td>Exercise Price</td>
<td>Fixed cost of extraction, production (no variable costs courted here)</td>
</tr>
<tr>
<td>r</td>
<td>Risk-free rate</td>
<td>Risk-free rate</td>
</tr>
<tr>
<td>T</td>
<td>Contract horizon</td>
<td>Investment option horizon, given maintenance costs, depletion, competitive forces</td>
</tr>
<tr>
<td>σ</td>
<td>Return volatility</td>
<td>Return volatility resulting from future price uncertainty, reserve uncertainty, etc.</td>
</tr>
</tbody>
</table>
The option values (calls and puts) are added to the static NPV in determining the total worth of the investment.

**Exercise 5:**

a) Consider an oil company with 400,000 barrels of known reserves. They can extract and produce the reserves at a cost of $25 per barrel over the next five years. Oil is priced in the market today at $100 a barrel, and the contribution margin averages 45%. The risk-free rate is 5%, and volatility of oil prices; believed to be the only source of uncertainty influencing the value of the reserves; is estimated at 25% per year. What is the value of producing the oil today?

b) Use the Black-Scholes framework to estimate the value of the reserves.

c) How much is the ability to delay production worth to the company?

**Solution:**

a) $8,000,000

b) $18,401,139

c) $10,401,139

**Exercise 6:**

a) Consider a company that is bidding on a mine. For various reasons, they cannot delay or ramp up production given the evolution of commodity prices in the market, but they can abandon the investment in three years by selling off the property and equipment for $5,000,000. It estimates the value of the mine today to be $10,000,000. The risk-free rate is 5%, and the volatility of commodities prices; believed to be the only source of uncertainty influencing the value of the reserves; is estimated at 20% per year. Use the Black-Scholes framework to estimate the maximum acceptable bid for the mine.
b) How much is the ability to abandon worth to the company?

Solution:

a) $10,405,352

b) $405,352

Using Binomial Lattices to Value Real Assets

The Binomial Lattice approach can be applied to price the option to expand, delay, and abandon using the very same inputs required for the Black-Scholes model, as discussed in Section I. An interesting application of the Binomial Lattice valuation has the lattice pricing the securities in the firm’s capital structure using the principle of limited liability. Shareholders cannot lose anything more than their initial investment, but must pay off outstanding debt before realizing the value of their equity. In this sense, their stock is like a call option on the firm’s assets—by paying off the debt outstanding, they earn the right to receive the cash flows from the underlying assets. They will only “exercise” when the value of the assets exceeds the debt outstanding. The position of lenders can be looked at through an options lens as well. They essentially hold risk-free debt, and have sold shareholders the option to default on the loan; a put option.

Exercise 7:

a) A firm’s assets, currently valued at $500, vary with an annual volatility of 30%. If the risk-free rate is 5% and the firm holds $250 of debt, is bankruptcy likely? Use a one-year horizon with three-month steps. Also, price the stock as a call option on the firm’s assets using the Binomial Lattice.
Solution:

a)

The firm’s equity is worth $262.19. Even in the worst projected outcome, the firm is solvent and the stock is worth $24.41.

III. ADVANCED OPTION PRICING APPLICATIONS FOR REAL ASSETS

Academic research continues to advance the quantitative techniques for pricing contingent claims on financial and real assets alike, but the use of such techniques remains found within a very small, sophisticated audience. A 2003 CFO Magazine survey found that just 9–11% of senior executives are using the valuation technique; and a large proportion of early adopters have abandoned it. The well-publicized demise of Enron, once considered to be championing the integration of real option decision-making into the 21st century, has only served to build an aura of distrust around the field.

While such preconceptions will wane over time, there are bona fide objections to refocusing the strategic investment process that will need to be overcome. The most significant of which is the
divide between what many perceive to be the worth of their investment opportunities, and what
the basic application of pricing models tells them. This often results when the assumptions
underlying the financial pricing techniques do not hold in a corporate investment setting. The
intent is not to critique the extension of pricing financial derivatives to real options; but to
understand where the underlying conditions differ, and to adjust pricing models as best as
possible. This section provides commentary on the critical strategic and economic considerations
surrounding investment opportunities; areas where much of the existing literature has been
agnostic. The aim is to equip students and practitioners with a set of approaches that address the
discrepancies between financial market and corporate investment settings. The most relevant
discrepancies include:

1) The manner in which volatility is estimated
2) The act of option exercise
3) The legal right to option payoffs.

Volatility
According to the Black-Scholes model, the value of a financial option is influenced by the
uncertainty of returns on the underlying stock. Even though many systematic and nonsystematic
factors influence returns, a reasonable estimate of volatility for the purposes of computing the
option value can be calculated by simply measuring the variation in historical returns on the
traded stock. Techniques for doing this may vary, but the volatility implied by call or put prices
set in a competitive financial market are most likely going to reflect some reasonable estimate of
future return volatility. On the other hand, there is no market exchange for real option
opportunities that can be referenced to produce an implied volatility. Though some models of
estimating volatility using certainty equivalents have been proposed (Copeland 2005), there is no widely accepted technique that captures the systematic and nonsystematic risks affecting the cash flows of real investments. The lack of a simple, practical method for getting the volatility input makes price estimates more likely to drift from their economic value to the firm, and therefore be less credible.

Exercise

Financial investors can exercise options almost instantaneously by calling their broker or using an online trading account. Real investment opportunities can be much more complex and time-consuming to act on. The investment decisions we identified earlier—extracting a commodity from a mine; ramping investment in advertising and research; and releasing follow-on products—all involve a certain level of time and resources to follow through. Companies, for many reasons, maintain varying degrees of agility or control; and this will affect their ability to exercise and capture the option payoffs from their project.

Capturing Option Payoffs

A related and equally important distinction between financial and real investment settings rests in the ownership of option payoffs. While financial investors hold legal rights to the profits on their option, the same protection does not exist for firms in competitive corporate markets. Firms can converge on one another’s markets with little or no recourse for the loser. This can be prevented in markets where physical property rights (the commodities seller owns its mine) or intellectual property rights (the pharmaceutical company files patents) exist and are enforced but, in many cases, the information and competitive setting is available for all rivals to capitalize on.
While employees are being mobilized, marketing programs launched, and distribution channels filled during the exercise process; rivals are reacting and new information arriving in ways that make the original option value estimate meaningless.

Intel’s initial processor releases surely generated demand for faster subsequent releases, but the company ended up sharing much of the profits from these future releases with Advanced Micro Devices and others. The risk of such convergence happening is especially high when there are weak isolating mechanisms, and competitors have varying degrees of agility to act on their investment opportunities. Another competitive factor that heavily influences option payoffs is the extent to which a firm can cope with the very uncertainty that drives (at least mathematically) option values (Williams 2006). Those such as Wal-Mart and McDonald’s that compete on the basis of scale are more likely to see risk as disruptive rather than a source of value creation within their process-driven organizations. In light of these more prominent discrepancies, we have attempted to modify and expand upon traditional models to better capture the strategic realities of the firm.

**Black-Scholes Model: Dividend Yield Adjustment**

In many settings, possessing a first-mover or organizational learning advantage may enhance the value of future projects. Then, as barriers to entry decline and competing firms enter the market, the profitability of these projects becomes less secure and more uncertain. Extreme forms of convergence to commodity status of products and services occur in so called fast-cycle markets, where cash-flow half-life is on the order of one year or less (Williams 1999). Examples include microchips (Intel), hard drives for computers (Seagate), cell phones (Nokia), the fashion industry
(Benetton), and in innovation-driven sports markets such as golf (Callaway). These markets experience high marginal utility of early adoption, but simultaneously are characterized by weak isolating mechanisms, with the result that considerable profit can be made for companies that move quickly. At the same time, delays in production or distribution are typically very costly, as competitors quickly enter first-mover markets with look-alike products at a fraction of the first-mover’s price.

One way to model the trajectories of such fast-cycle option payoffs is to assess a dividend yield that erodes the value of the expected underlying cash flows (Damodaran). In a similar sense that dividend payments represent foregone income for the financial call option holder, the assessed dividend yield captures the economic cost of having to share option payoffs with fast-following competitors.

**Exercise 8:**

a) Future releases for a microchip manufacturer are expected to be worth $25 million; with development costs of $25 million, a two-year investment horizon, and an annual volatility of returns on the investment of 25%. Competitors are likely to follow the releases with lower quality, knock-off products in an attempt to capture market share. What do various dividend yields imply about option values?
Solution:

a)

While assessing a dividend yield on a real asset may seem opaque; to the extent investment value erosion can be estimated, the approach can be informative. Estimates may be drawn from projections of excess returns or monopoly profits available during the competitive advantage period, consumer demand and price forecasts, and past investment experience.

Stochastic Variables

For many financial derivatives, market conditions evolve by a sufficient margin to justify modeling the inputs to the pricing model dynamically. Volatility can increase over time, or vary with the price of the underlying security. The underlying may tend to exhibit mean reversion—common in interest rate derivatives—or experience random shocks from time to time, as in energy markets. Traders have reverse-engineered the Black-Scholes model; stripping out the assumed static volatility and Geometric Brownian Motion assumptions; and recalibrating their models for assumptions more relevant to the given security.
The variables that drive real option value may change over time as well. An excellent example of this in the context of real options is the tipping of the high-definition video storage market towards the Blu-ray Disc platform, where the HD DVD Optical Disc platform ultimately lost out. Preceding the release of the machines, both Sony and Toshiba invested heavily in research and development, and spent significant time courting movie studios and distributors for exclusivity deals. The money spent on these endeavors was expected to be more than recovered on the discs that would be sold in the future and used on the platform. Given the high stakes and uncertainty for both Sony and Toshiba at this stage, their development options were both likely “in the money.” However, the subsequent events—the rate of consumer adoption, and the level of success in landing exclusivity deals—would determine the ultimate payoffs. The degree of uncertainty would decrease as these events were to unfold, and one competitor would surpass the other. However, a manager at Sony or Toshiba doesn’t have a crystal ball, and must ex ante do his/her best to capture the possibilities and state-dependent outcomes in their decision tools. To the extent they are equipped with quantitative approaches rooted in dynamic rather than static settings, they will be making more informed decisions.

Quantitative Approaches

Monte Carlo simulations have proven to be a popular and useful tool for conducting scenario analysis for investment decisions. The tool allows for a wide variety of variables to be modeled with a distribution; and the range of option payoffs are produced over hundreds or thousands of trials. Managers may be adept at forecasting the cash flow drivers of their businesses, but there are further adjustments to the underlying price process and volatility assumptions worth incorporating into the simulations.
Recall that the Black-Scholes model assumes stock prices are governed by Geometric Brownian Motion, a form of continuous time stochastic process. In order to look at the option in a Binomial Lattice; or assign a different stochastic process to the underlying; the continuous price path must be converted into a discrete step-by-step model. The Euler discretization allows for the price changes to be simulated in small-time increments as follows:

\[ dS_t = \mu S_t dt + \sigma S_t dw_t, \]

Where

- \( dS_t \) = the change in the value of the underlying from one period to the next
- \( \mu \) = the annualized mean return
- \( dt \) = the time increment
- \( \sigma \) = the annualized standard deviation of returns
- \( S_t \) = the present value of the underlying at time \( t \), and
- \( dw_t \) = the instantaneous increment to a Wiener process, which captures the random arrival of information

**Exercise 9:**

a) Model the price path over 12 months of a stock with \( \mu \) of 8%, \( dt \) of one month, initial price of $24, and annual \( \sigma \) of 20% using the following random numbers generated from a normal distribution:

<table>
<thead>
<tr>
<th>Period</th>
<th>Random Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2369</td>
</tr>
<tr>
<td>2</td>
<td>2.3593</td>
</tr>
<tr>
<td>3</td>
<td>1.1456</td>
</tr>
<tr>
<td>4</td>
<td>-1.0193</td>
</tr>
<tr>
<td>5</td>
<td>0.1606</td>
</tr>
<tr>
<td>6</td>
<td>1.8706</td>
</tr>
<tr>
<td>7</td>
<td>-0.8572</td>
</tr>
<tr>
<td>8</td>
<td>0.3757</td>
</tr>
<tr>
<td>9</td>
<td>-1.9000</td>
</tr>
<tr>
<td>10</td>
<td>0.8561</td>
</tr>
<tr>
<td>11</td>
<td>0.9516</td>
</tr>
<tr>
<td>12</td>
<td>0.9795</td>
</tr>
</tbody>
</table>

b) Assume the strike price on a 12-month call option is $23.50. If the underlying stock follows the path from a), what is the value of the option at expiration?
Solution:

a)

\[
\begin{array}{cccccccccccc}
\text{Period} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
\text{Price} & $24.00 & $24.45 & $26.24 & $26.85 & $29.06 & $32.54 & $31.97 & $31.79 & $30.40 & $32.28 & $34.33 \\
\end{array}
\]

b) \(\text{Max}(0, S - X) = 10.83\)

There are alternative processes that can be modeled for situations where a normally distributed return process or constant volatility does not apply.

A Markov Ito price process allows for changing annual returns and heteroscedasticity by

\[
dS_i = \mu(S_i,t)S_i dt + \nu(S_i,t)S_i dw_i
\]

Where

\[
\nu = \text{the annual variance of returns}
\]

An Ornstein-Uhlenbeck process allows for mean reversion by

\[
dS_i = \alpha(\theta - S_i) dt + \sigma dw_i
\]

Where

\[
\alpha = \text{the parameter denoting the speed of mean reversion, and}
\]

\[
\theta = \text{the mean}
\]

Exercise 10:

a) A mining company has 10,000 tons of XYZ deposits that it can bring to market at any point. The current market price per ton is $650, and prices tend to mean revert with properties \(\alpha = .005\) and \(\theta = $580\). If the annual price volatility is 20%, model the price path using one-month steps with the following randomly generated numbers:

\[
\begin{array}{cccccccccccc}
\text{Period} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
\text{Random Normal} & 0.4990 & 0.5113 & -1.5999 & 1.1199 & -0.7580 & 1.0363 & -1.0604 & -0.0668 & 1.9674 & 0.5088 & -0.5628 & -1.1259 \\
\end{array}
\]
b) Assuming that the contribution margin is 60% and the fixed cost of extraction per ton is $300, value the opportunity to bring the metals to market at the end of 12 months using the price path in a).

**Solution:**

a)

<table>
<thead>
<tr>
<th>Period</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$850.00</td>
</tr>
<tr>
<td>1</td>
<td>$697.58</td>
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<tr>
<td>2</td>
<td>$735.78</td>
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<td>4</td>
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<tr>
<td>5</td>
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<tr>
<td>6</td>
<td>$876.46</td>
</tr>
<tr>
<td>7</td>
<td>$525.63</td>
</tr>
<tr>
<td>8</td>
<td>$536.94</td>
</tr>
<tr>
<td>9</td>
<td>$810.72</td>
</tr>
<tr>
<td>10</td>
<td>$814.97</td>
</tr>
<tr>
<td>11</td>
<td>$6650.28</td>
</tr>
<tr>
<td>12</td>
<td>$509.81</td>
</tr>
</tbody>
</table>

\( b) \quad Max(0, S - X) = Max(0, 10,000 - ((509.81 * 0.60) - 300) = 58,844 \)

A Stochastic Variance Model allows for variance to evolve according to some specified relationship

\[ dS_t = \mu(S_t, t)S_t dt + \nu(S_t, h_t, t)S_t dw_t \]

Where

\[ h_t = \text{secondary volatility factor where heteroscedasticity is not driven by } t \text{ or } S_t \]

A Poisson Jump Diffusion process allows for spikes and regressions in price path

\[ dS_t = \mu(S_t, t)S_t dt + \nu(S_t, t)S_t dw_t + A_t S_t dq_t \]

Where

\[ A_t = \text{the variable accounting for random jumps, and} \]

\[ dq_t = \text{the change in the level of a Poisson process, which is either 1 or 0} \]

Managers can configure their business case simulation with the appropriate statistical underpinnings and evolving decision rules or payoff functions that reflect competitive dynamics. For example, the achievement of a certain level of customer adoption can serve as the market tipping point, where volatility and the risk of competitive convergence severely dissipate. Such a
payoff rule resembles that of a barrier option in financial markets, where payoffs only kick in when a predetermined threshold has been reached.

Introducing even the most basic forms of scenario analysis requires training and planning process redesign. Yet despite the negative sentiment surrounding real options expressed in senior executive surveys, some promising trends have emerged. The use of sensitivity analysis and scenario analysis is widespread, appearing at 85% and 67% of companies in the aforementioned CFO Magazine survey. While managers may perceive real option pricing exercises to be occurring in black boxes, they may soon realize that, by incorporating sensitivity analysis and scenario analysis into the decision process, they are indirectly assigning a value to the firm’s real options. We hope the approaches outlined in this section bring them closer to embracing more explicit assessments of real options.

Advanced Binomial Lattice Approach – Cost of Delay

For some firms, it is particularly important to be able to maintain flexibility to ramp and shutter operations, because the goal is to match production capacity of the firm to rapidly shifting changes in supply and demand. The case of Pulse Engineering is illustrative here. In 1982, Pulse became a component supplier to IBM in the rapidly growing personal computer industry, and could build delay lines at a rate of 15,000 units a week. But demand quickly jumped to 35,000 units per week, leaving Pulse unable to meet demand. Management responded with increased investment, which increased available capacity by late 1984 when demand shrunk to 6,000 units per week, requiring Pulse to lay off much of its production force and take large inventory write-offs. Then, in 1986, demand for a second-generation PC component rose from 20,000 units per
week to 120,000 units per week over a nine-month period; only to fall off in demand by 1988 in a pattern similar to 1984. Yet during this period spanning rapid growth and volume decline, Pulse Engineering enjoyed several years of profits, due in no small part to its ability to rapidly adjust its capacity to changing market demand.

In situations like these, rather than using Monte Carlo simulations to simulate the underlying price path and payoffs, managers can explicitly model decision-triggering events in a Binomial Lattice. This technique is particularly useful for estimating the cost of delayed exercise, and provides a useful visual roadmap for tracking the life of the investment. The critical step of this approach is to distinguish between the exercise decision point and when the exercise actually happens. The decision point occurs when the organization commits to following through with investment. “Exercise” can be defined as the point at which the firm has made the necessary resource allocations to make the investment possible, as discussed earlier in this section. The wider the gap between decision and exercise, the greater the expected option value lost because of the firm’s inability to react quickly and capture payoffs before competitors.

**Exercise 11:**

a) A mobile phone maker has the option of releasing an extension of an existing model with new Web browsing and music applications. Fixed development costs are $1 billion; and the present value of expected cash flows from the extension are $850 million, with annual volatility of 35%. The applications and features built into the phone are anticipated to be popular for 15 months before future releases make the model obsolete. Price the option using a Binomial Lattice; assuming the risk-free rate is 5%.
b) Now assume that the company plans on making its exercise decision three months from now, but only has the wherewithal to mobilize on the decision in nine months. It expects to have foregone income of $300 million at the 6-month point, and an additional $115 million at the 9-month point due to missing out on sales to early adopters. Retrace the project value through the Binomial Lattice.

c) What is the cost of delay to the firm?

Solution:

a) 

b) The foregone income can be subtracted from the project value at the 6-month and 9-month nodes; similar to the dividend yield treatment in Exercise 4.
Advanced Binomial Lattice Approach – Cost of Commitment

Firms in many industries face competitive situations where it is necessary to commit to a course of action over extended periods. In his pioneering work on commitment, Ghemawat shows how the strategies of firms have a tendency to persist over time (Ghemawat). In the case of Boeing, for example, the company was fully committed to the development of the 747 because of the high levels of investment that precluded the company from developing in other aircraft over the period. In the case of Reynolds Aluminum, a decision to shut down a facility precluded the company from ever starting up that facility again because the high costs of restarting exceeded that of a newer facility. In the case of Coors, the company’s decision to move from regional to national distribution would take a full decade to be fully realized due to lags in marketing, market penetration, and large-scale facilities start-up.

During these periods of inertia and irreversibility, decision-making is complicated by changing levels of uncertainty, the arrival of new information, competitive behavior, and compounding...
exit barriers associated with decisions over time. As Ghemawat makes clear, an important factor over periods of commitment is the degree to which investment decisions can be reversed. If, during the gap between decision and exercise, information arrives that makes the investment no longer worthwhile (or worthwhile escalating), option value can be recovered only when the firm can act on the new information. The ex ante difference between the traditional option price and the price accounting for exercise and decision reversal constraints can be interpreted as the “cost of commitment.”

**Exercise 12:**

a) A pharmaceutical company has the right to develop a drug at any point over the next two and a half years. The present value of expected cash flows and exercise price related to development are both $200 million; the annual volatility of expected cash flows is 20%; and the risk-free rate is 5%. Price the option.

b) Now assume the company plans to make a decision on option exercise, and stick with it six months from now. In what scenarios will the firm wish it had made a different decision, and how (qualitatively) will this affect the real option value?

c) What is the cost of commitment to the firm?

**Solution:**

a)
b) When decisions can’t be reversed, the firm will regret exercising when subsequent information is unfavorable, and not exercising when subsequent information is favorable. The inability to act on new information makes the possibility of development less valuable to the firm.
The cost of commitment is $14.61 million.

The scale orchestration constraints common in oligopolistic industries serve to decrease much of the appeal of real option investments. Such a control-oriented focus may lead firms to be unable to quickly react to favorable developments; or abandon investments that subsequently turn for the worse. Furthermore, to the extent the organization’s culture discourages risk-taking and innovation, real option thinking may be discouraged.

**Volatility Estimation Approaches**

Both external and internal sources can be tapped for the volatility input required for Black-Scholes and Binomial Lattice valuation. If a given investment or strategic decision is typical for the firm; such as the development opportunities of a pharmaceutical company; the standard deviation of the enterprise value can be used. When the company is privately held, a peer set of firms with comparable assets and activities can be constructed; and volatility can be similarly estimated. The variation in cash flows of past projects may also be a good indicator of the risk of future projects, as would the implied variation from Monte Carlo simulations. When there are
multiple sources of project risk that can be readily identified and estimated, a portfolio variance model can be introduced to estimate volatility:

\[ \sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij} \]

Where

- \( \sigma_p \) = project volatility
- \( \sigma_i \) = volatility of factor \( i \)
- \( w_i \) = weighting of volatility factor \( i \)
- \( \rho_{ij} \) = correlation between factors \( i \) and \( j \)

**Do All Investments Have Embedded Options?**

The bulk of this section highlights investment opportunities where real option pricing tools focus and empower valuation exercises. When can these same tools be misleading or inappropriate to apply? One of the biggest credibility barriers preventing real options valuation from being adopted on a widespread basis is the notion that any project can be justified by supplementing the stand-alone NPV with the option value for an unlimited number of projects. While part of embracing real option thinking is looking for sources of project value outside of the expected course of action, some warnings are in order:

- Favorable market settings alone do not necessarily mean a given investment has option value. If a company is unwilling to pursue a new strategy regardless of the potential payoffs, that strategy has no value to them. Similarly, if a firm lacks the agility or competence to capture potential option payoffs available, no option value exists. Retail chains with substantial real estate investments such as McDonald’s should not be
including the option value of selling underlying property if prices rise when they have no intention or capability of relocating or leasing.

- Premeditated strategic moves do not count as options. When a firm has dedicated itself to escalating or unwinding an investment, the very “right” to react to new information that characterizes real options does not exist. This situation resembles the commitment setting discussed earlier, with the important difference that with commitment it isn’t option exercise that is certain; but rather the follow-through if exercise occurs. Premeditated moves should be evaluated with NPV analysis of most likely outcomes and Monte Carlo simulations, but not options analysis.

Being skilled at identifying when not to apply real options valuation tools will help prevent managers from attempting to justify unprofitable projects, and investors from overvaluing shares.

CONCLUSION AND DISCUSSION OF FUTURE RESEARCH

Advances in pricing methods for financial securities have served to benefit managers in the valuation of the strategic options of their firms. Research in asset pricing is rapidly evolving; and the development of valuation models driven by quantitative software platforms is transforming the capital markets. There is great potential for leveraging asset pricing technology to the evaluation of real investments, and helping managers make capital allocation decisions, value their businesses, and assess performance. Pricing real options arguably involves as much art as science, and the application of traditional models can produce misleading output. This chapter has outlined a number of valuation approaches designed to bring about heightened understanding
of strategic capabilities and limitations of the firm in relation to its real option opportunities. By incorporating insights from competitive strategy into the valuation exercise; and using a variety of approaches to triangulate on investment values; the real options management process will be better informed.
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